Solutions I.

1.1. Energy/time emitted by Sun $\equiv L_\odot$
Mass of Sun $\equiv M_\odot$
$L_\odot = 3.86 \times 10^{26}$ W $\quad M_\odot = 2 \times 10^{30}$ kg.

Hence energy/time/mass of Sun

$$\frac{L_\odot}{M_\odot} \simeq 2 \times 10^{-4} \text{ W kg}^{-1}$$

$$= \frac{2}{10{,}000} \times 1 \text{ W kg}^{-1}$$

Humans emit energy at rate 1 W kg^{-1} so the Sun produces $\sim \frac{1}{10{,}000}$ of rate of human per mass

Note it is rate of energy emitted/mass.

1.2. Surface temperature of stars may be approximated by that of a Black Body that emits at same rate per area. This is the ~~surface~~ effective temperature

$$L_* = 4\pi R^2 \sigma_{SB} T_{eff}^4 \qquad (1.3).$$

$$R^2 = \frac{L_*}{\sigma_{SB} T_{eff}^4} \cdot \frac{1}{4\pi}$$

$\sigma_{SB} \simeq 6 \times 10^{-8}$ MKS

Substitute in values ⇒

$$R_{\odot}^2 = \frac{4\times 10^{26}}{4\times\pi\times 6\times 10^{-8}\times (36)^2\times 10^{12}} \ m^2$$

$$\sim \frac{10^{26+8-12}}{3\times 6\times 15\times 100} \ m^2$$

$18\times 15 \simeq 270$
$\simeq \frac{1}{4}\times 10^3$

$$\sim 4\times 10^{-5}\times 10^{22} \ m^2$$

$$\sim 4\times 10^{17} \ m^2 \sim 40\times 10^{16} \ m^2$$

$$\Rightarrow \underline{R_{\odot} \sim 7\times 10^8 m.}$$

Gravitational acceleration at surface : $g = \frac{GM}{R^2}$

$$g \propto \frac{M}{R^2} \Rightarrow \frac{g_{\odot}}{g_E} \propto \frac{M_{\odot}}{M_E}\left(\frac{R_E}{R_{\odot}}\right)^2$$

Values given : $\frac{M_{\odot}}{M_E} \sim 3.3\times 10^5$ $\frac{R_E}{R_{\odot}} \sim 10^{-2}$

$$\Rightarrow \frac{g_{\odot}}{g_E} \sim 3.3\times 10^5\times 10^{-4} \simeq \underline{\underline{30}}.$$

(C)

1.4. Text says when convert Δm_H mass to Helium from Hydrgen release energy 0.007 $\Delta m_H c^2$. If ~~rate~~ convert Δm_H /sec $\Rightarrow$

$$\text{Energy/sec} = 0.007\,\Delta m_H c^2 = L_\odot$$

$$\Rightarrow \quad \Delta m_H = \frac{4\times 10^{26}}{0.007\times (3\times 10^8)^2}\ \text{kg s}^{-1}$$

$$= \frac{4\times 10^{26}}{7\times 10^{14}}\ \text{kg/s} \simeq \underline{5.8\times 10^{11}}\ \text{kg/s.}$$

~~If~~ For total mass $M_\odot$ can then survive $\frac{M_\odot}{\Delta m_H}$ seconds.

$$t_{\text{survive}} \sim \frac{2\times 10^{30}\ \text{kg}}{6\times 10^{11}\ \text{kg s}^{-1}} \simeq 3\times 10^{18}\ \text{s.}$$

$$1\text{y} = 3\times 10^7\ \text{s.}$$

$$\simeq 3\times 10^{11}\ \text{y.}$$

NB age of Universe $\simeq 1.4\times 10^{10}$ y.

Since cannot burn total mass, but only that material in core, get lifetime in phase H $\rightarrow$ He in core (main sequence) of $\sim \underline{3\times 10^{10}\ \text{yr}}$..

