Solution Homework

1. A cylinder with a piston contains 1 kg of argon gas at a temperature of 300 K and a pressure of 1 bar. Initially the gas occupies a volume of 0.73 m^3. The temperature of the gas is raised to 500 K with the addition of 120 kJ of heat while the pressure in the cylinder remains constant. The volume of the gas is 1.73 m^3 after the reversible expansion.

(a) Calculate the work associated with the process.

$$W = -\int PdV = P(V_1 - V_2) = -100kJ$$

(b) Draw a $p - V$ diagram that schematically shows the work calculation.

Area under the curve.

(c) Is the work done on the gas positive or negative? Why?

The work done on the gas is negative because the work done by the gas is positive.

(d) Calculate the change in energy, ΔU.

(Notice that the constraint on the heat $Q = 120$ kJ doesn't allow the gas to be ideal but this is a mistake. The problem in this case just gave too many informations.) Using $Q = 120$ kJ

$$\Delta U = W + Q = 120 - 100 = 20kJ$$

Without the constraint

$$\Delta U = \frac{3}{2}Nk_bT = 62.42kJ$$

(e) Calculate the change of enthalpy, ΔH.

The pressure is kept constant therefore we have

$$\Delta H = \Delta U + P\Delta V = \Delta U - W = 120kJ$$

2. A Carnot cycle removes 10 kW of heat from a 200 K thermal reservoir and exhausts heat to a 300 K thermal reservoir.

a) Calculate the coefficient of performance of the Carnot refrigerator.

$$\gamma = \frac{T_l}{T_h - T_l} = \frac{200}{100} = 2$$

b) Calculate the rate of work, dW/dt (in kW) required for by the Carnot refrigeration process.

$$\frac{dW}{dt} = \frac{1}{\gamma}\frac{dQ_l}{dt} = \frac{10}{2} = 5kW$$

c) Calculate the rate at which heat is exhausted to the hot (300 K) thermal reservoir.

$$\frac{dQ_h}{dt} = \frac{T_h}{T_l}\frac{dQ_l}{dt} = 15kW$$

A different reversible device removes 10 kW of heat from a 200 K thermal reservoir and exhausts 20 kW of heat to a 300 K thermal reservoir.

d) Calculate the rate of work, (in units of kW) required for this refrigeration process.

$$\frac{dW}{dt} = \frac{dQ_h}{dt} - \frac{dQ_l}{dt} = 10kW$$

e) Calculate the coefficient of performance for this refrigerator.

$$\gamma = \frac{Q_l}{W} = 1$$

A third reversible device removes 10 kW of heat from a 200 K thermal reservoir and exhausts 20 kW of heat to a 300 K thermal reservoir. However, this cycle does not produce or consume work. Instead, this cycle operates by using heat, dQ_s/dt, from a third thermal reservoir at temperature T_s.

f) Calculate dQ_s/dt (in units of kW) required for this second refrigeration process.

$$\frac{dQ_s}{dt} = \frac{dQ_h}{dt} - \frac{dQ_l}{dt} = 10kW$$

g) Find T_s in units of K.

Since the process is reversible we have $S_s + S_l = S_h$ or $\frac{Q_s}{T_s} + \frac{Q_l}{T_l} = \frac{Q_h}{T_h}$. We have

$$T_s = \frac{Q_s}{\frac{Q_h}{T_h} - \frac{Q_l}{T_l}} = 600K$$

3. A power plant is to be constructed at the location of an enormous (2 km by 3 km) lava field that contains porous rocks down to a depth of 5 km. The rocks have a total mass of $M = 10^{14}$

kg and are initially hot($T_i = 600$ C). The objective is to generate electricity by flooding the rock field with water and using the resulting steam to run a turbine by use of a Carnot cycle process. The rocks will cool and their heat loss is given by $dQ_h = -MCdT_h$ where $C = 1$ J g^{-1}K^{-1} is the temperature-independent specific heat of the rocks. This power plant will shut down when the rocks reach 110 C. A nearby cold river that is fed by high elevation ice melt is used as the lower reservoir temperature, with $T_l = 20$ C . Assume that this temperature remains constant.

(a) If the power plant operates at the Carnot limit, what is the total amount of electrical energy (J) available from the rocks? [Show your work, explain the steps, and include units.]

We have $\eta = \frac{\partial W}{\partial Q_h}$ therefore

$$W = \int_{T_i}^{T_f} \eta dQ_h = -MC(\int_{T_i}^{T_f} dT_h - T_l \int_{T_i}^{T_f} \frac{dT_h}{T_h}) = MC(T_i - T_f - T_l \ln(\frac{T_i}{T_f})) \simeq 2.5 \times 10^{19} J$$

(b) The total amount of energy generated in the world was $\sim 10^{14}$ kWh in 1976. How does your result compare?

$1kWh = 3.6 \times 10^6 J$ and in 1976 it was $\sim 3.6 \times 10^{20} J$, a bit more than ten times more than now.

4.The reaction $2Ag_2S + 2H_2O \rightarrow 4Ag + 2H_2S + O_2$ has a change of enthalpy of $\Delta H = +595.5$ kJ. What is the change of enthalpy of the reaction $Ag + \frac{1}{2}H_2S + \frac{1}{4}O_2 \rightarrow \frac{1}{2}Ag_2S + \frac{1}{2}H_2O$?

This reverse reaction with 1/4 of the elements has a a change of enthalpy of $\Delta H' = -\frac{1}{4}\Delta H = -148.875$ kJ.

5. A heat engine follows the following cycle with respect to entropy and temperature (see homework):

Compare the efficiency of this heat engine with a Carnot cycle engine that operates between entropy σ_1 and σ_2 and temperature τ_1 and τ_2.

The efficiency is $\eta = \frac{W}{Q_h}$. The work is the enclosed surface area:

$$W = (\sigma_3 - \sigma_1)(\tau_2 - \tau_1) + (\sigma_2 - \sigma_1)(\tau_3 - \tau_2)$$

and

$$Q_h = \tau_3(\sigma_2 - \sigma_1) + \tau_2(\sigma_3 - \sigma_2)$$

We have

$$\eta = 1 - \frac{\tau_1(\sigma_3 - \sigma_1) + \tau_2(\sigma_2 - \sigma_1)}{\tau_3(\sigma_2 - \sigma_1) + \tau_2(\sigma_3 - \sigma_2)}$$