Introduction to Quantum Mechanics 171.304
Midterm Exam 3/22/07 1-2

Check the attached formula pages. Start each problem on a fresh page and give detailed reasoning. Please ask your proctor for clarification if the text is unclear.

Problem 1 (30 points)

Consider a hydrogen like bound state between two particles with mass m_1 and m_2 and charges q_1 and q_2.

(a) Write an expression for the ground state energy and wave function

(b) Provide an expression for the characteristic radius of the atom. Make sure your result is consistent with the equal sizes of positronium and hydrogen.

Problem 2 (35 points)

In this problem you will use perturbation theory to account for the effect of the finite size of the proton on the energy levels of hydrogen (the volume effect). The proton is described as a uniformly charged sphere with radius b. The electrical potential is then

$$V(r) = \begin{cases} -\dfrac{e^2}{r} & \text{for } r > b \\ \dfrac{e^2}{2b}\left[\left(\dfrac{r}{b}\right)^2 - 3\right] & \text{for } r > b \end{cases}$$

(a) Relative to the point charge model for the nucleus write an expression for the perturbing term in the Hamiltonian.

(b) Write an expression for energy shift of an arbitrary energy level.

(c) Show that to lowest order, only s-levels are affected by this perturbation.

(d) Considering that the nuclear radius is much smaller than the Bohr radius, a_0, show that the ground state energy shift is $\frac{4}{5}|E_1|\left(\dfrac{b}{a_0}\right)^2$. Here E_1 is the ground state energy.

Problem 3 (20 points)

Consider a quantum system described by the Hamiltonian, $\mathcal{H}$. Denote the corresponding eigen-states and eigen-values $|\psi_n\rangle$ and E_n where $n = 1,2,3,\ldots$.

(a) Show that a trial wave function $|\varphi\rangle$ that is orthogonal to the ground state wave function can be used to provide an upper bound on the energy of the first excited state.

Problem 4 (15 points)

Does H_2 respond strongly to a magnetic field? Provide detailed reasoning.

Formulae

Hydrogen atom

$$E_n = -\frac{\mu e^4}{2\hbar^2 n^2}$$

$$\psi_{n=0}(\mathbf{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

$$a_0 = \frac{\hbar^2}{\mu e^2}$$