(d)

1.7. Each F5 dwarf star has absolute mag in V-band of + 3.3 (Table 1.4) and each K0III giant has $M_V = +0.7$ (Table 1.5).

$$M_{V\odot} = +4.83 \Rightarrow L_{F5*} = 10^{0.4(4.83-3.3)} L_\odot$$
$$= 4 L_\odot$$

(using $M_1 - M_2 = -2.5 \log L_1/L_2$)

$$L_{K0III} = 10^{0.4(4.83-0.7)} = 45 L_\odot$$

Thus a cluster of 200 F5 and 20 K0III has a total V-band luminosity of

$$(200 \times 4 + 20 \times 45) L_{\odot,V} = 800 L_{\odot,V} + 900 L_{\odot,V}$$
$$= 1700 L_{\odot,V}$$

(note $\simeq$ equal contributions from F* and K*)

$$M_V = -2.5 \log L/L_\odot + M_{V\odot}$$

$$= -8 + 4.83 = -3.246 \simeq -3.25 \checkmark$$

(c)

$$M_{B,F\star} = M_{V,F\star} + (B-V)_{F\star}$$

$$= 3.3 + 0.41 = 3.71$$

$$M_{B,K\star} = 0.7 + 1.02 = 1.72$$

$$M_{B,\odot} = 4.83 + 0.65 = 5.48$$

Therefore

$$\frac{L_{B F\star}}{L_{\odot,B}} = \frac{L_{V,F\star}}{L_{V\odot}} \times 10^{-0.4(0.41-0.65)}$$

$$= 10^{-0.4(3.71-5.48)}$$

$$= \underline{5.1} \quad \text{(either way)}$$

Similarly

$$\frac{L_{K\star,B}}{L_{\odot,B}} = 10^{-0.4(1.72-5.48)}$$

$$= \underline{32}$$

Combine to form cluster $\Rightarrow$

$$\frac{L_{B,\text{cluster}}}{L_{B\odot}} = 200 \times 5.1 + 20 \times 32 = 1660$$

ie $(B-V)_{\text{cluster}} = -2.5 \log\left(\frac{L_{B,\text{cluster}}}{L_{V,\text{cluster}}}\right)$

$$= -2.5 \log\left\{\frac{L_{B,\text{cluster}}}{L_{B\odot}} \cdot \frac{L_{V\odot}}{L_{V,\text{cluster}}} \cdot \frac{L_{B\odot}}{L_{V\odot}}\right\}$$

$$= -2.5 \log\left\{\frac{L_{B,\text{cluster}}}{L_{B\odot}} \cdot \frac{L_{V\odot}}{L_{V,\text{cluster}}}\right\} \underbrace{-2.5 \log \frac{L_{B\odot}}{L_{V\odot}}}_{(B-V)_\odot}$$

$$\Rightarrow (B-V)_{\text{cluster}} = -2.5 \log \frac{1660}{1700} + (B-V)_\odot$$

$$= 0.026 + 0.65$$

$$= 0.676 \simeq \underline{0.68} \quad \text{qed.}$$

(g)

1.14. Use definition of distance modulus, ie relation between apparent & absolute magnitudes.

$$m - M = 5 \log \frac{r}{10pc} \qquad r = \text{distance}$$

We have $\quad r = d\,\text{Mpc} = d \times 10^6 \text{pc}$

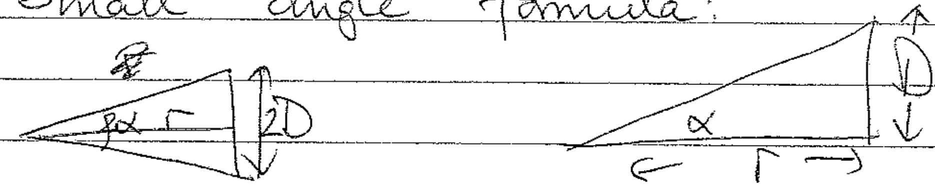
The Sun has abs. mag in B-band $M_{B\odot} = 5.43$, so in this galaxy:

$$\Rightarrow \quad m = 5 \log(10^5 d) + 5.43.$$

$\log(ab) = \log a + \log b$

$$m = 25 + 5\log(d) + 5.43$$
$$= 30.43 + 5\log(d).$$

Small angle formula:



See Fig 2.1. $\qquad \frac{D}{r} = \alpha$ in radians.

