171.603 **Final Exam** **December 16, 2002**

This is a closed book exam. However, feel free to use your own notes. You will not need a calculator, since there are no significant numerical estimates to be performed. In fairness to your classmates, if you have a symbolic calculator, please, do not use it: others may not have one. Please, be sure to show all essential steps of your work. All problems are worth 20 points to a maximum of 120 points. The duration of the exam is 3 hours.

Use any unit system you are comfortable with, but state the system of units you are using at the beginning.

Good luck!

1. An electromagnetic wave is propagating along a pair of wires. If λ denotes the wave length of the wave propagating, assume that the diameter of the wires, $d \ll \lambda$, whereas the total length, L satisfies $L \gg \lambda$.

   (i) Can a TEM mode propagate along the pair of wires? (Explain your answer.)

   (ii) If your answer to part (i) is "yes", calculate the electric and magnetic fields between the wires. Else, ignore part (ii).

2. The imaginary part of a dielectric constant as a function of the frequency is given by the expression:

$$Im \quad \epsilon(\omega) = \frac{(\epsilon_1 - 1)\omega\tau}{1 + \omega^2\tau^2},$$

   where ϵ_1 and τ are known constants. (Gaussian units are used, ϵ is a dimensionless number.)

   Determine the complex dielectric constant as a function of the frequency.

3. A beam of protons of energy E (as measured in the laboratory system) hits a stationary hydrogen target. You want to produce a pair of K mesons, *i.e.* the reaction

$$p + p \longrightarrow p + p + K + \bar{K}$$

has to take place. (Denote the mass of the proton by M and the mass of the K-meson by m.) Note that the masses of K and $\bar{K}$ are equal.)

Calculate the minimal value of E at which the reaction shown above can take place. (This energy is called the "threshold energy" for the reaction.)

4. Consider a closed curve, C in space-time and the integral

$$\Psi = \int_C A_\mu dx^\mu,$$

where A_μ is the vector potential of some electromagnetic field.

Determine whether Ψ is an observable quantity. (Explain.)

5. Consider the Lorentz invariant,

$$F_{\alpha\beta}F^{\beta\gamma}F_{\gamma\delta}F^{\delta\alpha},$$

where $F_{\alpha\beta}$ is the field tensor of the electromagnetic field. Express this invariant as a function of the fundamental quadratic invariants.

6. Inside a superconductor, instead of Ohm's law ($\mathbf{j} = \sigma\mathbf{E}$, valid for a normal metal) London's equations are approximately valid:

$$c\nabla \times (\lambda\mathbf{j}) = -\mathbf{B}, \qquad \frac{\partial}{\partial t}(\lambda\mathbf{j}) = \mathbf{E}.$$

(Gaussian units) Otherwise, Maxwell's equations are valid and the boundary conditions are unchanged. In the superconductor, you may put $\epsilon = \mu = 1$. The quantity λ is constant in the superconductor.

Consider now an infinite superconducting slab of thickness $2d$ ($-d \leq z \leq d$), outside of which there is a given constant magnetic field parallel to the surface:

$$H_x = H_z = 0 \qquad H_y = H_0,$$

and $\mathbf{E} = 0$ everywhere. There are no surface charges or surface currents.

Calculate $\mathbf{H}$ and $\mathbf{j}$ inside the slab.