171.312 Statistical Physics and Thermodynamics
Homework due Wednesday, October 5 to Damien Benveniste in section.

1. Given that the temperature of the cosmic microwave background radiation is 2.725 K,
(a) How many photons are there for every cubic centimeter of the universe? Show your derivation!
(b) What is the entropy density of the cosmic microwave background?
(c) If all of the matter in the universe can be approximated as one hydrogen atom per cubic meter, what is the entropy density of the matter in the universe?
(d) Compare the answers of (b) and (c) and comment on this.
(e) What is the energy density of the cosmic microwave background today?
(f) What is the energy density of the matter today? [Use the approximation from part (c)]
(g) Compare the answers of (e) and (f) and comment on this, including a comparison with what you said in (d).
(h) What is the free energy density of the cosmic microwave background radiation today?
(i) What was the free energy in the universe back in the epoch when the photons and matter decoupled, assuming the temperature was 3000 K.
(j) Assuming that the expansion of the universe has been isentropic, how much smaller was the radius of the observable universe then compared with now?
(k) How much work per unit volume did the cosmic microwave background photons do since the epoch when the temperature was 3000 K?

2. Consider a solid at temperature T as composed of N spin=1 nuclei that do not interact with one another. There are three quantum states accessible to each nucleus, with quantum numbers m = -1, 0, or +1. Due to electric interactions with the fields in the solid, the m=-1 and the m=+1 states have the same energy, ε , and the m=0 state has zero energy.
(a) What is the partition function?
(b) What is the free energy?
(c) What is the entropy?
(d) What is the total energy for E<<kT?
(e) What is the heat capacity for E<<kT?

3. Each atom in a fixed lattice of hydrogen ions can be in any one of four possible states: (i) the ground state (H), with 1 electron and an energy of -13.6 eV; (ii) an excited state (H*) with one electron and an energy of -3.4 eV; (iii) a positive ion state (H+) with no electrons and an energy of 0.0 eV; and (iv) a negative ion state (H-) with 2 electrons and an energy of -0.7 eV. What is the chemical potential at which the average number of electrons per atom is unity?

4. A certain Johns Hopkins University teaching assistant has opened a business, "Daring Damien's Parking Garage." He charges c dollars to park a car for an entire day in one of his N parking spaces. Each parking space holds only one car at a time. The garage is only open to Johns Hopkins students. The TA wants to estimate his profit in advance. To do so, he assumes that the entire Johns Hopkins student body acts as a joint money and car reservoir for the garage. The average daily pocket money of a student then acts as an effective value of "temperature" (kT), while the abundance of cars can be

represented as a "chemical potential", μ . The average pocket money is known to be kT=p dollars, and the abundance of cars can be estimated as $\mu = 0.69\,p$ dollars.
(a) What does the TA estimate for the average number of cars parked in the garage per day?
(b) As the average pocket money increases, does the estimated number of cars parked in the garage increase or decrease?
(c) How many dollars per day, d, does the TA expect to bring in?
(d) The daring TA wishes to maximize his profits. The garage is fully automated and has no operating expenses. What does Damien choose for the price per car, c?

5. A molecule of hemoglobin contains 4 distinguishable sites capable of binding O_2 molecules. Each site can bind no more than one molecule. The binding energy of an empty site is zero. When occupied, the energy level of a site is $-\varepsilon_0 - n\varepsilon_1$ where n is the number of remaining sites on the same molecule that are also occupied by O_2 molecules. A collection of N_0 hemoglobin molecules are brought into thermal and diffusive contact with a reservoir of O_2 molecules with chemical potential μ and fixed temperature T (i.e., they are immersed in a large volume of blood plasma with a fixed concentration of dissolved $O_{2)}$.
(a) How many O_2 molecules are bound to the hemoglobin molecules after equilibrium is attained?
(b) What is the probability that a particular hemoglobin molecule has exactly three oxygen molecules bound to it?
(c) What is the probability that a particular site on a given hemoglobin molecule is occupied?