(1)

Final

$\frac{18}{18}$

1) a) The fundamental assumption of thermal physics is that a closed system is equally likely to be in any of the quantum states accessible to it.

2

0th law

b) If two systems are in thermal equilibrium with a third system, they must be in thermal equilibrium with each other.

2

1st law

c) Heat is a form of energy. Energy is conserved $dU = đQ + đW$.

2

2nd law

d) If a closed system is in a configuration that is not the equilibrium configuration, the most probable consequence will be that the entropy of the system will increase monotonically in successive instants of time.

2

3rd law

e) The entropy of a system approaches a constant value as the temperature approaches zero

2

f) $$n_Q = \left(\frac{M\tau}{2\pi\hbar^2}\right)^{\frac{3}{2}}$$

2

Fermion and boson gases behave like a classical gas if

$$n \ll n_Q$$

2 g) The Carnot efficiency of a heat engine depend only on the temperatures and not on the material.

2 h) $3N$

2 i) Infinite number of mode

2) 2 a) $P_3\left(h=\frac{2}{3}\right) = C_3^1 \frac{1}{2}^2 \frac{1}{2} = \frac{3!}{2!} \frac{1}{2^3} = \frac{3}{8}$

2 b) $P_6\left(h=\frac{2}{3}\right) = C_6^2 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2 = \frac{6!}{2!\,4!} \frac{1}{2^6} = \frac{6\times 5}{2} \frac{1}{2^6} = \frac{15}{64}$

$\frac{10}{10}$

3 c) $P_N\left(h=\frac{2}{3}\right) = 0$

3 d) At equilibrium for large N, the system will be in the most probable configuration. The multiplicity function will approach a δ-function around the most probable configuration

3) $\frac{5}{5}$

$$\frac{P_A}{P_B} = \frac{e^{-\beta \varepsilon_A}}{e^{-\beta \varepsilon_B}} = e^{-\beta(\varepsilon_A - \varepsilon_A - \delta\varepsilon)} = \underline{e^{\beta\delta\varepsilon}}$$

4) a) $S = k_B \ln \Omega$ where $\Omega = \frac{N!}{n_0!\, n_1!}$ $\quad N_A + N_B = N$

3

$$\ln \Omega \simeq N\ln N - \cancel{N} - n_0 \ln n_0 + \cancel{n_0} - n_1 \ln n_1 + \cancel{n_1}$$

$\frac{10}{10}$

$U = n_1 E$

$\Rightarrow n_1 = \frac{U}{E}$

$\Rightarrow n_0 = N - \frac{U}{E}$

$$\Rightarrow \ln \Omega = N\ln N - \left(N - \frac{U}{E}\right)\ln\left(N - \frac{U}{E}\right) - \frac{U}{E}\ln\frac{U}{E}$$

②

b) we have $\frac{1}{T} = \left.\frac{\partial S}{\partial U}\right|_N$

$$\Rightarrow \frac{1}{k_B}\left.\frac{\partial S}{\partial U}\right|_N = \left[\left(-\frac{1}{\epsilon}\right)\ln\left(N-\frac{U}{\epsilon}\right) + \cancel{N-\frac{U}{\epsilon}}\,\frac{\frac{1}{\epsilon}}{\cancel{N-\frac{U}{\epsilon}}}\right] - \left[\frac{1}{\epsilon}\ln\frac{U}{\epsilon} + \frac{\cancel{U}}{\epsilon}\frac{1}{\cancel{U}}\right]$$

$$= +\frac{1}{\epsilon}\ln\left(N-\frac{U}{\epsilon}\right) - \frac{1}{\epsilon}\ln\frac{U}{\epsilon} = \frac{1}{\epsilon}\ln\left(\frac{N\epsilon}{U}-1\right)$$

$$\Rightarrow \boxed{T = \frac{\epsilon}{k_B\ln\left(\frac{N\epsilon}{U}-1\right)}}$$

c) $$T = \frac{\epsilon}{k_B\ln\left(\frac{N\epsilon}{U}-1\right)} = \frac{\epsilon}{k_B\ln\left[\frac{\epsilon}{U}\left(N-\frac{U}{\epsilon}\right)\right]} = \frac{\epsilon}{k_B\ln\left(\frac{n_0}{n_1}\right)} = \frac{\epsilon}{k_B\ln\left(\frac{n_0}{N-n_0}\right)}$$

$\exists\; T<0$ if $\frac{n_0}{N-n_0} < 1$

$$\Rightarrow n_0 < N - n_0$$

$$\Rightarrow \boxed{n_0 < \frac{N}{2}}$$

5) a) Fermi gas can only have one particle per state therefore the energy levels are filled up to high energy level and the average kinetic energy becomes very high even in the ground state. The bose gas in the ground state has a very low kinetic energy as all the particles get in the ground state.

