

This is a closed book exam. However, feel free to use your own notes. You will not need a calculator, since there are no significant numerical estimates to be performed. In fairness to your classmates, if you have a symbolic calculator, please, do not use it: others may not have one. Please, be sure to show all essential steps of your work. All problems are worth 20 points to a maximum of 120 points. The duration of the exam is 3 hours.

Use any unit system you are comfortable with, but state the system of units you are using at the beginning.

Good luck!

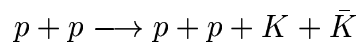
1. An electromagnetic wave is propagating along a pair of wires. If λ denotes the wave length of the wave propagating, assume that the diameter of the wires, $d \ll \lambda$, whereas the total length, L satisfies $L \gg \lambda$.
 - (i) Can a TEM mode propagate along the pair of wires? (Explain your answer.)
 - (ii) If your answer to part (i) is "yes", calculate the electric and magnetic fields between the wires. Else, ignore part (ii).
2. The imaginary part of a dielectric constant as a function of the frequency is given by the expression:

$$\text{Im } \epsilon(\omega) = \frac{(\epsilon_1 - 1)\omega\tau}{1 + \omega^2\tau^2},$$

where ϵ_1 and τ are known constants. (Gaussian units are used, ϵ is a dimensionless number.)

Determine the complex dielectric constant as a function of the frequency.

3. A beam of protons of energy E (as measured in the laboratory system) hits a stationary hydrogen target. You want to produce a pair of K mesons, *i.e.* the reaction



has to take place. (Denote the mass of the proton by M and the mass of the K -meson by m .) Note that the masses of K and \bar{K} are equal.)

Calculate the minimal value of E at which the reaction shown above can take place. (This energy is called the “threshold energy” for the reaction.)

4. Consider a closed curve, C in space-time and the integral

$$\Psi = \int_C A_\mu dx^\mu,$$

where A_μ is the vector potential of some electromagnetic field.

Determine whether Ψ is an observable quantity. (Explain.)

5. Consider the Lorentz invariant,

$$F_{\alpha\beta} F^{\beta\gamma} F_{\gamma\delta} F^{\delta\alpha},$$

where $F_{\alpha\beta}$ is the field tensor of the electromagnetic field. Express this invariant as a function of the fundamental quadratic invariants.

6. Inside a superconductor, instead of Ohm's law ($\mathbf{j} = \sigma \mathbf{E}$, valid for a normal metal) London's equations are approximately valid:

$$c \nabla \times (\lambda \mathbf{j}) = -\mathbf{B}, \quad \frac{\partial}{\partial t} (\lambda \mathbf{j}) = \mathbf{E}.$$

(Gaussian units) Otherwise, Maxwell's equations are valid and the boundary conditions are unchanged. In the superconductor, you may put $\epsilon = \mu = 1$. The quantity λ is constant in the superconductor.

Consider now an infinite superconducting slab of thickness $2d$ ($-d \leq z \leq d$), outside of which there is a given constant magnetic field parallel to the surface:

$$H_x = H_z = 0 \quad H_y = H_0,$$

and $\mathbf{E} = 0$ everywhere. There are no surface charges or surface currents.

Calculate \mathbf{H} and \mathbf{j} inside the slab.