171.312 Statistical Physics and Thermodynamics
Homework due November 23

1. A cylinder with a piston contains 1 kg of argon gas at a temperature of 300 K and a pressure of 1 bar. Initially the gas occupies a volume of 0.73 $m^3$. The temperature of the gas is raised to 500 K with the addition of 120 kJ of heat while the pressure in the cylinder remains constant. The volume of the gas is 1.73 $m^3$ after the reversible expansion.

(a) Calculate the work associated with the process.
(b) Draw a p-V diagram that schematically shows the work calculation.
(c) Is the work done on the gas positive or negative? Why?
(d) Calculate the change in energy, $\Delta U$.
(e) Calculate the change of enthalpy, $\Delta H$.

2. A <u>Carnot cycle</u> removes 10 kW of heat from a 200 K thermal reservoir and exhausts heat to a 300 K thermal reservoir.

a) Calculate the coefficient of performance of the Carnot refrigerator.
b) Calculate the rate of work, $dW / dt$ (in kW) required for by the Carnot refrigeration process.
c) Calculate the rate at which heat is exhausted to the hot (300 K) thermal reservoir.

A different <u>reversible device</u> removes 10 kW of heat from a 200 K thermal reservoir and exhausts 20 kW of heat to a 300 K thermal reservoir.

d) Calculate the rate of work, (in units of kW) required for this refrigeration process.
e) Calculate the coefficient of performance for this refrigerator.

A third <u>reversible device</u> removes 10 kW of heat from a 200 K thermal reservoir and exhausts 20 kW of heat to a 300 K thermal reservoir. However, this cycle does not produce or consume work. Instead, this cycle operates by using heat, $dQ_s / dt$, from a third thermal reservoir at temperature $T_s$.

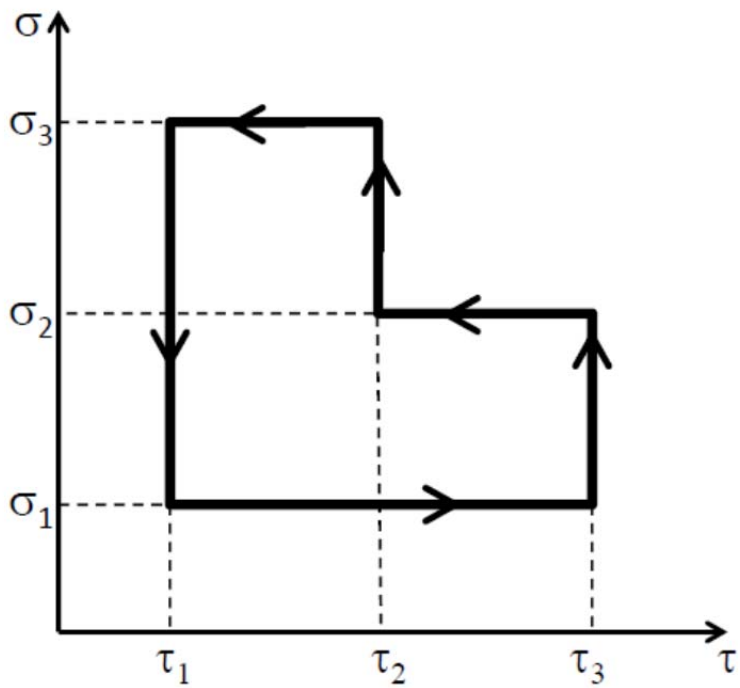
f) Calculate $dQ_s / dt$ (in units of kW) required for this second refrigeration process.
g) Find $T_s$ in units of K.

3. A power plant is to be constructed at the location of an enormous (2 km by 3 km) lava field that contains porous rocks down to a depth of 5 kg. The rocks have a total mass of $M = 10^{14} kg$ and are initially hot ($T_i = 600\, C$). The objective is to generate electricity by flooding the rock field with water and using the resulting steam to run a turbine by use of a Carnot cycle process. The rocks will cool and their heat loss is given by $dQ_h = -MCdT_h$ where $C = 1\ \mathrm{J\ g^{-1}K^{-1}}$ is the temperature-independent specific heat of the rocks. This power plant will shut down when the rocks reach 110 C. A nearby cold river that is fed by high elevation ice melt is used as the lower reservoir temperature, with $T_l = 20C$. Assume that this temperature remains constant.

(a) If the power plant operates at the Carnot limit, what is the total amount of electrical energy (W) available from the rocks? [Show your work, explain the steps, and include units.]
(b) The total amount of energy generated in the world was $\sim 10^{14}$ kWh in 1976. How does your result compare?

4. The reaction $2Ag_2S + 2H_2O \rightarrow 4Ag + 2H_2S + O_2$ has a change of enthalpy of $\Delta$H=+595.5 kJ. What is the change of enthalpy of the reaction $Ag + \frac{1}{2}H_2S + \frac{1}{4}O_2 \rightarrow \frac{1}{2}Ag_2S + \frac{1}{2}H_2O$ ?

5. A heat engine follows the following cycle with respect to entropy and temperature:




Compare the efficiency of this heat engine with a Carnot cycle engine that operates between entropy $\sigma_1$ and $\sigma_2$ and temperature $\tau_1$ and $\tau_2$.