

Do this exam by yourself. No calculators or books are needed or permitted. However, a formula sheet (two pages of US letter size paper). **Problems are on BOTH SIDES of this page. Please explain your reasoning. Draw sketches and free body diagrams.** Even if you don't know how to solve the problem, state in words what you think is happening and why. Express all answers in terms of the variables defined in the problem.

1. [35 points] A flat object (of arbitrary shape) is uniformly charged by charge density  $\sigma$ . Let us consider the electric field  $\vec{E}$  in point  $P$  outside of the plane containing the object. The solid angle with which the object is seen from  $P$  is  $\Omega$ .

Electric field  $\vec{E}$  at point  $P$  can be decomposed into two components, one perpendicular to the plane,  $\vec{E}_\perp$  and one parallel to the plane,  $\vec{E}_\parallel$ :

$$\vec{E} = \vec{E}_\perp + \vec{E}_\parallel$$

- (a) [10 points] Prove that  $E_\perp = k\sigma\Omega$ . [Hint: subdivide the flat object into many infinitesimal pieces, and show that  $dE_\perp = k\sigma d\Omega$ , from which the answer follows directly.]
- (b) [10 points] To check that your answer make sense, compute the field of an infinite plane with surface charge density  $\sigma$ . [Hint: pick a point outside the plane, and compute the solid angle from which the whole infinite plane is seen.]
- (c) [15 points] A cube has one side removed, and the remaining five are charged with a surface charge density  $\sigma$ . What is the electric field in its center? (We need both the direction and the magnitude.)
- (d) [EXTRA CREDIT = 15 points] A regular tetrahedron (composed of four equilateral triangles) has one of its triangles removed, and the remaining three are charged with a surface charge density  $\sigma$ . What is the electric field in its center?
2. [35 points] A conducting sphere of radius  $R$  contains a small cavity of radius  $r < R/2$ . The center of the cavity is located at  $r = R/2$ , and in its center contains a charge  $Q$ .
- (a) [5 points] What is the total charge on the wall of the cavity?
- (b) [5 points] How is the charge distributed on the surface of the conducting sphere? Why?
- (c) [10 points] If the sphere is neutral, what is the charge density on its surface?
- (d) [15 points] If the sphere has charge  $q$ , compute the electric field at the distance  $L > R$  of its center, oriented along the direction of the cavity. Will this electric field depend on the position of the cavity? Will it depend on its shape?

3. [40 points] A thin cylindrical rod of length  $L$  and radius  $R \ll L$  is charged with a uniform volume charge density  $\rho$ .
- [20 points] What is the electric field inside and outside the rod, away from the ends?
  - [20 points] Assuming that the potential of the surface of the rod is  $\phi_0$ , compute the potential  $\phi(r)$  inside the rod (i.e.,  $r < R$ ).
  - [EXTRA CREDIT = 15 points] The rod is placed inside a hollow conducting cylinder of radius  $R$  and same length. The conducting cylinder is grounded (and thus its potential is zero). What is the total charge that moves from the ground to the cylinder when its connected to the ground?

### Possibly useful formulas

- $\int \frac{dx}{x} = \ln x + C$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + C$  for  $n \neq -1$
- $\int \sin x dx = -\cos x + C$  and  $\int \cos x dx = \sin x + C$
- $\sin^2 \alpha + \cos^2 \alpha = 1$
- Solutions of the quadratic equation  $ax^2 + bx + c = 0$  are

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Scalar product:  $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$ .
- A second order linear differential equation  $\ddot{x} + \omega^2 x = 0$  has the solution in the form of  $x(t) = A \sin(\omega t) + B \cos(\omega t)$ .