171.303
Quantum Mechanics I.

PROBLEM SET # 13
November 30, 2011
(due 12/05/11)

Final Exam: December 13 (Tuesday) 9:00-12:00; Room:# 274

1. Coherent states of the harmonic oscillator are eigenstates of the annihilation operator with complex eigenvalues α:

$$\hat{a}|\alpha >= \alpha|\alpha > .$$

   (a) Expand $|\alpha >$ in terms of the energy eigenstates of the harmonic oscillator.

$$|\alpha >= \sum_{n=0}^{\infty} C_n |n > .$$

   Get the recursion relation between the expansion coefficients, express them in terms of the coefficient C_0 and normalize the state to 1.

   (b) Assume that at $t = 0$ the state of the system is $|\alpha_0 >$, determine the state at a later time t.

   (c) Show that the coherent states are not orthogonal, $< \alpha_1|\alpha_2 > \neq 0$ if $\alpha_1 \neq \alpha_2$.

2. Use the WKB approximation to determine the bound states for

   (a) the harmonic oscillator,

   (b) the half harmonic oscillator

$$V(x) = \begin{cases} \frac{1}{2}m\omega^2 x^2 & : \quad x > 0 \\ \infty & : \quad x < 0 \end{cases}$$

3. Calculate the WKB approximate transmission probability for a particle of energy E that encounters a finite square barrier of height $V_0 > E$ and width a. In Chapter 6 the exact transmission coefficient was calculated. Show that the exact result goes over to the the WKB coefficient, when $T \ll 1$, *i.e.* the barrier is very high and/or wide and when only the dominant E dependence is kept.