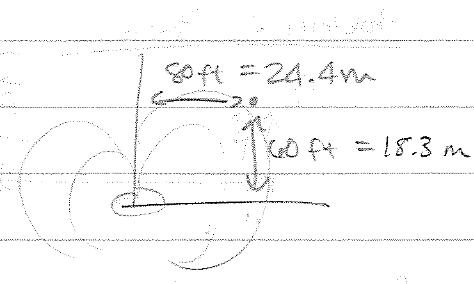
HOMEWORK #12

Problem 1 (Purcell 11.5)

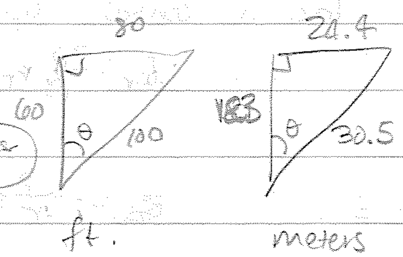
simple magnetic dipole



$$B_x = \frac{3m\sin\theta\cos\theta}{r^3}$$

$$B_x = \frac{3m\left(\frac{8}{10}\right)\left(\frac{6}{10}\right)}{(3050)^3} = .0061 \text{ gauss}$$

↖ in cm



$$B_z = \frac{m(3\cos^2\theta - 1)}{r^3} = \frac{m\left(3\left(\frac{6^2}{10^2}\right) - 1\right)}{(3050)^3} = .00034 \text{ gauss}$$

↖ in cm

to use for $\vec{m}$ (solenoid from section 11.1)

$$B = 30{,}000 \text{ gauss} = \frac{2\pi I}{cb}$$

$$\text{so } I = \frac{Bbc}{2\pi}$$

$$\vec{m} = \frac{Ia}{c} = \frac{Bbc}{2\pi}\frac{a}{c} = \frac{B\pi b^3}{2\pi} = \frac{(30{,}000)(20)^3}{2} = 1.2\times10^8$$

→ These values are much smaller than the Earth's field, so the physicist should not complain!

11.9 Since it is a paramagnetic sample,

$$F \propto B_z \frac{\partial B}{\partial z}. \qquad \left(F = m \nabla B = m\frac{\partial B}{\partial z} \text{ for a sample with a constant magn. moment} \right)$$

For a semi-infinite solenoid

$$B_z = \frac{2\pi I n}{c}(\cos\theta_1 - \cos\theta_2) = \frac{2\pi I n}{c}\left(\frac{z}{\sqrt{r_0^2+z^2}} + 1\right)$$

$$\frac{\partial B}{\partial z} = \frac{2\pi I n}{c}\left(\frac{-z^2}{(r_0^2+z^2)^{\frac{3}{2}}} + \frac{1}{\sqrt{r_0^2+z^2}}\right)$$

$$= \frac{2\pi I n}{c}\left(\frac{r_0^2}{(r_0^2+z^2)^{\frac{3}{2}}}\right)$$

$$\frac{\partial^2 B}{\partial z} = \frac{2\pi I n}{c}\left(\frac{-3 z r_0^2}{(r_0^2+z^2)^{\frac{5}{2}}}\right)$$

$$\frac{\partial F}{\partial z} = 0 \Rightarrow \frac{\partial}{\partial z}\left(B_z \frac{\partial B_z}{\partial z}\right) = 0 \Rightarrow \left(\frac{\partial B}{\partial z}\right)^2 + B_z \frac{\partial^2 B}{\partial z} = 0$$

$$\Rightarrow \cancel{\left(\frac{2\pi I n}{c}\right)^2}\left(\frac{r_0^2}{(r_0^2+z^2)^{\frac{3}{2}}}\right)^2 + \cancel{\left(\frac{2\pi I n}{c}\right)}\left(\frac{z}{\sqrt{r_0^2+z^2}} + 1\right)\left(\frac{-3 z r_0^2}{(r_0^2+z^2)^{\frac{5}{2}}}\right) = 0$$

