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Physics of Human Energy Use
Solutions to Assignment 10

1) Consider the energy budgets for different car designs. For fiducial numbers, suppose that a typical sedan weighs 1.5 metric tons and has an effective cross sectional area of 0.75 m^2 (i.e., C_dA), while a typical SUV weighs 2.5 tons and has an effective cross sectional area 1.4 m^2 (these actually are fairly representative). Also suppose that half a car's mileage is done on long road-trips at an average speed of 100 km/hr, and half on short trips in which the average speed (while moving) is 50 km/hr, but the car stops once every km.

a) Which contributions scale in proportion to mass and which in proportion to area?

Both the trip force, $(1/2)Mv^2 N_{\rm stops}/D$, and the rolling resistance, $C_{rr}Mg$, are proportional to mass. Air drag, $(1/2)C_dA\rho_{\rm air}v^2$, is proportional only to effective cross section.

b) What fraction of the total energy can be attributed to each contribution for sedans? For SUVs?

The contribution to the total energy from any of these forces is proportional to the total distance during which the force is exerted. Call the total mileage driven D. We will scale it in km, the speed in units of 10 m/s, and the mass in metric tons. Then we have

$$E_{\rm trip} = (1/2)Mv_{\rm slow}^2(D_{\rm km}/2) = 2.5\times 10^4 M_{\rm T} v_{10}^2 D_{\rm km}\ {\rm J}$$

$$E_{\rm rr} = C_{rr}MgD = 9.8\times 10^4 M_{\rm T} D_{\rm km}\ {\rm J}$$

$$E_{\rm drag} = (1/2)C_dA\rho_{\rm air}v^2 D = 6\times 10^4 C_dA_{\rm msq} v_{10}^2 D_{\rm km}\ {\rm J}$$

The trip energy is proportional only to $D/2$ because the problem assumes that low-speed trips account for half the total distance travelled, and we have made the approximation that the single stop in each of the long high-speed trips is a negligible contribution to the total energy. Written this way, it is clear that each of the three is proportional to $D_{\rm km}$, so it will have no effect on their relative importance. Note, however, that the air drag contribution must be broken up into two pieces, one for the slow-speed trips and one for the high-speed. Translating 50 km/hr to 14 m/s, we have

$$E_{\rm trip} = 5\times 10^4 M_{\rm T} D_{\rm km}\ {\rm J}$$

and

$$E_{\rm drag} = 6\times 10^4 C_dA_{\rm msq} D_{\rm km}(2+8)/2 = 3\times 10^5 C_dA_{\rm msq} D_{\rm km}.$$

Plugging in the specific values of M_T and C_dA for the sedan, we have

$$E_{\rm trip} = 7.5\times 10^4 D_{\rm km}\ {\rm J}$$

$$E_{\rm rr} = 1.5\times 10^5 D_{\rm km}\ {\rm J}$$

$$E_{\rm drag} = 2.3\times 10^5 D_{\rm km}\ {\rm J}$$

$$E_{\rm tot} = 4.5\times 10^5 D_{\rm km}\ {\rm J}.$$

The relative shares are: 17% (trip), 33% (rolling resistance), and 50% (drag).

On the other hand, $E_{\rm trip} + E_{\rm rr}$ scales with mass, so for the SUV it rises to $3.75 \times 10^5 D_{\rm km}$ J, while $E_{\rm drag}$, which scales with effective cross section, becomes 4.2×10^5 J, for a total of $8.0 \times 10^5 D_{\rm km}$ J. The shares in this case are 16% (trip), 31% (rolling resistance), 53% (drag).

Note that relative to American driving habits, these numbers are weighted a bit toward low-speed trips, but not by a great deal. Even so, the trip energy share is relatively small because such a large part of the total distance is on highways, where braking is rare (or at least we hope it is).

c) What fraction of the total energy might be recouped with regenerative brakes, as in hybrids?

Only the trip energy is recaptured by regenerative brakes, so at most about 17% for both sedans and SUVs. In practical systems, the amount saved is more like half that, or $\simeq 8\%$.

2) Imagine an idealized electrical regenerative braking system on a vehicle. A dual purpose electric motor/generator is connected by mechanical linkages to the drive axle. Inside that motor/generator a magnet rotates inside a coil of wire. For the purposes of calculation, suppose that the magnetic field strength is 1 Tesla and that the coil radius is 3 cm.

a) If the vehicle is moving at 30m/s, its wheels have a diameter of 0.75 m, and there are no gears in the mechanical linkage (so that the magnet rotates at the same rate the wheels do), how many loops of wire must run around the coil in order for the generator to create an electrical potential of 500 V? This number is, by the way, the actual operating voltage in a Toyota Prius.

The magnitude of the voltage created by a changing magnetic field passing through a loop of wire is

$$V = N_{\rm loops} d(BA)/dt,$$

where B is the magnetic field strength and A is the area of the loop. In this case, the field passing through the loop is modulated so that $BA = (1\ {\rm T})(0.0028\ {\rm m}^2)\cos(2\pi f t)$, where f is the number of rotations per unit time. For the numbers given, the circumference of the wheel is $C = \pi d = 2.4$ m, so $f = v/C = 12.5$ Hz. When the voltage oscillates sinusoidally, its rms value (the one that's usually quoted) is its maximum value divided by $\sqrt{2}$. Then

$$V_{\rm rms} = N_{\rm loops}\sqrt{2}\pi f BA = 0.16 N_{\rm loops}\ {\rm V}.$$

That means $N_{\rm loops}$ must be $\simeq 3200$.

b) If the mass of the car is 1.5 metric ton, and you wish to stop the vehicle in 10 s, how much current must flow through the circuit?

The kinetic energy to be removed is $(1/2)Mv^2 = 6.75 \times 10^5$ J. If it is to be taken away in 10 s, the power $P = 6.75 \times 10^4$ W. With $P = VI$, the current will be 135 Amp.

3) Consider a gravitational regenerative braking system for subway cars.

a) If each subway car weighs 40T and can carry 250 people (packed), does the weight of the people exceed the weight of the empty car?

Taking an average mass of 70 kg per person (averaged over the age distribution, the mean mass is $\simeq$ 80 kg for men, $\simeq$ 60 kg for women), 250 people sum to 17.5T, rather less than the empty subway car.

b) How far must the tunnel drop in order to accelerate the car from a standing start to 50 km/hr?

By conservation of energy, $(1/2)Mv^2 = Mgh$, so $h = (1/2)v^2/g$, independent of mass. The speed 50 km/hr translates to $\simeq$ 14 m/s, so that's a drop of 10 m.

4) The lift/drag ratio in an airplane is $[A_w/(C_d A_p)]^{1/2}$, where the effective cross section for drag presented by the plane's fuselage and wings is $C_d A_p$ and the area of the wings (viewed from above) is A_w. Both effects involve pushing air aside. How is it, then, that air passing through a cross section as large as A_w can be deflected to make lift, while the effective cross section for drag is so much smaller?

To create drag, the motion of air along the plane's direction of travel must be slowed down. This happens when some of the air's momentum (in the reference frame of the airplane) is lost by the formation of a turbulent wake, vortices, etc. Only air flowing quite close to the airplane's body is affected in this way.

On the other hand, to create lift, air must be given a downward component of velocity, but its horizontal component isn't necessarily affected. This can be accomplished across a much wider cross section because the wings deflect the air downward with a turning radius comparable to the linear dimensions of the wings. The pressure contrasts associated with that deflection are therefore expressed over a similar scale, affecting air flow across a cross sectional region comparable in size to the wings.