Problem Set 5
(due November 25)

1. Most of the energy released in the collapse of a massive star to a neutron star (a core-collapse supernova) is in the form of neutrinos.

   (a) If the just-formed neutron star has a mass $M = 1.4M_\odot$ and a radius $R = 10$ km, estimate the mean nucleon density, in m^{-3}. Find the mean free path, in meters, of a neutrino inside the neutron star, assuming that the cross section for the scattering of neutrinos on neutrons is $\sigma = 10^{-46}$ m^2.

   (b) How many seconds does it take a typical neutrino to emerge from the neutron star in a random walk? Assume that neutrinos travel at a velocity close to the velocity of light.

2. The white dwarf Sirius B has a mass $M \approx 1.0M_\odot$ and a radius $R = 6000$ km. Estimate the ideal gas pressure and the radiation pressure at the center of this star assuming that its core is all ^{12}C with a central temperature of $T_c = 3 \times 10^7$ K. Compare these values with the central pressure needed to support the star.

3. Consider a neutron star of mass $1.4M_\odot$, whose central density is 1.5×10^{18} kg m^{-3}. Show that the degenerate neutrons are non-relativistic and estimate the degeneracy pressure at the center. Compare this to the estimated pressure at the center of Sirius B (previous problem).

4. What is the minimum rotation period for a $1.4M_\odot$ neutron star (the fastest that it can spin without flying apart)? Assume that it remains spherical with a radius of 10 km.

5. During an observed glitch, the period of the Crab pulsar decreased by $|\Delta P| \approx 10^{-8}P$. If the increased rotation was due to an overall contraction of the neutron star, find the change in the star's radius. Assume that the pulsar is a rotating sphere of uniform density with an initial radius of 10 km.

6. Consider a pulsar that has a period P_0 and period derivative $\dot{P}_0$ ($\equiv dP_0/dt$) at $t = 0$. Assume that the product $P\dot{P}$ remains constant. Integrate to obtain an expression for the pulsar's period at time t.