Midterm Solutions

1a) The definition of the effective temperature of a star of luminosity, L, and radius, R, is given by

$$L = 4\pi R^2 \sigma_{SB} T_{eff}^4$$

ie star emits same flux as the black body with $T_{BB} = T_{eff}$.

$$\frac{L}{4\pi R^2} = \sigma_{SB} T_{eff}^4$$

b) Stars are not uniform in temperature, but are hotter in their interiors ie there is a temperature gradient. This gives rise to spectral lines; Black body radiation is continuous ie no lines. The stellar lines are in general absorption lines due to cooler material in the outer parts.

c) We are given the absolute magnitude of the Sun, and we know that its distance is 1AU.
We are given the distance modulus, which is the difference between the apparent and absolute magnitudes.
The absolute magnitude is the apparent

magnitude a star would have at a distance of 10pc.

$$(m - M)_0 = 5 \log \frac{d}{10pc}$$

m : apparent mag
M : absolute mag.

denotes corrected for dust extinction.

$$m = 5 \log \frac{d}{10pc} + M$$

Sun - Earth
$1AU = d = \frac{1pc}{206,265}$

$$m_{\odot,V} \doteq 5 \log \frac{1}{2\times10^6} + M_{\odot,V}$$

$\log_{10} 5 \doteq 0.7$.

$$= 5 \log (5\times10^{-7}) + M_{\odot,V}$$

$$\doteq 5 (-7 + 0.7) + M_{V,\odot}$$

$M_{V,\odot} = +4.83$

$$\doteq -31.5 + M_{V,\odot} = -31.5 + 4.83$$

$$\underline{m_{V,\odot} \doteq -26.67}$$

1 d) We have two stars: one with $L_V = 1 L_{V\odot}$, $L_B = 1 L_{B\odot}$, and another whose luminosity we need to work out.

We are told that $M_{V\star} = +3.3$, and we know $M_{V\odot} = +4.83$, $(B-V)_\odot = 0.65$, $(B-V)_\star = 0.41$

Using $M_1 - M_2 = -2.5 \log L_1/L_2$

we can derive $L_\star$: L_1, L_2 same units

$$M_\odot - M_\star = -2.5 \log L_\odot/L_\star$$

$L_{V\star}$: $4.83 - 3.3 = -2.5 \log 1L_\odot / L_\star L_\odot$

($L_\star$ in $L_\odot$).

ie $L_{V\star} = 10^{0.4(1.53)} L_{V,\odot}$

$= 4 L_{V\odot}$

Similarly, $M_{B\odot} = M_{V\odot} + (B-V)_\odot$

$= +4.83 + 0.65 = 5.48$

$M_{B\star} = 3.3 + 0.41 = 3.71$

$L_{B\star}$:

$5.48 - 3.71 = -2.5 \log L_{B\odot}/L_{\star,B}$

$\Rightarrow L_{B\star} = 10^{0.4(1.77)} L_{B\odot}$

$= 5.1 L_{B\odot}$.

$\Rightarrow$ Unresolved, so simply add: $L_{V,TOT} = 5 L_{V\odot}$

$L_{B,TOT} = 6.1 L_{B\odot}$.

2 (a) Ellipticals are elliptical in shape on the sky ie isophotes are elliptical. There is no disk. The starlight is smooth – no dust clouds to obscure parts. There are no bright star-forming regions, due to fact there is no cold gas. There are no spiral arms. Stars are old ~ 10 Gyr. Little angular momentum – not rotating.

b) The stars in ellipticals are old, so the brightest stars are those on the evolved red giant branch, which are typically K-stars. These brightest stars dominate the luminosity and hence the spectrum looks like that of a K giant. Note that the spectral lines will be much broader than in a single star, since there will be a range of line of sight speeds of stars within the galaxy (similar to the halo stars in Table 2.1 $\sigma_{1\text{-}D} \simeq 100\,km/s$).

c)(i) A stars have strong Hydrogen Balmer lines

(ii) A stars are more massive than the turn-off of a 10 Gyr population and have relatively short main sequence lifetimes. There must have been a recent star formation event.