$$\Rightarrow \frac{r_0^4}{(r_0^2+z^2)^3} + \frac{3z^2 r_0^2}{(r_0^2+z^2)^3} - \frac{3z r_0^2}{(r_0^2+z^2)^{\frac{5}{2}}} = 0$$

$$\Rightarrow \frac{r_0^4 - 3z^2 r_0^2 - 3z r_0^2\sqrt{r_0^2+z^2}}{(r_0^2+z^2)^3} = 0$$

$$\Rightarrow r_0^4 - 3z^2 r_0^2 - 3z r_0^2 \sqrt{r_0^2+z^2} = 0$$

$$\Rightarrow r_0^2 - 3z^2 = 3z\sqrt{r_0^2+z^2}$$

$$\Rightarrow r_0^4 - 6z^2 r_0^2 + \cancel{9z^4} = 9z^2(r_0^2 + \cancel{z^2})$$

$$\Rightarrow r_0^4 = 15 z^2 r_0^2$$

$$\Rightarrow r_0^2 = 15 z^2$$

$$\Rightarrow z = \frac{r_0}{\sqrt{15}} \qquad \square$$

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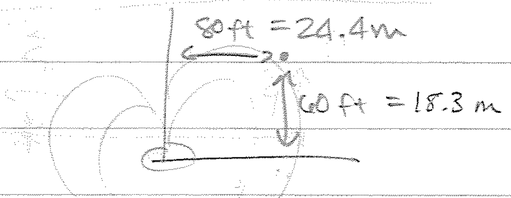

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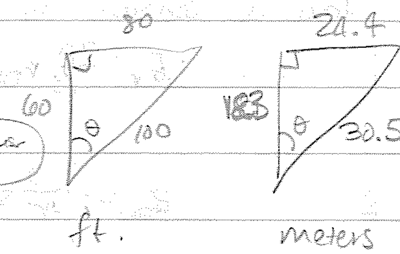

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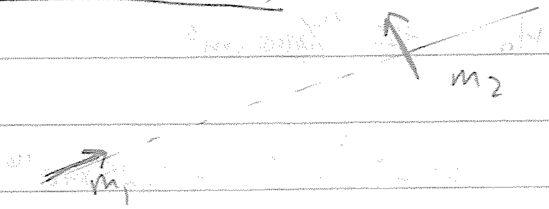
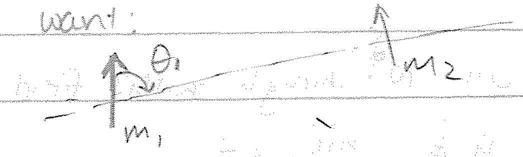
Problem 3 (Purcell 11.12)



a) work to rotate m_1 while holding m_2 fixed

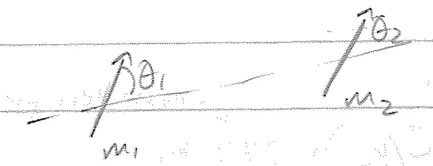
want:



$$W_1 = \Delta U = \int -\vec{m}\cdot\vec{B} = m_1\int B\cos\theta d\theta = m_1 \frac{m_2}{r^3}\int_0^{\theta_1}\cos\theta\, d\theta = \frac{m_1 m_2}{r^3}\sin\theta_1$$

work to rotate m_2 while holding m_1 fixed

$$W = \Delta U = -\int \vec{m}\cdot\vec{B} = \ldots$$



$$\vec{B} = \hat{r}\left(\frac{2m_1}{r^3}\cos\theta_1\right) + \hat{\theta}\left(\frac{m_1}{r^3}\sin\theta_1\right)$$

$$\ldots = \frac{m_1 m_2}{r^3}\left[\int_{\pi/2}^{\theta_2} 2\cos\theta_1\cos\theta d\theta + \int_{\pi/2}^{\theta_2}\sin\theta_1\cos\theta d\theta\right]$$

$$= \frac{m_1 m_2}{r^3}\left[2\cos\theta_1(\sin\theta_2 - \sin\tfrac{\pi}{2}) + \sin\theta_1(\sin\theta_2 - \sin\tfrac{\pi}{2})\right]$$

$$= \frac{m_1 m_2}{r^3}\left[2\cos\theta_1\sin\theta_2 - \sin\theta_1\sin\theta_2 - \sin\theta_1\right]$$

total work: $\frac{m_1 m_2}{r^3}\left[2\cos\theta_1\sin\theta_2 - \sin\theta_1\sin\theta_2\right]$

