Physics of Human Energy Use
Solutions to Assignment 1

1)*Suppose that your car gets 30 miles per gallon when travelling at 60 miles per hour. What is its fuel-energy consumption rate in Watts?*

The rate at which your car consumes gasoline is

$$\frac{60\text{mi/hr}}{30\text{mi/gal}} = 2\text{gal/hr}.$$

The energy contained in 1 gal of gasoline is $\simeq 1.3 \times 10^8$ J (a figure given in lecture but also derivable from the data in Table 1.1). So the fuel-energy consumption rate is

$$P = 2\text{gal/hr} \times \frac{1}{3600}\text{hr/s} \times 1.3 \times 10^8\text{J/gal} = 7 \times 10^4\text{W}.$$

2) *Look up on the web (choose a reputable source!) the energy intensity and energy used per capita for the following countries: US, Canada, Switzerland, Russia, Bahrein, Turkey, Paraguay, and Kenya. Is there a broader range in energy intensity or in energy per capita? What might account for the relative magnitudes of these quantities found in these countries?*

The Department of Energy maintains a very good energy information webiste. The tables provided at `www.eia.doe.gov/emeu/international/energyconsumption.html` contain the information needed. When the question is, "How much energy is used in order to reach a given standard of living?", the "purchasing power parity" definition of GDP is probably most appropriate for international comparisons. Ordered by their energy intensity, these countries had the following ranking for 2006:

Country	Energy Intensity (kBTU/\$) $\simeq 10^6 J/\$$	Energy per Capita (MBTU/yr/person) $\simeq 10^9 J/\text{yr/person}$
Paraguay	29.2	65.5
Bahrein	28.3	695
Russia	18.8	214
Canada	13.1	427
US	8.8	335
Turkey	5.7	55.5
Switzerland	5.2	171
Kenya	3.4	5.6

First, it's apparent that there's a much larger range in energy per capita (within this set a factor of more than 100) than in energy intensity (the ratio of maximum to minimum is ~ 9). Note that for answering a question like this, one can only compare ratios, not differences because the two quantities have different units—there is no way to say whether a difference of 100 W/person is greater or smaller than a difference of 100 J/$, but (dimensionless) ratios of maximum to minimum can always be compared.

This contrast between energy intensity and energy per capita reflects the fact that in an industrial economy, using more energy at some level buys more GDP, but it doesn't necessarily lead to a larger population. In line with this fact, the contrast in energy per capita is mostly due to the contrast between developed economies and undeveloped. Even the lowest energy per capita developed country on this list (Switzerland) uses 30 times as much energy per person as a truly undeveloped country (Kenya). There are also secondary effects due to specialized economies (Bahrein depends almost entirely on oil and gas production, which is both energy-intensive and requires few workers) and climate (contrast Canada with the US).

These secondary effects and others account for the contrasts in energy intensity. Russia, for example, suffers from the double-whammy of a very cold climate, requiring lots of energy for space heating, and also an outmoded industrial plant whose energy use is very inefficient. Switzerland is like the US in terms of its economy, standard of living, and climate, but the bulk of its population live in very compact cities not very distant from one another. Efficient public transport within the cities and short inter-city travel distances minimize energy expended on transportation. Its economy also has an especially large financial services sector, which requires less energy than industry. Turkey is an example of a country in a development transition: its energy per capita is an order of magnitude greater than Kenya's (in part also because of its much less temperate climate), but its energy intensity is closer to the mid-range. Kenya has a very mild climate and little industry, so little energy is used for anything. Finally, Paraguay tops the chart in energy intensity even though its energy per capita is relatively modest; the reason is that it is also extremely poor, so the ratio to GDP is very large.

Note: A number of students used figures compiled by the World Resources Institute. There appear to be several discrepancies between their figures and those of the DoE. For example, WRI lists Kenya as consuming 20 GJ/person-yr, whereas DoE shows only 5 GJ/person-yr; similarly, WRI shows 430 GJ/person-yr for Bahrein, while DoE gives 695 GJ/person-yr.

3) *Look up the population density of Manhattan. Assuming that per capita energy usage of New Yorkers (all forms, including non-residential use) is the same as the national average (in fact, they use rather less), estimate the mean power used on the island. Next estimate the mean power delivered to Manhattan by sunlight, first assuming that the Sun is directly overhead during the daytime. Then correct this estimate by assuming that the mean angle of the Sun from the zenith during daytime is* 60°. *How does the human power consumption per unit area compare to the rate at which solar power arrives?*

The population density of Manhattan in 2000 was 2.7×10^4 people/km^2 = 0.027 people/m^2 (from any number of internet references, all making use of 2000 census data). If total US energy consumption is 10^{20} J/yr $\simeq 3 \times 10^{12}$ W and the US population is 3×10^8 people, the per capita energy consumption rate is 1×10^4 W. The Manhattan energy consumption rate per unit area is therefore 270 W/m^2.

By contrast, the Solar constant is 1400 W/m^2. However, the Sun shines on average only half the time (daytime). In addition, if its angle from the zenith is on average 60°, then the rate at which its energy arrives per unit area on the ground is reduced by another factor of $\cos 60^\circ = 0.5$. So in the end the mean Solar power per unit area is $\simeq 350$ W, not much larger than the rate per unit area at which people expend power. It's a good thing that the actual per capita energy use of New Yorkers is actually a couple of times smaller than the national average and that the mean windspeed in Central Park is $\simeq 10$ mi/hr, so a lot of that heat gets quickly blown out to sea.

4) *Imagine that you're building a new skyscraper, 300 m tall (that's roughly Empire State Building height). Compare the energy it takes to lift its materials to the top with the energy it would take to raise their temperature from ambient to room temperature on a cold winter day (-10 C). Assume that the mean mass per atom is that of concrete (N CaO + SiO_2), where $N \simeq 2$–3).*

The atomic weight of Ca is 40, O is 16, Si is 28. So the mean mass per atom is $(2.5 \times 40 + 4.5 \times 16 + 28)/(2.5 + 4.5 + 1) = 25$ AMU. To raise each atom to the top therefore takes $25 \times 1.7 \times 10^{-27} \times 9.8 \times 300 = 1.25 \times 10^{-22}$ J. To raise each atom to room temperature means an increase of 30 C. At an energy cost of $3 \times (1/2)k_B$ per degree, that costs 6×10^{-22} J. So warming the concrete only once on a cold day requires roughly four times the energy of lifting it up to the top of one of the tallest buildings anywhere.

5) *Imagine a box that you can put a number of hard balls into and shake. As you do work on the box by shaking it, the balls acquire energy by collisions with the walls of the box. Would you describe the balls' energy as organized kinetic energy or heat? Does your choice depend on the number of balls in the box?*

A clear answer can be given only in extreme limits. When there are only a few balls in the box, you can track the energy of individual balls; in fact, if you knew the details of the shaking, you could predict their energies given their initial positions and states of motion. Moreover, the number of balls having a particular energy at any given time would not be particularly well given by predictions based on averaging over very large numbers of objects because there would be small number fluctuations. On the other hand, in the limit of a truly large number of balls (10^{20} perhaps), there's no question that very soon after you began shaking you could describe their collective energy as heat because there would be no way you could distinguish any individual ball's energy history and you'd be able to speak of the balls' energy only in statistical and probablistic terms. In between–10^3 balls or 10^6 balls or . . .—the hallmarks of organized energy would gradually fade with increasing number, but the dividing line is a fuzzy one that depends on the quality of your measuring instruments.