171.312 STATISTICAL PHYSICS AND THERMODYNAMICS

MID-TERM EXAM

Friday, October 21, 2011

1. [10 points] State the "Fundamental Assumption" of statistical and thermal physics.

2. [10 points] A freshly laid egg remains in contact with a heat reservoir (i.e., a hen) until it hatches. During this process, is heat absorbed or given off by the egg? Explain your answer.

3. A physical model of a rubber band is a chain of N independent segments, each of which can have one of two possible lengths, 0 or λ. If a segment has a length λ, then its energy is zero. If a segment has zero length, then its energy is ε. The rubber band is in thermal contact with the atmosphere at temperature T.

(a) [5 points] Find the average length, L, of the rubber band, in terms of N, T, λ, and ε.

(b) [5 points] As the temperature is increased, does the average length increase or decrease?

(c) [5 points] What is L in the low temperature limit ($kT \ll \varepsilon$)?

(d) [5 points] What is L in the low temperature limit ($kT \gg \varepsilon$)?

4. Two identical systems S_1 and S_2 are both in thermal contact with a large reservoir and in diffusive contact with one another. For both systems, the free energy, F, is related to the particle number in the same manner: $F_1 = cN_1^2$ and $F_2 = cN_2^2$ where c is a constant (and the same constants for both identical systems).

(a) [15 points] A battery is now put in place that maintains a chemical potential difference

$$\Delta = \mu_{2,\text{ext}} - \mu_{1,\text{ext}} > 0$$

between the two systems. In diffusive equilibrium, find the number N_1 of particles in S_1 and the number N_2 of particles in S_2, expressed in terms of Δ, c, and the total particle number $N = N_1 + N_2$.

(b) [10 points] Now the battery is disconnected, and useful work is extracted isothermally as the particles flow slowly from S_1 to S_2 until diffusive equilibrium is reestablished. How much work is extracted?

5. A container of N atoms of type A low density gas at temperature T and volume V is in thermal equilibrium with an identical container of N atoms of type B of low density gas at temperature T and identical volume V. A constraining wall is removed between the two containers to allow particle diffusion.

(a) [10 points] After equilibrium is achieved, what is the total change of entropy?

(b) [10 points] What would the total change of entropy be if the type A= type B?

6. The following questions relate to general properties of a Fermi gas.

(a) [5 points] If the density of a Fermi gas is raised by a factor of a thousand, by what factor does the Fermi energy, ε_F, increase or decrease?

(b) [5 points] If the temperature of a Fermi gas is raised by a factor of two, by what factor does the Fermi energy, ε_F, increase or decrease?

(c) [5 points] Sketch a qualitative graph that illustrates the general behavior of $f(\varepsilon)$ vs. ε for a Fermi gas. Include a curve for absolute zero temperature, and two slightly higher temperatures to indicate the nature of the trend.