Problem 7 (Purcell 9.13)

parallel:

$$E'^2 - B'^2 = \vec{E}_{\parallel}' \cdot \vec{E}_{\parallel}' - \vec{B}_{\parallel}' \cdot \vec{B}_{\parallel}' = \vec{E}_{\parallel} \cdot \vec{E}_{\parallel} - \vec{B}_{\parallel} \cdot \vec{B}_{\parallel}$$

perpendicular:

$$E_{\perp}' \cdot E_{\perp}' - \vec{B}_{\perp}' \cdot \vec{B}_{\perp}' = \gamma^2 \cdot (\vec{E}_{\perp} + \vec{\beta} \times \vec{B}_{\perp}) \cdot (\vec{E}_{\perp} + \vec{\beta} \times \vec{B}_{\perp})$$

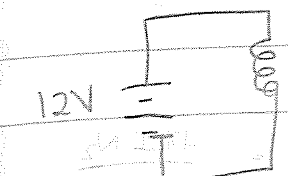
$$- \gamma^2 (B_{\perp}^2 - \vec{\beta} \times \vec{E}_{\perp}) \cdot (\vec{B}_{\perp} - \vec{\beta} \times \vec{E}_{\perp})$$

$$= \gamma^2 (E_{\perp}^2 (1-\beta) - B_{\perp}^2 (1-\beta^2)) = E_{\perp}^2 - B_{\perp}^2$$

$$E'^2 - B'^2 = E_{\parallel}^2 - B_{\parallel}^2 + E_{\perp}^2 - B_{\perp}^2 = E^2 - B^2$$

HOMEWORK #9

Problem 1 (Purcell 7.13)



current: $I = I_0 \left(1 - \exp\left(-\frac{Rt}{L}\right)\right)$

$$.9 = 1 - e^{-Rt/L}$$

$$t_0 = .115 s$$

energy stored in magnetic field:

$$U = \tfrac{1}{2} L I^2 = \tfrac{1}{2} (.5 \times 10^{-3}) \left(.9 \times \frac{12}{.01}\right)^2 = \tfrac{1}{2} (.5 \times 10^{-3})(1080)^2$$

$$= 292 \text{ J}$$

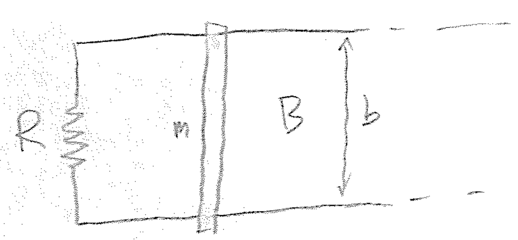
energy withdrawn from battery:

$$U = \mathcal{E} \int I \, dt$$

$$= \mathcal{E} \frac{\mathcal{E}}{R} \int_0^{t_0} (1 - e^{-Rt/L}) \, dt$$

$$= 1008 \text{ J}$$

7.14



a) $\mathcal{E} = -\frac{1}{c}\frac{d\Phi}{dt} = -\frac{1}{c}\frac{d}{dt}(Bbx) = -\frac{Bbv}{c}$

$\Rightarrow I = \frac{\mathcal{E}}{R} = -\frac{Bbv}{cR}$

$\Rightarrow \vec{F} = \frac{I\vec{l}\times\vec{B}}{c} = -\frac{IbB}{c}\hat{x}$

$\Rightarrow a = \frac{|\vec{F}|}{m} = -\frac{IlB}{cm} = -\frac{b^2B^2v}{c^2Rm} = \frac{dv}{dt}$

$\Rightarrow v = v_0 e^{-\frac{b^2B^2}{c^2Rm}t}$

$v \to 0$ as $t \to \infty$ but v is never 0.

b) $x = \int_0^\infty v(t)\,dt = \int_0^\infty v_0 e^{-\frac{b^2B^2}{c^2Rm}t} = -\frac{v_0c^2Rm}{b^2B}e^{-\frac{b^2B^2t}{c^2Rm}}\Big|_0^\infty$

