Final Exam Physics 106 May 18, 2011

Do this exam by yourself. No calculators or books are needed or permitted. However, a formula sheet (four pages of US letter size paper) is allowed. **Problems are on BOTH SIDES of this page. Please explain your reasoning. Draw sketches and free body diagrams.** Even if you don't know how to solve the problem, state in words what you think is happening and why. Express all answers in terms of the variables defined in the problem.

1. [20 points] A thick flat plastic sheet with a dielectric constant ε is placed in the homogeneous electric field $\vec{E}_0$, so that the axis perpendicular to the surface of the plastic sheet makes an angle α with $\vec{E}_0$. Draw a sketch of the electric field lines outside and inside of the plastic sheet. What is the magnitude of the electric field inside it?

2. [20 points] An infinite flat metal plate of thickness $2h$ carries the current density j.

(a) [10 points] What is the orientation and magnitude of $\vec{B}$ at a distance x from the midplane of the plate? Consider both inside and outside of the plate, and sketch $B(x)$.

(b) [10 points] The same infinite flat metal plate contains a cylindrical hole of radius h, with its axis lying in the midplane. The current density j is running parallel to the cylindrical hole, as shown in the figure. Let the x axis start at the axis of the hole. What is $B(x)$?

3. [20 points] A cone of height H and radius of base R is permanently magnetized with uniform magnetization M. The vector $\vec{M}$ is collinear with the axis of the cone. Let that axis be z, and the apex of the cone the coordinate origin.

What is the magnetic field along the z axis, at the distance h from the apex, $\vec{B}(h)$? (Here $h+z$ is larger than the thickness of the cone at the distance z from its apex.) *[Hint: model the cone as a stack of infinitesimal disks.]*

4. [20 points]

(a) [10 points] Compute the total resistance of the resistor network shown in the figure.

(b) [10 points] For what value of r (expressed as a function of R) the total resistance of the whole network will also equal r?

(c) [EXTRA CREDIT = 10 points] What is the resistance of the infinite network shown in the figure? *[Hint: if one cell from the network is removed, its resistance is still r.]*

5. [20 points] A horizontal steel bar of length ℓ and mass m can slide down two vertical rails. The system is in a homogeneous magnetic field B perpendicular to both the rails and the bar. The top ends of the rails are connected to a capacitor C. At $t = 0$, the bar is at the top of the rails, and the capacitor is not charged.

(a) [5 points] If the bar is falling with an instantaneous velocity v_y, what is electromotive force in the loop?

(b) [5 points] What is the force on the bar? Is it pulling or pushing the bar?

(c) [5 points] What is the relationship between the electromotive force and the voltage on the capacitor? (Note that the resistance in this circuit is negligible.) Use the obtained formula to express the relationship between the charge on the capacitor and the vertical velocity of the bar, v_y. What is the current through the bar?

(d) [5 points] Using the 2nd Newton's Law, obtain the acceleration of the bar. Solve the differential equation and obtain the velocity of the bar as a function of time.

6. [EXTRA CREDIT = 20 points] One of the designs for a perpetual motion machine[1] involves a parallel plate capacitor with holes on both plates, and a large plastic ring which is ran through both holes. The ring which rests on ball bearings, and can rotate without friction.

The inventor's idea is the following: the ring is charged, so that the charge is uniformly distributed around the ring. When the capacitor is also charged, a strong electric field exists between the plates and is negligible outside the plates. The part of the ring which is at any given moment between the plates is thus being pushed by the field – and the wheel turns, forever.

(a) [5 points] What is the problem with this argument?

(b) [15 points] Prove that the torque on the ring about its center is exactly zero.

[1] From Wikipedia: "Perpetual motion describes hypothetical machines that operate or produce useful work indefinitely and, more generally, hypothetical machines that produce more work or energy than they consume, whether they might operate indefinitely or not. There is undisputed scientific consensus that perpetual motion would violate either the first law of thermodynamics, the second law of thermodynamics, or both."

Possibly useful formulas

- $\int \frac{dx}{x} = \ln x + C$, and $\int e^{\beta x} dx = \frac{1}{\beta} e^{\beta x} + C$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad \text{for } n \neq -1$
- $\int \sin x dx = -\cos x + C$ and $\int \cos x dx = \sin x + C$
- $\sin^2 \alpha + \cos^2 \alpha = 1$
- $\sinh(x) = \frac{e^x - e^{-x}}{2}$
- $\cosh(x) = \frac{e^x + e^{-x}}{2}$
- Solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Scalar product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$.
- A second order linear differential equation $\ddot{x} + \omega^2 x = 0$ has the solution in the form of $x(t) = A\sin(\omega t) + B\cos(\omega t)$.