171.312 STATISTICAL PHYSICS AND THERMODYNAMICS

FINAL EXAM

Monday December 12, 2011

[18 points]

1) Provide brief answers to all of the following questions:

(a) What is the "Fundamental Assumption"?

(b) What is the 0th law of thermodynamics?

(c) What is the 1st law of thermodynamics?

(d) What is the 2nd law of thermodynamics?

(e) What is the 3rd law of thermodynamics?

(f) Write an expression for the "quantum concentration" and state its relevance for ideal and degenerate gases.

(g) What choice of reservoir material maximizes the Carnot efficiency of a heat engine?

(h) For a three dimensional solid with N atoms, what is the maximum number of phonon modes?

(i) For a cube of volume $V = L^3$, what is the maximum number of electromagnetic modes?

[10 points]

2) The following question involves tosses of a fair coin with a 50% chance of heads and a 50% chance of tails.

(a) If a coin is flipped three times, what is the probability that heads will come up 2/3 of the coin tosses and tails 1/3 of the coin flips?

(b) For 6 flips, what is the probability that the coin will come up 2/3 heads and 1/3 tails?

(c) For N=10^22 flips what is the probability of 2/3 heads and 1/3 tails? [Please provide only a very rough approximation here! No detailed calculations, please!]

(d) What is the implication of (a) through (c) for equilibrium and the determination of the multiplicity of large thermodynamic systems?

[5 points]

3) A system at temperature T includes two particular states, **A** and **B**. The energy of state **A** is ε_A and the energy of state **B** is $\varepsilon_B = \varepsilon_A + \delta\varepsilon$. What is the relative probability of the system being in state **A** relative to being in state **B**?

[10 points]

4) Consider a system of $N \gg 1$ non-interacting particles in which the energy of each particle can have two (and only two) distinct values: 0 and $E\ (E > 0)$. The occupation numbers of the zero and E energy levels are n_0 and n_1 respectively. The fixed total energy of the system is U.

(a) Find the entropy of the system.

(b) Find the temperature as a function of U.

(c) For what range of values of n_0 is $\tau < 0$?

[7 points]

5) Two separate containers, P and Q, contain different kinds of degenerate gases. Both of the different types of degenerate gases are in their ground state. The kinetic energy of the degenerate gas in P is much higher than that of Q.

(a) What kind of degenerate gas is P and what kind is Q? Explain.

(b) Which will have the higher thermal conductivity?

(c) One of the two gases dramatically changes its behavior below 2.17 K. Name this gas.

[10 points]

6) The entropy of a substance is σ_l when in its liquid phase and σ_g when in its gas phase, both for temperature τ and constant pressure, p.

(a) What is the heat of vaporization of the substance?

(b) If the enthalpy of the gas is H_g, provide an expression for the enthalpy of the liquid, H_l.

(c) What is the Gibbs function of the liquid, G_l?

(d) What is the Gibbs function of the gas, G_g?

(e)Prove that the Gibbs function does not change in a reversible isothermal and isobaric process.

[10 points]

7) On a single graph, sketch the distribution function $f(\varepsilon)$ for a degenerate Bose gas, a degenerate Fermi gas, and a classical gas, all as a function of $(\varepsilon - \mu)/\tau$ (which can be positive or negative). Assume that the temperature is somewhat above absolute zero. Label all important features.

[15 points]

8) Consider a gas of N noninteracting, spin ½ fermions of mass M, initially in a box of volume V_i at temperature $\tau_i = 0$. Let the gas expand irreversibly into a vacuum, without doing work, to a final (thermally isolated) volume V_f.

(a) What is the average energy per particle <u>before</u> the expansion $\langle \varepsilon_i \rangle / N$ as a function of the initial density N/V_i?

(b) What is the average energy per particle <u>after</u> the expansion $\langle \varepsilon_f \rangle / N$ as a function of the final temperature τ_f, assuming that the gas expanded enough to be an ideal gas after the expansion?

(c) Is energy conserved? I.e., does $\langle \varepsilon_i \rangle = \langle \varepsilon_f \rangle$?

(d) What is the final temperature, τ_f, of the gas after expansion as a function of mass M and <u>initial</u> density N/V_i?

(e) What range of final temperature of the gas τ_f after expansion, as a function of mass M and <u>final</u> density N/V_f, is consistent with the assumption that the gas expanded enough to be treated as an ideal gas after the expansion?

(f) What is the minimum final volume, V_f?

[15 points]

9) A simple theory of the thermodynamics of a ferromagnet expresses the free energy, F, as a function of the magnetization, M in the following form:

$$F = -HM + F_0 + a(T - T_c)M^2 + bM^4$$

where H is the magnetic field, and F_0, a and b are all positive constants, T is temperature, and T_c is the critical temperature.

(a) Explain what condition on the free energy F determines the thermodynamically most probable value for the magnetization M in equilibrium?

(b) Determine the equilibrium value for M assuming that $T - T_c \gg 2bM^2 / a > 0$

(c) Sketch a graph of M versus T from (b)

(d) Instead of the assumption in part (b), determine the equilibrium value for M for the assumption that $H = 0$. Explicitly check that your equilibrium solution(s) is/are stable.