$= \boxed{\frac{v_0c^2Rm}{b^2B}}$

c) $\frac{1}{2}mv_0^2 \overset{?}{=} \int_0^\infty I^2R\,dt$

$= \int_0^\infty \left(\frac{Bbv}{cR}\right)^2 R\,dt = \frac{B^2b^2}{c^2R}\int_0^\infty v^2\,dt$

$= \frac{B^2b^2}{c^2R}\int_0^\infty v_0^2 e^{-\frac{2b^2B^2}{c^2Rm}t} = \frac{\cancel{B^2}\cancel{b^2}}{\cancel{c^2}\cancel{R}}\left(v_0^2\frac{\cancel{c^2}\cancel{R}m}{2\cancel{b^2}\cancel{B^2}}\right)$

$= \frac{1}{2}mv_0^2$ ✓

Problem 3 (Purcell 7.21)

$$\varepsilon_0 = \frac{1}{\mu_0 c^2}$$
$$\varepsilon_0 \mu_0 = \frac{1}{c^2}$$
} for unit conversions

field inside a solenoid: $\dfrac{4\pi I N}{c\ell} = \dfrac{4\pi I N_2}{b_2 c}$

inductance $\quad \mathcal{E}_{21} = M_{21} \dfrac{dI_1}{dt}$

$$M_{21} = \frac{\Phi}{cI} = \frac{4\pi N_2}{b_2 c^2} \pi a_1^2 N_1$$

$$= \frac{4\pi^2 N_2 N_1 a_1^2}{b_2 c^2}$$

8.2 $R = 2000\,\Omega$ $\quad 120\bar{V}$ rms
$C = 1\,\mu F$ $\quad 60\,Hz$

a) Impedance $= Z$

$$Z = R - \frac{i}{\omega C} = 2000\,\Omega - \frac{i}{60\,Hz \cdot 1\,\mu F} = \boxed{(2000 - 1.6\times10^{4} i)\,\Omega}$$

b) RMS value of current

$$I_{rms} = \sqrt{|YV|^2}$$
$$= V_{rms}\left(\left(\frac{1}{R}\right)^2 + (\omega C)^2\right)^{\frac{1}{2}}$$
$$= 120V\left(\left(\frac{1}{2000\,\Omega}\right)^2 + (60\,Hz \cdot 10^{-6}F)^2\right)^{\frac{1}{2}}$$
$$= \boxed{0.0604\,A}$$

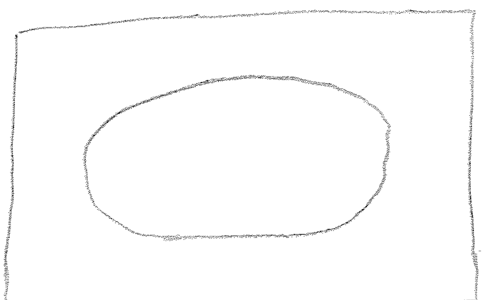
c) $$\bar{P} = \frac{V_{rms}^2}{R} = \frac{(120V)^2}{2000\,\Omega} = \boxed{7.2\,W}$$

d) $$V_C = \frac{V_{rms}}{R - \frac{i}{\omega C}}\left(-\frac{i}{\omega C}\right) = \frac{120V}{(2000 - 1.6\times10^{4} i)\,\Omega}\left(-\frac{i}{60\,Hz \cdot 10^{-6}F}\right)$$

$$= \boxed{(118.758 - 14.3i)\ V}$$

$$V_R = 120 - V_c = \boxed{(1.242 + 14.3i)\ V}$$

e)



← An oval

Problem 3 (Purcell 7.21)

$$\left.\begin{array}{l} \varepsilon_0 = \frac{1}{\mu_0 c^2} \\ \varepsilon_0 \mu_0 = \frac{1}{c^2} \end{array}\right\} \text{ for unit conversions}$$

