Physics 171.313 Introduction to Stellar Physics Fall 2008

Problem Set 3
(due October 16)

1. The pressure inside a normal star is given by:

$$P = P_g + P_{rad} = \frac{\rho kT}{\mu m_H} + \frac{1}{3}aT^4.$$

Using parameters appropriate to the Sun, show that throughout the Sun, including the core, where the internal temperature is 1.5×10^7 K, the kinetic pressure dominates.

2. Consider a hypothetical star of mass $M = 10M_\odot$, composed entirely of fully ionized ^{12}C. Its core temperature is $T_c = 6 \times 10^8$ K.

   (a) What is the mean particle mass, μ, in units of m_H?

   (b) Use the classical ideal gas law, the dimensional relation between mass, density, and radius, and the virial theorem to find the *scaling* of the stellar radius, R, with total mass M, mean particle mass μ, and core temperature T_c. Using the values of these parameters for the Sun, derive the radius of the star.

   (c) If the luminosity of the star is $L = 10^7 L_\odot$, what is the effective surface temperature?

   (d) Suppose the star produces energy via the reaction ^{12}C + ^{12}C $\rightarrow$ ^{24}Mg. The atomic weight of ^{12}C is 12, and that of ^{24}Mg is 23.985. (The atomic weight of a nucleus is defined as the ratio of its mass to 1/12 the mass of a ^{12}C nucleus). What fraction of the star's mass can be converted into thermal energy?

3. The central density of the Sun is 1.5×10^5 kg m^{-3} and the Rosseland mean opacity at the center is 0.2 m^2 kg^{-1}.

   (a) What is the mean free path of a photon at the center of the Sun?

   (b) Assume that the opacity remains constant throughout the Sun (it does not). What is the average time that it would take for a photon to escape from the Sun?

4. Estimate the Eddington luminosity of a 0.072 $M_\odot$ star. Assume the opacity $\kappa = 0.001$ m^2 kg^{-1}. From the observational mass-luminosity relationship, what is the star's actual luminosity? Do you expect radiation pressure to be a significant factor in the stability of a low mass main sequence star?

(more)

5. Recall the hypothetical star of radius R, with constant density ρ (i.e., independent of radius), that we examined in Problem 4 of the first problem set. The star is composed of a classical, non-relativistic, ideal gas of fully ionized hydrogen.

   Assume that the nuclear energy production rate depends on temperature as $q \propto T^4$ (this is the approximate dependence of the rate for the p-p chain at the temperature in the core of the Sun). At what radius does q decrease to 0.1 of its central value, and what fraction of the star's mass is included within this radius?