2π radians $= 360^\circ$
$= 360 \times 60 \times 60''$

$$1'' = \frac{\pi}{648{,}000} \text{ radians}$$

(h)

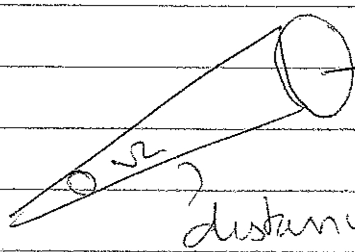
Thus length that subtends $1''$ at a distance of d Mpc is

$$D = 1'' \times \text{distance}$$

$$= \frac{1}{206{,}265} \times 10^6 \, d \, \text{pc}$$

$$= 4.848 \, \text{pc} \times d \simeq 5 \, \text{pc} \times d. \quad \underline{\text{qed.}}$$

For area need to look at solid angle ⇒ ~~[illegible] 360°~~ → need <u>steradians</u>.



Solid angle $\Omega = \dfrac{\text{area}}{\text{distance}^2}$

4π steradians is all sky.

$1 \text{ radian} = \dfrac{180^\circ}{\pi}$ $\quad$ $1 \text{ steradian} = \left(\dfrac{180}{\pi}\right)^2 \text{ sq. degree}$

$$1 \text{ square arcsec} = \left(\frac{\pi}{180}\right)^2 \times \frac{1}{(360 \times 60 \times 60)^2} \text{ steradian}$$

$$= 1.8 \times 10^{-16} \text{ ster.}$$

(i)

Surface brightness is here given first as magnitudes/solid angle.

$$I_B = 27 \text{ mag}/\square''.$$

ie area = 1 sq. arsec provides received light equivalent to 27 mag. in B-band

One star like the sun gives ~~that~~

$$m = 30.43 + 5\log d$$

Hence
$$m_{\square''} - m = -3.43 - 5\log d$$
$$= -2.5\log L_{\square''}/L_\odot$$

$$\Rightarrow \quad L_{\square'',B} = 10^{+0.4(3.43+5\log d)}$$

$$= (10^{1.37}) \times 10^{2\log d} \, L_{\odot,B}$$

$$= (10^{1.37}) \times d^2 \, L_{\odot,B}$$

$$\Downarrow$$

$$= 23.55 \, d^2 \, L_{\odot,B}$$

Need to translate arcsec^2 to parsec^2

$1'' = 5 d$ pc

$\Rightarrow \quad 1'' \times 1'' = 25 d^2$ pc.

So 27 mag/$\square''$ in B band $\equiv \dfrac{23.55 \, d^2 \, L_{\odot,B}}{25 \, d^2}$

(g)

$I_B = 27 \, mag/\square'' \leq 1 \, L_\odot/pc^2$.

The 4π has been incorporated into the $\left(\frac{\pi}{180}\right)^2$ between 1 sq degree and one steradian.
(4π stead = all sky).
$(\pi/180) \rightarrow$ degree a radian.
For I band, use

$$M_{I_\odot} = M_{\cancel{B}\odot} - (V - I)_\odot$$
V

$$= M_{B_\odot} - (B - V)_\odot - (V - I)_\odot$$

$$= M_{B_\odot} - 0.65 - 0.72$$

$$= M_{B_\odot} - 1.37. \quad \text{(brighter in I band)}$$

Hence the same ~~mag~~ surface brightness is a lower number of sun's luminosities ⇒

$$I_I = 27 \, mag/\square'' \rightarrow 1 \, L_\odot pc^{-2} \times 10^{-0.4(+1.37)}$$

$$= 1 \, L_\odot/pc^2 \times 10^{-0.55}$$

$$\underline{\underline{= 0.28 \, L_\odot/pc^2}}.$$