Quantum Mechanics I, Problem Set 7

Due Wednesday, October 31

1. Townsend Chapter 4, Problem 10.

2. Townsend Chapter 5, Problem 2.

3. Townsend Chapter 5, Problem 3.

4. Townsend Chapter 5, Problem 5. Note that the electron and the positron have opposite electric charges.

5. Townsend Chapter 5, Problem 7.

6. Compute the commutator $[S^2, S_{1z}]$, where $\vec{S} = \vec{S}_1 + \vec{S}_2$. Generalize your result to show that

$$[S^2, \vec{S}_1] = 2\mathrm{i}\hbar(\vec{S}_1 \times \vec{S}_2).$$

Think of this equation component by component.

7. Consider a system of two spin angular momenta $\vec{S}_1$ and $\vec{S}_2$, with quantum numbers $s_1 = 2$ and $s_2 = \frac{3}{2}$. The sum $\vec{S} = \vec{S}_1 + \vec{S}_2$ of the two spins is characterized by the quantum number s.

a. What are the possible values of s?

b. What is the state $|sm\rangle = |\frac{7}{2}, -\frac{7}{2}\rangle$, in terms of the states $|m_1 m_2\rangle$? (This is easy.)

c. What is $|sm\rangle = |\frac{7}{2}, -\frac{5}{2}\rangle$, in terms of the states $|m_1 m_2\rangle$? (This requires some work.)

d. What is $|sm\rangle = |\frac{5}{2}, -\frac{5}{2}\rangle$, in terms of the states $|m_1 m_2\rangle$? Use your answer for (c), and don't worry about the overall phase.

e. What is $|m_1 m_2\rangle = |-2, -\frac{1}{2}\rangle$ in terms of the states $|sm\rangle$? Use your answers to the previous two questions.

f. The answers to (b) through (e) can be obtained from the attached table of Clebsch-Gordan coefficients. Check your answers by doing this. Depending on the overall phase you chose in (d), your answers might look a little different. If so, change your phase convention in (d) to make them look the same.