

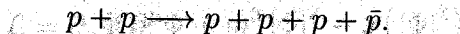
171.603 Final Exam December 13, 2001

This is a closed book exam. However, feel free to use your own notes. You will not need a calculator, since there are no significant numerical estimates to be performed. In fairness to your classmates, if you have a symbolic calculator, please, do not use it: others may not have one. Please, be sure to show all essential steps of your work. All problems are worth 20 points. The duration of the exam is 3 hours.

In this exam, "natural" units are used: $c = 1$.

Good luck!

1. Antiprotons were first produced in 1956 at the Berkeley, CA "Radiation Laboratory" (now called the Lawrence Berkeley Laboratory). A beam of protons was shot onto a stationary Hydrogen target. The reaction taking place was:



Determine the minimal energy of the proton beam for this reaction to take place. (If, for fun, you want a number too, recall that the rest energy of a proton is $m \approx 0.94\text{GeV}$.)

Important. In relativistic quantum theory one can prove that the masses of a particle and its antiparticle are exactly equal. (This was already known in 1956 when the solution of your problem was implemented in the "Rad Lab".)

2. For a particular electromagnetic field, the angle between the electric and magnetic fields is given by θ_0 . This angle is invariant to all inertial observers.

What is the value of θ_0 ?

3. Let U and \mathbf{S} be the energy density and Poynting vector of the electromagnetic field, respectively.

Show that $U^2 - \mathbf{S}^2$ is Lorentz invariant.

4. We have seen that a set of field equations for the electromagnetic field can be derived from an action principle based on the Lagrangian density:

$$\mathcal{L} = \frac{-1}{16\pi} F_{\alpha\beta} F^{\alpha\beta} - j_{\alpha} A^{\alpha},$$

where j_{α} is an external current. Someone proposes an action principle based on a new Lagrangian density,

$$\mathcal{L}' = \frac{-1}{16\pi} F_{\alpha\beta} F^{\alpha\beta} + K F_{\alpha\beta} {}^*F^{\alpha\beta} + j_{\alpha} A^{\alpha},$$

where K is some numerical constant.

What are the field equations derived from this new action principle? (As before, use Lorentz gauge for the vector potential.)

5. A complex scalar field interacting with itself is described by the Lagrangian density:

$$\mathcal{L} = \partial_{\mu} \Phi^* \partial^{\mu} \Phi + m^2 |\Phi|^2 - g (|\Phi|^2)^2.$$

Here, g is some constant measuring the strength of the self interaction and m is the mass of the quanta if the theory is quantized.

Determine the field equations obeyed by Φ and its (canonical) energy-momentum tensor.

(Note. Such a complex scalar field serves as a simplified model in order to study a variety of phenomena in the "Standard Model" of electroweak interactions.)