11.14

$(11-39)$: $\vec{M} = \chi_m' \vec{B}$

$(11-56)$: $\vec{M} = \chi_m \vec{H}$

$(11-57)$: $\vec{B} = (1 + 4\pi\chi_m)\vec{H}$

$$\Rightarrow \chi_m \vec{H} = \chi_m'(1 + 4\pi\chi_m)\vec{H}$$

$$\Rightarrow \chi_m - 4\pi\chi_m'\chi_m = \chi_m'$$

$$\Rightarrow \chi_m = \frac{\chi_m'}{(1 - 4\pi\chi_m'} \quad \square$$

11.16

$M = 750$ erg/gauss$\cdot$cm^3

a) At a point above a pole just outside the sphere, only B_r of (11-15) applies:

$$B_r = \frac{2m}{r^3} = \frac{2}{r_o^3}\cdot\left(\frac{4\pi}{3} r_o^3 M\right) = \frac{8\pi}{3} \overset{750\,G/cm^3}{M} = \boxed{\begin{array}{c}2000\pi \text{ Gauss} \\ \shortparallel \\ 6283 \text{ Gauss}\end{array}}$$

b) On the equator, only B_θ applies

$$B_\theta = \frac{m}{r^3} = \frac{1}{2}(2000\pi\ G) = \boxed{1000\pi \text{ Gauss} = 3142 \text{ Gauss}}$$

c) $$F = m\frac{\partial B}{\partial r} = m\cdot\underbrace{\frac{\partial}{\partial r}\left(\frac{2m}{r^3}\right)}_{\text{Only } B_r \text{ applies}} = -\frac{6m^2}{r^4} = -\frac{6}{(2r_o)^4}\left(\frac{4\pi}{3} r_o^3 M\right)^2$$

$$\Rightarrow F = -\frac{\cancel{6}^2}{\cancel{16} r_o^{\cancel{4}}}\cdot\frac{\cancel{16}\pi^2}{\cancel{9}_3} r_o^{\cancel{6}^2} M^2 = -\frac{2}{3}\pi^2 r_o^2 M^2$$

$$= -\frac{2}{3}\pi^2(750)^2 \text{ Dynes}$$

$$= -375000\,\pi^2 \text{ Dynes}$$

$$= \boxed{-3.7\times10^6 \text{ Dynes}}$$

Problem 5 (Purcell 11.15)

magnetization M_0 - 480 ergs/gauss cm^3

crystal (cube) $(5\times10^{-6})^3$ cm^3 $= 1.25\times10^{-16}$ cm^3

bacterium contains ~15 × 1.25×10^{-16} cm^3 such crystals

rotate this cell 90° through Earth's field ~.5 gauss

energy $U = \vec{m}\cdot\vec{B} = |\vec{m}\vec{B}|\cos\theta$

$$\Delta U = \int_0^{\pi/2} mB\cos\theta\, d\theta = mB[\sin\theta]_0^{\pi/2} = mB$$

$$= 15\times 1.25\times10^{-16}\,\text{cm}^3 \times 480\,\frac{\text{ergs}}{\text{g}\cdot\text{cm}^3} \times .5\,\text{g}$$

$$\cong 4.5\times10^{-13}\ \text{ergs}$$

thermal agitation kT ← room temperature

$$= (1.38\times10^{-23}\,\text{J/K})\times 273\,\text{K}$$

$$= 3.77\times10^{-21}\,\text{J} = 3.76\times10^{-14}\ \text{ergs}$$