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Physics of Human Energy Use
Solutions to Assignment 9

1) *If you run a heat pump with a SEER of 15 and power it with electricity generated in a coal-burning plant with thermal efficiency 40%, what is the ratio between the energy input into this system and the heat brought into the house?*

The Coefficient of Performance (CoP) of a heat pump is SEER/3.4, in this case, 4.4. That is, the heat brought into the house is 4.4 times the electrical power used to run the pump. But the 40% thermal efficiency of the generating station means that the total heat produced by burning its coal was 2.5 times the electrical energy delivered to the grid, so the the net CoP is only $4.4 \times 0.4 = 1.8$. The ratio of input energy to heat delivered is the inverse of this, or 0.57.

2) *A typical American house has about 250 m^2 of floor space spread over two floors. Suppose that its footprint is square, each story requires 3m of height, and (for simplicity) that its roof is flat. Windows generally occupy about 4% of the wall area. For the following questions, assume that the inside-outside temperature difference is 20 C. You can also use these facts and relations:*

The minimum ventilation requirement is 15 (A/100m^2)l/s + 3.5 l/(s-person), where the relevant area for the first term is the total floor area. The specific heat of air is 1×10^3 *J/(m^3-C) and its thermal conductivity is 0.024 J/(m-s-C).*

a) If there are four people inside, what is the minimum heating power associated with ventilation?

The ventilation requirement is $15 \times 2.5 + 4 \times 3.5 = 51.5$ l/s. One liter is 0.001 m^3, so that's 5.15×10^{-2} m^3/s. The energy per cubic meter required to warm the air from the outside temperature to the inside is 2×10^4 J/m^3, so that's a power of 1.0 kW.

b) If you insulate all the walls and roof with a material having a thermal conductivity of 0.02 J/(m-s-C) (e.g., sprayed insulation), how thick must it be in order to reduce the heat lost through the walls and roof to less than the minimum heat required to warm the ventilating air?

The house's footprint area is 125 m^2; if square, the sides are 11.2 m long. The wall area is therefore $4 \times 11.2 \times 6 = 268$ m^2 and the roof area is 125 m^2, for a total of 393 m^2. Assuming that the material of the walls has a much higher thermal conductivity than the insulating material, we can suppose that all the "thermal resistance" is in the insulation. The total heat flux is $393 \times 0.02 \times 20/s$ W for thickness s. To match the 1kW of the ventilation heating, $s = 0.16$ m.

c) *Similarly, if the windows are double-paned glass, how thick must their trapped air space be in order to keep the heat lost through the windows less than the minimum ventilation heating power?*

Again, the trapped air completely dominates the thermal resistance. In this case, the window area is $0.04 \times 268 = 10.7\ \mathrm{m}^2$, so the air space required is only $10.7 \times 0.024 \times 20/10^3 = 0.52$ cm thick.

d) *If the mean daytime angle of incidence between sunlight and a south-facing window is 45° and half the sunlight arriving at the top of the atmosphere reaches ground level, how large must that window be so that during 10 hours of daylight the solar energy entering the house is equal to the ventilation heating energy for a full 24 hr day?*

At the top of the atmosphere, the solar energy flux is 1400 W/m^2. With only half reaching ground level, that's a flux of 700 W/m^2. Projecting at a 45° angle means that the flux per unit window area is smaller by a factor $\cos 45° = 0.71$, or 500 W/m^2. In 10 hours, the accumulated energy per unit window area is 1.8×10^7 J/m^2. On the other hand, the ventilation heating energy per day is 1 kW$\times 8.64 \times 10^4$s/day$= 8.64 \times 10^7$ J. So the window needs to have an area of 4.8 m^2. For a standard of comparison, the area of the whole south-facing wall is 67 m^2, so this is about 7%: more than the typical window fraction, but not enormously. Of course, this is still only 1/3 of the total power if the window and wall losses are equal to the ventilation loss, and you will need a system for both storing the energy and then distributing it during the night.