

Quantum Mechanics I, Problem Set 9

Due Wednesday, November 14

1. Consider a free particle whose position-space wave function is given at $t = 0$ by

$$\Psi(x, 0) = \langle x | \psi \rangle = \begin{cases} A(a - |x|), & \text{if } -a < x < a \\ 0, & \text{otherwise,} \end{cases}$$

where A and a are positive real constants.

a. Determine A by normalizing $\Psi(x, 0)$.

b. Determine $\Phi(p, 0) = \langle p | \psi \rangle$ at $t = 0$. Sketch it. Comment on the behavior of $\langle p | \psi \rangle$ for very small and very large values of a . How does this relate to the uncertainty principle?

d. Find $\Phi(p, t)$.

e. Find $\Psi(x, t)$. You don't have to do the last integral.

f. (Optional) Use Mathematica or another numerical technique to make sketches of the solution. Set $a = \hbar = 1$. Use the numerical integration command **NIntegrate[...]** to compute $\Psi(x, t)$. For numerical reasons it is useful to replace the integral over $-\infty < p < \infty$ with a pair of integrals, for negative and positive p , over the range $p_{\min} < |p| < p_{\max}$, and take $p_{\min} = 0.001$ and $p_{\max} = 25$. Plot the probability density $|\Psi(x, t)|^2$, over the range $-4 < x < 4$, for the following values of t : $t = 0, 0.25, 0.50, 0.75, 0.1, 0.2, 0.5, 1, 2$. With Mathematica, this is easy to automate. You will see the wave packet spread out!

g. (Optional) What is wrong with your plot at $t = 0$? Where did the extra wiggles come from? The extra wiggles are apparent for small values of t .

2. Griffiths, Chapter 2, Problem 1.

3. Griffiths, Chapter 2, Problem 22.

4. Griffiths, Chapter 2, Problem 43.