3 a) Following §1.3.1 of the textbook, and homework problem 1.14,

$$I = \frac{\text{received flux}}{\text{angle squared.}} = \frac{\text{emitted energy/sec}}{\text{area}}$$

(equation 1.23)

b) The surface brightness $I \equiv \Sigma$ (excuse fact I used different symbol from textbook)

$$\Sigma = \Sigma_0 e^{-R/h_R} \quad \text{for a disk galaxy.}$$

This is luminosity per area eg $L_\odot/pc^2$

The total luminosity was evaluated in homework problem 2.8

$$L_D = \int_0^\infty \Sigma_0 e^{-R/h_R} 2\pi R \, dR$$

$$= 2\pi \Sigma_0 h_R^2$$ – this is first step of solution.

We want

$$L(<R_{\frac{1}{2}}) = \frac{L_D}{2} = \frac{2\pi \Sigma_0 h_R^2}{2} = \pi \Sigma_0 h_R^2$$

Therefore

$$L(<R_{\frac{1}{2}}) = \pi \Sigma_0 h_R^2 = \int_0^{R_{\frac{1}{2}}} 2\pi R \Sigma_0 e^{-R/h_R} dR$$

Solve for $R_{\frac{1}{2}}$

$$\Sigma_0 2\pi \int_0^{R_{\frac{1}{2}}} R e^{-R/h_R} dR = 2\pi \Sigma_0 \left\{ -h_R e^{-R/h_R} (R + h_R) \right\} \Big|_0^{R_{\frac{1}{2}}}$$

standard integral with $x = R$, $a = -\frac{1}{h_R}$

$$\Rightarrow \pi \Sigma_0 h_R^2 = 2\pi \Sigma_0 h_R \left\{ e^0 . h_R - e^{-R_{1/2}/h_R} (R + h_R) \right\}$$

$$= 2\pi \Sigma_0 h_R^2 \left\{ 1 - \frac{1}{h_R} e^{-R_{1/2}/h_R} (R + h_R) \right\}$$

ie $\frac{1}{2} = 1 - \frac{1}{h_R} e^{-R_{1/2}/h_R} (R_{\frac{1}{2}} + h_R)$

$$\Rightarrow \frac{1}{2} = \frac{1}{h_R} e^{-R_{1/2}/h_R} (R_{\frac{1}{2}} + h_R)$$

Set $X = \frac{R_{1/2}}{h_R}$ rhs $= e^{-X}(1 + X)$

Find X s.t. $0.5 = e^{-X}(1 + X)$

Trial and error:

$X = 1$: rhs $= 0.74$
$X = 2$ rhs $= 0.4$
$X = 1.5$ rhs $= 0.55$
$X = 1.7$ rhs $= 0.49$ $\leftarrow$ close to required 0.5

$\Rightarrow$ $\underline{R_{1/2} \simeq 1.7\, h_R}$. (1.67 is correct answer).

4 a) Use the equation given

$$V^2_{circ}(R) = \frac{GM(<R)}{R}$$

plus fact that enclosed mass is proportional to density x volume.

$$V^2_{circ}(R) \propto \frac{\rho(R)\, R^3}{R} \propto \rho R^2$$

If V^2_{circ} is constant $\underline{\rho \propto R^{-2}}$.

b(i) Supermassive black hole so massive and compact that can model as a point mass and ignore all other matter. Hence

$$M(<R) = M_{bh} \text{ for all } R$$
$\rightarrow$ constant.

For enclosed mass invariant as change radius,

$$V^2_{circ}(R) \propto \frac{M(<R)}{R}$$

$$\propto \frac{1}{R}$$

$$\Rightarrow V_{circ} \propto \frac{1}{\sqrt{R}} \propto R^{-1/2}.$$

This is what we observe and also for planets around the Sun, hence this is "Keplerian Motion".

(ii) Cannot have a mass distribution more concentrated than a point mass, hence cannot have V_{circ} decline faster than $R^{-1/2}$.

$R^{-2/3}$ is a faster decline than $R^{-1/2}$ and hence the tracer (gas, stars) cannot be on circular orbits