3

7/7

P : Fermi gas

Q : Bose gas

b) ~~In the ground state the bose gas will have an infinite thermal conductivity.~~

2

2 c) the gas Q bose condensates. At $T_c = 2.17K$ Q is ^4He

6) 2 a) $L = \tau(\sigma_g - \sigma_\ell)$

10/10

2 b) $L = H_g - H_\ell \Rightarrow H_\ell = H_g - \tau(\sigma_g - \sigma_\ell)$

2 c) we have $G = U - TS + PV = H - TS$

$$\Rightarrow G_\ell = H_\ell - \tau\sigma_\ell = H_g - \tau(\sigma_g - \cancel{\sigma_\ell}) - \cancel{\tau\sigma_\ell}$$
$$= H_g - \tau\sigma_g$$

2 d) $G_g = H_g - \tau\sigma_g \Rightarrow G_g = G_\ell$

2 e) $G = U - TS + PV$

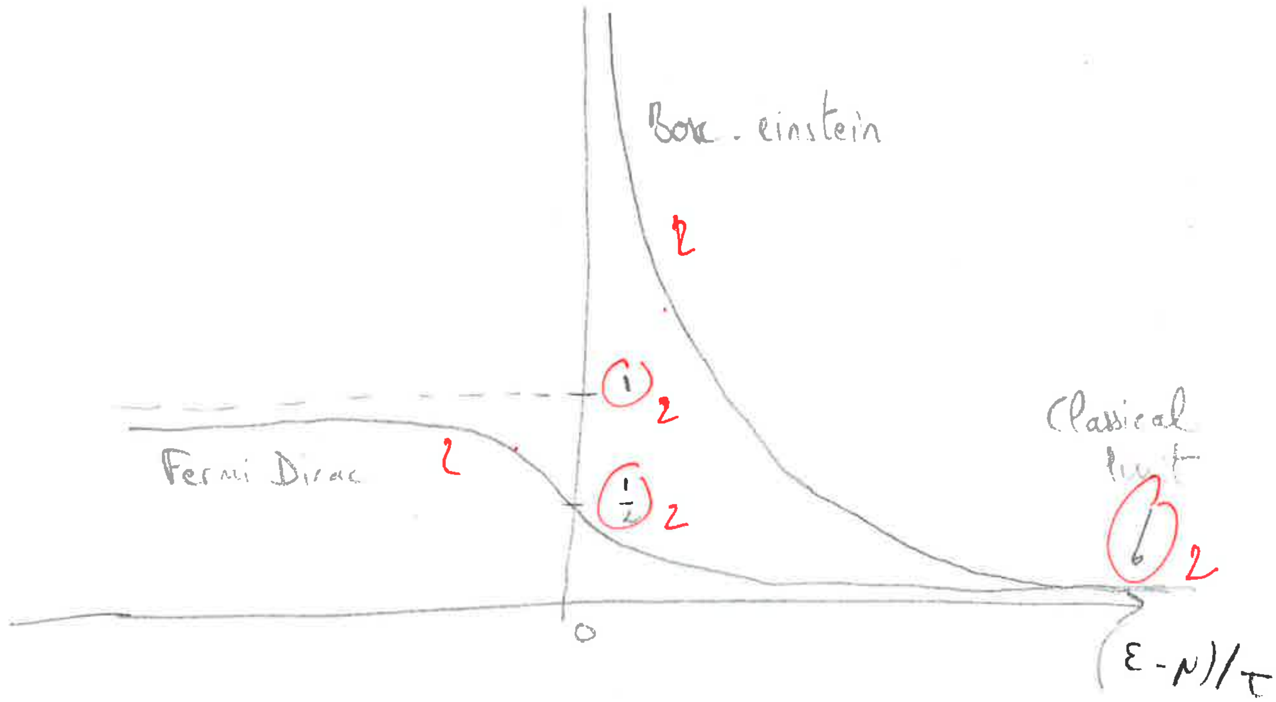
$$\Rightarrow dG = \cancel{TdS} - \cancel{PdV} - \cancel{TdS} - SdT + \cancel{PdV} + VdP$$

$$\Rightarrow dG = -SdT + VdP$$ therefore G is only function of T and P

Thus $\Delta G = 0$ if $\Delta T = \Delta P = 0$

③

7)



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8) a) $\langle \varepsilon_i \rangle = \int_0^\infty \varepsilon f(\varepsilon) D(\varepsilon)\, d\varepsilon$

3

15/15

$D(k)\,dk = \dfrac{2\, 4\pi k^2\, dk}{\left(\frac{2\pi}{L}\right)^3}$ $\quad \varepsilon = \dfrac{k^2\hbar^2}{2m} \Rightarrow k^2 = \dfrac{2m\varepsilon}{\hbar^2}$

