171.106: Electromagnetic Theory I
Midterm Exam 4/15/10 1:30-2:30

A letter sized paper with formulae on one side is permitted and check out the formula page as well. Start each problem on a fresh page and please give detailed reasoning. If in a later question you need a result from a question that you cannot solve then write formulae with the unknown as a symbol and you will get due credit. Ask your proctor for clarification if the text is unclear.

Problem 1 (30 points)

Consider Hall's experiment indicated in Fig. 1. The material is a metal with a density, n, of charge carriers each with charge, q and mass, m. The Hall voltage, V_H, is measured for a given applied field, B, current, I, and conductor thickness, t. From these numbers the so called Hall coefficient for the material is determined:

$$R_H = \frac{tV_H}{IB}.$$

(a) Show that the Hall coefficient $R_H = \frac{1}{nqc}$ (CGS units)

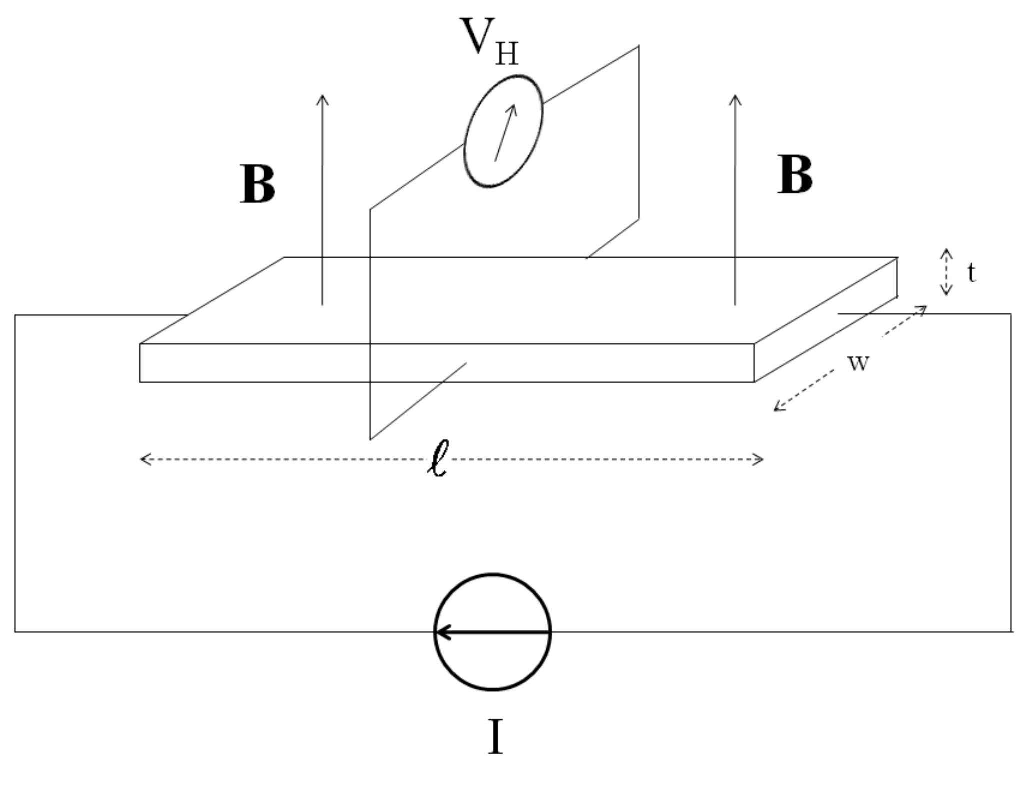

Figure 1.

Problem 2 (20 points)

In the following B is a constant with dimension of magnetic field and d is a constant of dimension length. $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$ is the position vector.

Which of the following could be expressions for a spatially varying magnetic field.

(a) $\mathbf{B}(\mathbf{r}) = \frac{B}{d}\mathbf{r}$

(b) $\mathbf{B}(\mathbf{r}) = \frac{B}{d}(x\hat{\mathbf{x}} + y\hat{\mathbf{y}} - 2z\hat{\mathbf{z}})$

Hint: There are no magnetic monopoles!

Problem 3 (25 points)

Consider the circuit shown in Fig. 2. Here we assume that the current source is providing a fixed current, I_S, and the battery has an EMF voltage $\mathcal{E}_s$.