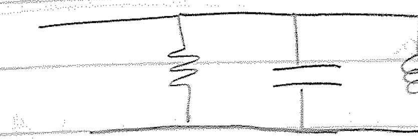
field inside a solenoid: $\frac{4\pi I N}{c\ell} = \frac{4\pi I N_2}{b_2 c}$

inductance $\mathcal{E}_{21} = M_{21} \frac{dI_1}{dt}$

$$M_{21} = \frac{\Phi}{cI} = \frac{4\pi N_2}{b_2 c^2} \pi a_1^2 N_1$$

$$= \frac{4\pi^2 N_2 N_1 a_1^2}{b_2 c^2}$$

Problem 5 (Purcell 8.3)



$R = 1000\,\Omega$

$C = 500\text{ pF}$

$L = 2\text{ mH}$

$\frac{1}{iL}\omega^{-1} \to -\omega^{-2}\frac{1}{iL}$

$$Z^{-1} = \left(\frac{1}{R} + \frac{1}{i\omega L} + \frac{\omega C}{-i}\right)$$

$$Z = \frac{1}{\left(\frac{1}{R} + \frac{1}{i\omega L} - \frac{\omega C}{i}\right)}$$

$\boxed{\begin{array}{l} +i^{-1} = -i \\ i^0 = 1 \\ i = i \\ i^2 = -1 \end{array}}$

a) $\omega = 10$ kilocycles/sec. $= 10{,}000$ Hz

$$Z = \frac{1}{\frac{1}{1000} + \frac{1}{i(10{,}000)(.002)} - \frac{(10{,}000)(5\times10^{-10})}{i}}$$

$$Z = \frac{1}{\frac{1}{1000} - i\left(\frac{1}{10000}\cdot\frac{1}{.002}\right) - i\left(10000\times5\times10^{-10}\right)}$$

$$Z = \frac{1}{.001 - i(.05 + .000005)} = \frac{1}{.001 - i(.050005)}$$

$$Z = \frac{1}{.001 - i(.050005)} \cdot \frac{.001 + i(.05005)}{.001 + i(.05005)}$$

$$= \frac{.001 + i(.05005)}{1\times10^{-6} + .0025005} = .4 + 20i$$

b) $\omega = 10^6$ Hz

same method as in part a

$$Z = 39.2 - 194.1i$$

c) maximize Z with respect to ω

$$\frac{dZ}{d\omega} = -1\left(\frac{1}{R} + \frac{1}{i}\left(\frac{1}{\omega L} - \omega C\right)\right)^{-2}\left[\frac{-\omega^{-2}}{iL} - \frac{C}{i}\right] = 0$$

$$\frac{1}{iL}\omega^{-2} - \frac{C}{i} = 0$$

$$\frac{1}{\cancel{i}L}\omega^{-2} = \frac{C}{\cancel{i}}$$

$$\omega^{-2} = LC$$

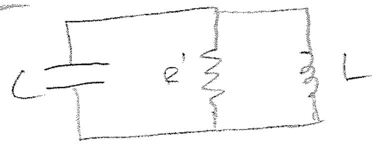
$$\frac{1}{\omega^2} = +LC$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \sqrt{\frac{1}{LC}}$$

resonant frequency

8.4



Work out equation analogous to (8-2):

$$\frac{d^2V}{dt^2} + \frac{R}{L}\frac{dV}{dt} + \frac{V}{LC} = 0$$

① $I = -\frac{dQ}{dt}$ ② $Q = CV$ ③ $V = I_1 R' = L\frac{dI_2}{dt}$ ④ $I_1 + I_2 = I$

