

171.312 Statistical Physics and Thermodynamics
Homework due TUESDAY, October 11 to Damien Benveniste in section.

1. We may define a temperature of a Fermi-Dirac gas, T_0 , at which the chemical potential of the gas is zero. Express T_0 in terms of the Fermi temperature $T_F = \varepsilon_F / k_B$.
2. Derive the temperature dependence of the Fermi-Dirac distribution function with respect to ε . What is the temperature dependence of this slope evaluated at $\varepsilon = \mu$? What is the limit of this at very low temperature?
3. Repeat problem 2 for the Bose-Einstein distribution.
4. For an ideal gas (in the classical limit) prove that the chemical potential decreases as the temperature increases.
5. For an ideal gas (in the classical limit) prove that the free energy decreases with temperature.
6. Treating the air in our classroom as an ideal gas, what is its approximate entropy? [Give a number.]
7. We have defined the chemical potential as $\mu = (\partial F / \partial N)_{\tau, V}$. Prove that $\mu = (\partial U / \partial N)_{\sigma, V}$ is an equivalent definition.
8. Consider an orbital with an energy δ above the Fermi level, $\varepsilon = \mu + \delta$. Prove that the probability that this level is occupied is equal to the probability that an orbital with an energy δ below the Fermi level is unoccupied.