First Midterm   Physics 106   March 10, 2011

Do this exam by yourself. No calculators or books are needed or permitted. However, a formula sheet (four pages of US letter size paper) is allowed. **Problems are on BOTH SIDES of this page. Please explain your reasoning. Draw sketches and free body diagrams.** Even if you don't know how to solve the problem, state in words what you think is happening and why. Express all answers in terms of the variables defined in the problem.

1. [45 points] A rectangular frame of length $2b$ along the x axis and a along the y axis is moving with constant velocity $\vec{v} = v\hat{x}$ from $x = -\infty$ into the magnetic field $\vec{B} = \hat{z}B_0 e^{\alpha x}$. The frame is made of wire with the resistance per unit length, ρ.

(a) [10 points] If the center of the frame is at the position $x = x_c$, what is the flux through the frame? Show that the flux is proportional to $\sinh(\alpha b)$.

(b) [10 points] When the center of the frame is at x_c, what is the electromotive force which is being generated in the frame as it is moving?

(c) [10 points] What is the power dissipated in the frame, as a function of x_c?

(d) [15 points] What is the total amount of heat generated by the current in the frame as it travels from $x_c = -\infty$ to $x_c = 0$?

2. [45 points]

(a) [5 points] An infinite wire carries the current I. What is the magnetic field at a point P at the distance a from the wire?

(b) [30 points] A very long wire runs along positive x axis from $-\infty$ and carries the current $2I$. The wire is spliced at $(0,0,0)$, with one branch turning into the positive y axis and carrying the current I, and another turning into the negative y axis and also carrying the current I.

What are the magnitude and the orientation of the magnetic field, $\vec{B}$, at a point P on the z axis at a height a from the origin (that is, at $(0,0,a)$)?

(c) [10 points] A very long wire is bent at 90° in the horizontal plane, so that the current I runs down the x axis to the origin, and from there along the positive y axis to infinity. What is $\vec{B}$ (as a vector) at $(0,0,a)$?

3. [10 points] An equilateral triangle with the side a is made of resistive wire. Each side has the resistance R. Voltage V is applied to two corners of the triangle.

(a) [5 points] What are the currents through each of the sides? Draw their direction on a sketch of the triangle.

(b) [5 points] What is the voltage between the third corner and the other two?

(c) [EXTRA CREDIT = 30 points] What are the orientation and the magnitude of the magnetic field $\vec{B}$ in the center of the triangle?

Possibly useful formulas

- $\int \frac{dx}{x} = \ln x + C$, and $\int e^{\beta x} dx = \frac{1}{\beta} e^{\beta x} + C$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$
- $\int \sin x dx = -\cos x + C$ and $\int \cos x dx = \sin x + C$
- $\sin^2 \alpha + \cos^2 \alpha = 1$
- $\sinh(x) = \frac{e^x - e^{-x}}{2}$
- $\cosh(x) = \frac{e^x + e^{-x}}{2}$
- Solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Scalar product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$.
- A second order linear differential equation $\ddot{x} + \omega^2 x = 0$ has the solution in the form of $x(t) = A \sin(\omega t) + B \cos(\omega t)$.