Physics 171.201
Final Exam

December 16th, 2003

Answer all **six** problems. Be sure that you pace yourself so that you have enough time to work on each problem. Note that the problems do not have equal weight. Partial credit will be given, so be sure to **show your work** as clearly as possible. Good luck!

List of potentially useful formulae from special relativity

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - (v/c^2)x}{\sqrt{1 - v^2/c^2}}$$

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2}$$

$$u_y' = \frac{u_x \sqrt{1 - v^2/c^2}}{1 - u_x v/c^2}$$

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$E = mc^2$$

$$\vec{p} = m\vec{v}$$

$$E^2 = m_0{}^2 c^4 + p^2 c^2$$

$$c^2 t^2 - x^2 - y^2 - z^2 = c^2 t'^2 - x'^2 - y'^2 - z'^2$$

$$E^2 - p^2 c^2 = E'^2 - p'^2 c^2$$

1) (40 points) Consider a method to search for oil in the desert that involves sending sound waves vertically into the ground. If the waves hit an oil deposit, they are partially reflected back to the surface. (See figs. below.) In this problem assume the waves obey the dispersion relation $\omega = (P/\rho)^{1/2}k$ and experience an impedance $Z = \sqrt{P\rho}$, where P is pressure and ρ is density. Also assume that the sand and the oil have the same pressure P but different densities. Call the density of sand ρ_S and the density of oil ρ_O with $\rho_O > \rho_S$.

a) If the sound wave emitted at the surface is a pulse Δt wide, derive an expression for the time delay between when the pulse is emitted and when the reflected pulse reaches the surface. Take the oil deposit to be a depth D below the surface.

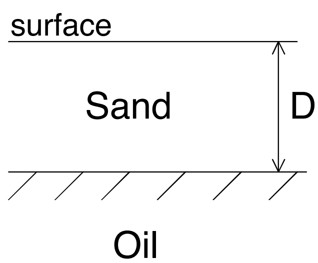
b) For the case considered in (a), will the reflected pulse that returns to the surface have the same width Δt as the emitted pulse, yes or no? Explain your answer. Under what circumstances would your answer change?

c) If instead the sound wave emitted at the surface is a traveling wave of amplitude A and frequency ω, $\psi(z,t) = A\cos(\omega t - kz)$, determine the amplitude of the reflected wave.

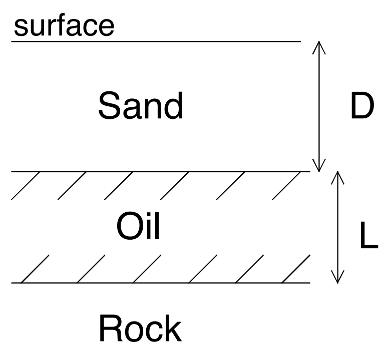
d) Assume now that the oil deposit has a finite thickness L below which there is rock of density ρ_R with $\rho_R > \rho_O$. (P is still the same.) For the case considered in (c) of a traveling wave, determine the values of L for which the wave reflected from the top of the oil and from the bottom of the oil interfere destructively.

e) Assuming there is destructive interference and $\rho_S \approx \rho_O \approx \rho_R$ so that the reflection amplitudes are small, find the relation among tne 3 densities for which the total reflected wave is zero. Ignore higher order reflections.

parts a - c



parts d & e



2) (30 points) Consider a drumhead in the shape of a rectangle with sides length L and $L/\sqrt{2}$. The phase velocity of the drumhead is v_p.

a) If all four edges of the drumhead are held fixed, determine the frequencies of the 5 lowest frequency normal modes of the drumhead.

b) If instead one of the sides of length L is allowed to move freely while the other three sides are held fixed, determine the frequencies of the 2 lowest frequency normal modes of the drumhead.

3) (30 points) Suppose a high-energy collision occurs in the earth's upper atmosphere producing an exotic particle called X. Particle X is at rest in the earth's reference frame but quickly decays into a neutrino and a muon. Take the neutrino's rest mass to be zero and the muon's rest mass to be 100 MeV

a) If the muon produced in the decay has a velocity in the earth's frame of $\frac{\sqrt{15}c}{4}$, what is the rest mass of particle X?

b) What is the momentum of the neutrino produced in the decay?

c) Imagine that particle X decays at a height 10 km above the earth's surface and that the muon is produced with a velocity that is directed vertically downward toward the earth. The muon has a lifetime in its rest frame of 2×10^{-6} seconds. What is the probability that the muon reaches the earth's surface before it disintegrates?