④ $I_1 + I_2 = I \overset{①}{\Rightarrow} \frac{V}{R'} + I_2 \overset{②}{=} -C\frac{dV}{dt}$

$$\Rightarrow I_2 = -C\frac{dV}{dt} - \frac{V}{R'}$$

$$\overset{③}{\Rightarrow} V = L\frac{d}{dt}\left(-C\frac{dV}{dt} - \frac{V}{R'}\right)$$

$$\Rightarrow V = -LC\frac{d^2V}{dt} - \frac{L}{R'}\frac{dV}{dt}$$

$$\Rightarrow \boxed{\frac{d^2V}{dt} + \frac{1}{R'C}\frac{dV}{dt} + \frac{V}{LC} = 0}$$

Using (8-3,4,5):

$$\Rightarrow (\alpha^2 - \omega^2)\cos\omega t + 2\alpha\omega\sin\omega t - \frac{1}{R'C}(\alpha\cos\omega t + \omega\sin\omega t)$$

$$+ \frac{1}{LC}(\cos\omega t) = 0$$

The coefficients of sin + cos must each be zero:

$\Rightarrow \alpha^2 - \omega^2 - \frac{\alpha}{R'C} + \frac{1}{LC} = 0$ and $2\alpha\omega - \frac{\omega}{R'C} = 0$

$$\Rightarrow \boxed{\alpha = \frac{1}{2R'C}}$$

$$\Rightarrow \omega^2 = \alpha^2 - \frac{\alpha}{R'C} + \frac{1}{LC} = \frac{1}{4R'^2C^2} - \frac{1}{2R'^2C^2} + \frac{1}{LC}$$

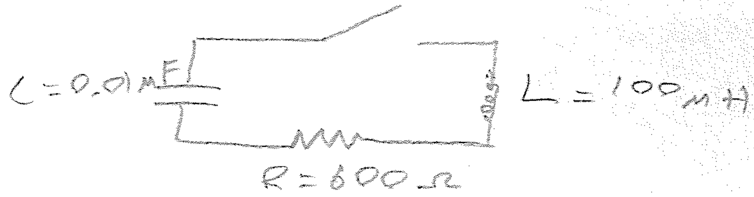
$$\Rightarrow \boxed{\omega^2 = \frac{1}{LC} - \frac{1}{4(R'C)^2}}$$

To be the same as a series RLC

$$\frac{R}{L} = \frac{1}{R'C} \Rightarrow \boxed{R' = \frac{L}{RC}}$$

8.6

$$V(t) = Ae^{-\beta t}$$



By (8-2)

$$\frac{d^2V}{dt^2} + \frac{R}{L}\frac{dV}{dt} + \frac{1}{LC}V = 0$$

$$\Rightarrow \left(A\beta^2 e^{-\beta t}\right) + \frac{R}{L}\left(-\beta A e^{-\beta t}\right) + \frac{1}{LC}\left(Ae^{-\beta t}\right) = 0$$

$$\Rightarrow \beta^2 - \frac{R}{L}\beta + \frac{1}{LC} = 0$$

$$\Rightarrow \beta = \frac{\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$= \frac{\frac{R}{L} \pm \frac{R}{L}\sqrt{1 - \frac{4L}{CR^2}}}{2}$$

$$= \frac{R}{2L}\left(1 \pm \sqrt{1 - \frac{4L}{CR^2}}\right)$$

$$= \boxed{\underbrace{5.82\times10^6 \text{ Hz}}_{\beta_1} \quad \text{or} \quad \underbrace{1.72\times10^5 \text{ Hz}}_{\beta_2}}$$

$$\Rightarrow V(t) = Ae^{-\beta_1 t} + Be^{-\beta_2 t} \qquad (8-16)$$

$$V(0) = A + B = V$$

$$\left.\frac{dV}{dt}\right|_{t=0} = 0 = -\beta_1 A - \beta_2 B$$

$$\Rightarrow \boxed{\frac{B}{A} = -\frac{\beta_1}{\beta_2} = -33.97}$$