171.603 **Final Exam** **December 13, 2001**

This is a closed book exam. However, feel free to use your own notes. You will not need a calculator, since there are no significant numerical estimates to be performed. In fairness to your classmates, if you have a symbolic calculator, please, do not use it: others may not have one. Please, be sure to show all essential steps of your work. All problems are worth 20 points. The duration of the exam is 3 hours.

In this exam, "natural" units are used: $c = 1$.

Good luck!

1. Antiprotons were first produced in 1956 at the Berkeley, CA "Radiation Laboratory" (now called the Lawrence Berkeley Laboratory). A beam of protons was shot onto a stationary Hydrogen target. The reaction taking place was:

   $$p + p \longrightarrow p + p + p + \bar{p}.$$

   Determine the minimal energy of the proton beam for this reaction to take place. (If, for fun, you want a number too, recall that the rest energy of a proton is $m \approx 0.94\text{GeV}$.)

   Important. In relativistic quantum theory one can prove that the masses of a particle and its antiparticle are exactly equal. (This was already known in 1956 when the solution of your problem was implemented in the "Rad Lab".)

2. For a particular electromagnetic field, the angle between the electric and magnetic fields is given by θ_0. This angle is invariant to all inertial observers.

   What is the value of θ_0*?*

3. Let U and $\mathbf{S}$ be the energy density and Poynting vector of the electromagnetic field, respectively.

   Show that $U^2 - \mathbf{S}^2$ *is Lorentz invariant.*

4. We have seen that a set of field equations for the electromagnetic field can be derived from an action principle based on the Lagrangian density:

$$\mathcal{L} = \frac{-1}{16\pi} F_{\alpha\beta} F^{\alpha\beta} - j_\alpha A^\alpha ,$$

where j_α is an external current. Someone proposes an action principle based on a new Lagrangian density,

$$\mathcal{L}' = \frac{-1}{16\pi} F_{\alpha\beta} F^{\alpha\beta} + K F_{\alpha\beta} \, {}^*F^{\alpha\beta} + j_\alpha A^\alpha ,$$

where K is some numerical constant.

What are the field equations derived from this new action principle? (As before, use Lorentz gauge for the vector potential.)

5. A complex scalar field interacting with itself is described by the Lagrangian density:

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi + m^2 \left|\Phi\right|^2 - g \left(\left|\Phi\right|^2\right)^2 .$$

Here, g is some constant measuring the strength of the self interaction and m is the mass of the quanta if the theory is quantized.

Determine the field equations obeyed by Φ and its (canonical) energy-momentum tensor.

(*Note.* Such a complex scalar field serves as a simplified model in order to study a variety of phenomena in the "Standard Model" of electroweak interactions.)