$dk = \left(\dfrac{2m}{\hbar^2}\right)^{\frac{1}{2}} \dfrac{1}{2\sqrt{\varepsilon}}\, d\varepsilon$

$\Rightarrow D(\varepsilon)\,d\varepsilon = V\left(\dfrac{2m}{\hbar^2}\right)^{\frac{3}{2}} \dfrac{\sqrt{\varepsilon}}{2\pi^2}\, d\varepsilon$

$\Rightarrow \langle \varepsilon_i \rangle = \dfrac{V}{2\pi^2}\left(\dfrac{2m}{\hbar^2}\right)^{\frac{3}{2}} \int_0^{\varepsilon_F} \varepsilon^{\frac{3}{2}}\, d\varepsilon = \dfrac{V}{2\pi^2}\left(\dfrac{2m}{\hbar^2}\right)^{\frac{3}{2}} \dfrac{2}{5}\varepsilon_F^{\frac{5}{2}}$

we have also $N = \dfrac{V}{2\pi^2}\left(\dfrac{2m}{\hbar^2}\right)^{\frac{3}{2}} \int_0^{\varepsilon_F} \varepsilon^{\frac{1}{2}}\, d\varepsilon = \dfrac{V}{2\pi^2}\left(\dfrac{2m}{\hbar^2}\right)^{\frac{3}{2}} \dfrac{2}{3}\varepsilon_F^{\frac{3}{2}}$

$\Rightarrow \quad \varepsilon_F = \left(\dfrac{3}{2}\dfrac{N}{A}\right)^{\frac{2}{3}} \Rightarrow \langle \varepsilon_i \rangle = A\dfrac{2}{5}\left(\dfrac{3}{2}\dfrac{N}{A}\right)^{\frac{5}{3}}$

$$\Rightarrow \langle \varepsilon_i \rangle = \frac{3}{5} N \left(\frac{3}{2}\right)^{\frac{5}{3}} \left(\frac{N}{A}\right)^{\frac{2}{3}}$$

$$\Rightarrow \frac{\langle \varepsilon_i \rangle}{N} = \frac{2}{5}\,\frac{3}{2}\left(\frac{3}{2}2\pi^2\left(\frac{\hbar^2}{2m}\right)^{\frac{3}{2}} \frac{N}{V_i}\right)^{\frac{2}{3}}$$

$$\Rightarrow \frac{\langle \varepsilon_i \rangle}{N} = \frac{3}{5}\,\frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{V_i}\right)^{\frac{2}{3}} = \frac{3}{5}\varepsilon_F$$

b) If Ideal gas $\langle \varepsilon_f \rangle = \frac{3}{2} N \tau_f$

3

$$\Rightarrow \frac{\langle \varepsilon_f \rangle}{N} = \frac{3}{2}\tau_f$$

c) we have no work $\Rightarrow W = 0$

2 Thermally isolated $\Rightarrow Q = 0$ $\Big\} \Rightarrow \Delta U = 0$

Energy conserved

d) we have $\frac{\langle \varepsilon_i \rangle}{N} = \frac{\langle \varepsilon_f \rangle}{N}$

2

$$\Rightarrow \frac{3}{2}\tau_f = \frac{3}{5}\,\frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{V_i}\right)^{\frac{2}{3}}$$

$$\Rightarrow \boxed{\tau_f = \frac{2}{5}\,\frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{V_i}\right)^{\frac{2}{3}}}$$

(4)

e) for the gas to be considered classical we have

3 $$\frac{N}{V_f} \ll \left(\frac{M\tau_f}{2\pi\hbar^2} \right)^{3/2}$$

$$\Rightarrow \left(\frac{N}{V_f} \right)^{2/3} \frac{2\pi\hbar^2}{M} \ll \tau_f$$

f) $$\Rightarrow \left(\frac{\cancel{N}}{V_f} \right)^{2/3} \frac{2\pi\cancel{\hbar^2}}{\cancel{m}} \ll \frac{2}{5} \frac{\cancel{\hbar^2}}{2\cancel{m}} \left(3\pi^2 \frac{\cancel{N}}{V_i} \right)^{2/3}$$

3/2 $$\Rightarrow \frac{1}{V_f} \ll \left(\frac{1}{5\pi} \right)^{3/2} \frac{3\pi^2}{V_i}$$

$$\Rightarrow V_i \ll 3\pi^2 \left(\frac{1}{5\pi} \right)^{3/2} V_f$$

$$\Rightarrow V_f \gg \frac{V_i}{3\pi^2} (5\pi)^{3/2}$$

9) a) $F = -HM + F_0 + a(T - T_c)M^2 + bM^4$

3 $\quad \dfrac{\partial F}{\partial M} = 0$

b) $\dfrac{\partial F}{\partial M} = -H + 2a(T-T_c)M + 4bM^3 = -H + 2a(T-T_c)M\left(1 + \dfrac{4bM^3}{2a(T-T_c)M}\right)$