(a) Determine the current, I_2, through the resistor.
(b) Determine the total power dissipated in the circuit

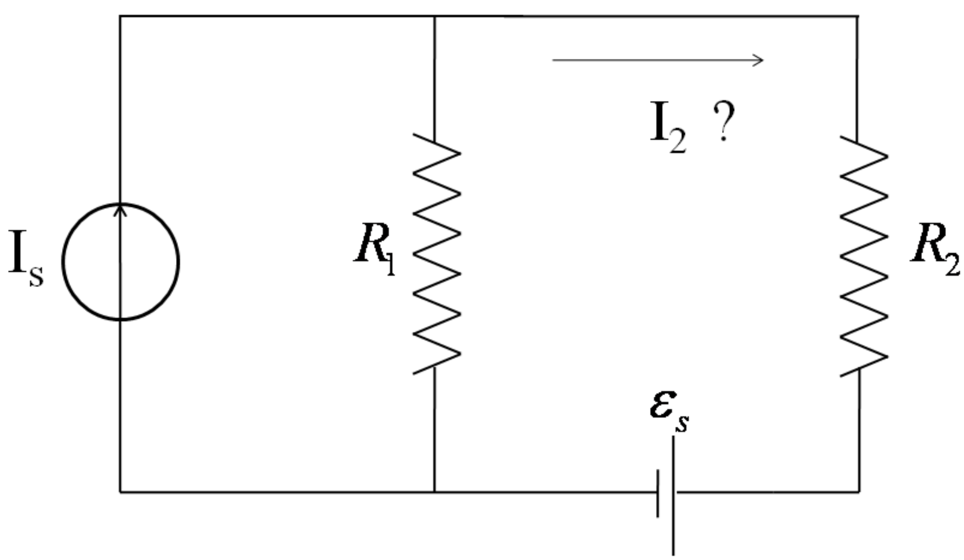

Figure 2.

Problem 4 (25 points)

Consider the circuit shown in Fig. 3. After a long period with the switched open (as shown), the switch is closed at time $t \equiv 0$.

(a) Determine an expression for the time dependent current through the inductor.
(b) Determine how much energy is dissipated in the resistor after the switched is closed.

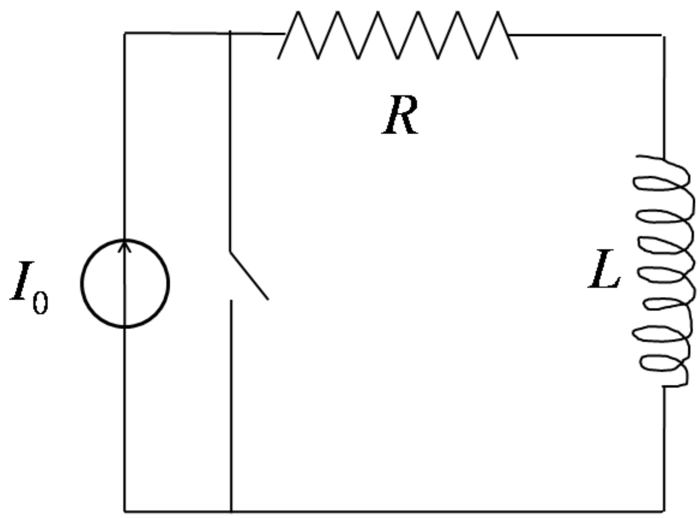

Figure 3.

Formulae

GCS units

1. $\oint \mathbf{E}\cdot \mathrm{d}\mathbf{a} = 4\pi Q_{enc}$ $\qquad \nabla\cdot\mathbf{E} = 4\pi\rho$
2. $\oint \mathbf{B}\cdot \mathrm{d}\mathbf{s} = \frac{4\pi}{c} I_{enc}$ $\qquad \nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J}$
3. $\varepsilon = -\frac{1}{c}\frac{\mathrm{d}\phi}{\mathrm{d}t}$ where $\phi = \int \mathbf{B}\cdot\mathrm{d}\mathbf{a}$ $\qquad \nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$
4. $\nabla\cdot\mathbf{B} = 0$

MKS units

5. $\oint \mathbf{E}\cdot \mathrm{d}\mathbf{a} = \frac{1}{\varepsilon_0} Q_{enc}$ $\qquad \nabla\cdot\mathbf{E} = \rho/\varepsilon_0$
6. $\oint \mathbf{B}\cdot \mathrm{d}\mathbf{s} = \mu_0 I_{enc}$ $\qquad \nabla\times\mathbf{B} = \mu_0\mathbf{J}$
7. $\varepsilon = -\frac{\mathrm{d}\phi}{\mathrm{d}t}$ where $\phi = \int \mathbf{B}\cdot\mathrm{d}\mathbf{a}$ $\qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$
8. $\nabla\cdot\mathbf{B} = 0$

CGS & MKS units

9. $I = \int \mathbf{J}\cdot\mathrm{d}\mathbf{a}$
10. $V_A - V_B = \int_A^B \mathbf{E}\cdot\mathrm{d}\mathbf{s}$
11. $V = L\frac{dI}{dt}$
12. $\varepsilon_{21} = -M_{21}\frac{\mathrm{d}I_1}{\mathrm{d}t}$
13. $M_{21} = M_{12}$
14. $Q = CV$
15. $I = \frac{\mathrm{d}Q}{\mathrm{d}t}$
16. $V = RI$
17. $P = IV$