4

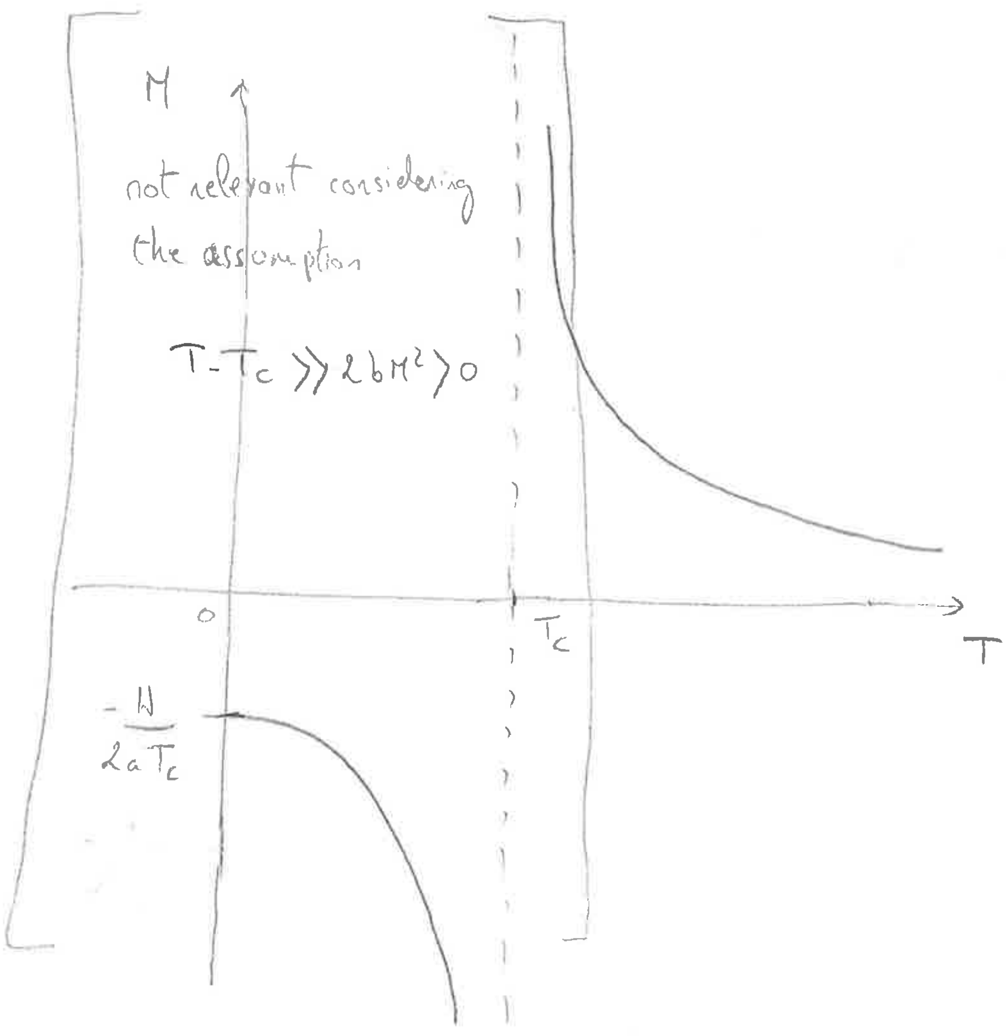
$$= -H + 2a(T-T_c)M\left(1 + \frac{2bM^2}{a(T-T_c)}\right)$$

$\frac{15}{15}$ with $T - T_c \gg \dfrac{2bM^2}{a} \Rightarrow \dfrac{\partial F}{\partial M} \simeq -H + 2a(T-T_c)M = 0$

$$\Rightarrow M = \frac{H}{2a(T-T_c)}$$

c)

3



5

d) $H=0$

$$\Rightarrow \frac{\partial F}{\partial M} = 2a(T-T_c)M + 4bM^3 = 0$$

3 solutions $\quad M=0$ and $\quad M = \pm\sqrt{\frac{a}{2b}(T_c - T)}$ $\quad T_c \geq T$

1 $\qquad$ 2

$$\frac{\partial^2 F}{\partial M^2} = 2a(T-T_c) + 12bM^2 \qquad \left.\frac{\partial^2 F}{\partial M^2}\right|_{M=M_s} \geq 0 \iff \text{stable solutions}$$

for $M=0$ $\quad \frac{\partial^2 F}{\partial M^2} = 2a(T-T_c) \geq 0$ for $T \geq T_c$ $\quad$ 1

for $M = \pm\sqrt{\frac{a}{2b}(T_c-T)}$ $\quad \frac{\partial^2 F}{\partial M^2} = 4b(T_c-T) \geq 0$ for $T \leq T_c$ $\quad$ 1