4) (30 points) Consider two pendula comprised a massless strings length L attached to masses of mass M. The masses are connected to each other by a spring with spring constant K.

a) Assuming small amplitude motion of the masses, write down the equations of motion for the angular displacements, θ_1 and θ_2, of the two pendula.

b) Derive the normal mode frequencies for the system and draw sketches of the relative motions of the masses for each mode.

c) Assume K is very small so that the masses are weakly coupled. Imagine that at time t=0 the masses are held at positions $\theta_1 = \theta_0$ and $\theta_2 = 0$ and then released. That is, consider the initial conditions:

$$\theta_1(t=0) = \theta_0 \qquad \frac{d\theta_1}{dt}(t=0) = 0$$

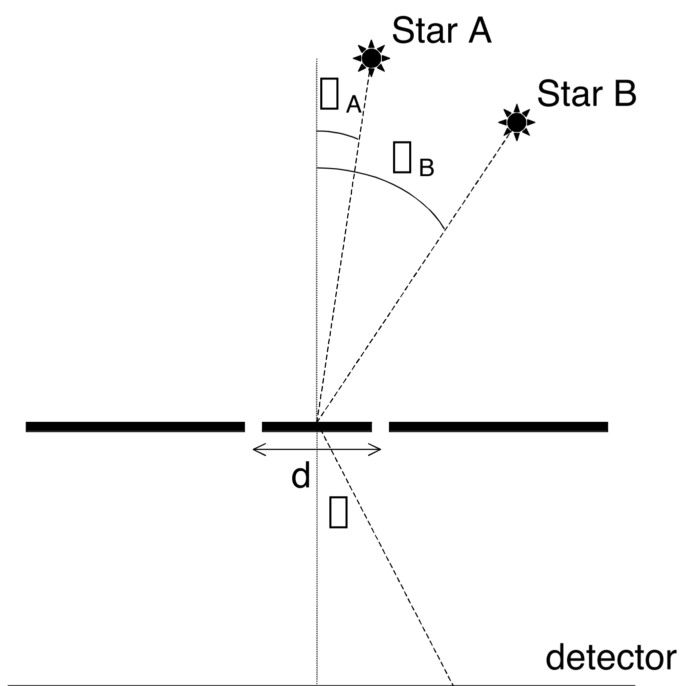
$$\theta_2(t=0) = 0 \qquad \frac{d\theta_2}{dt}(t=0) = 0$$

At approximately what time will $\theta_2 = \theta_0$. What will be the amplitude of θ_1 at that time?

5) (30 points) Consider the astronomical device known as the Michelson Stellar Interferometer shown in the sketch below. The device consists of two slits separated by a distance d and a detector screen a distance L behind the slits.

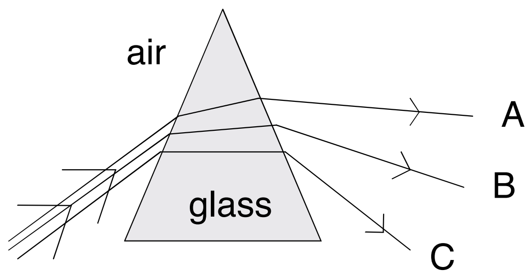
a) Light from a distant star A with wavelength λ is incident on the slits. The star is situated at an angle θ_A with respect to normal as shown in the figure. Assuming L >> d and L >> λ, determine the intensity of the interference pattern on the detector at $\theta = 0$ in terms of λ, d and θ_A.

b) Consider now in addition light from a second star B also with wavelength λ incident on the slits at an angle θ_B. Assuming that the two stars are equally bright, determine the value for d for which the intensity of the interference pattern at $\theta = 0$ is zero. (Experimentally, the Michelson Stellar Interferometer is used to measure the angular separation of stars in the sky by tuning d until such destructive interference is achieved.)

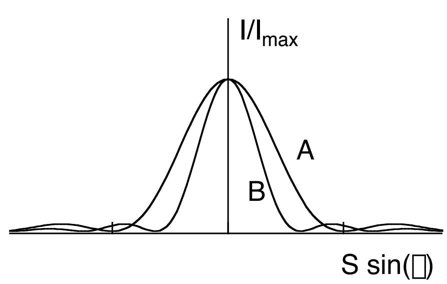


6) (40 points) Each of the following short problems is worth 10 points. In each case be sure to explain your answer in order to earn full credit.

a) Light consisting of three colors is incident on a prism. The colors are red (long wavelength), green (medium wavelength) and blue (short wavelength). The prism is made of glass with an index of refraction that varies with wavelength as $n(\lambda) = n_0 \lambda^{-2}$. Based on the picture, match each color to one of the three rays A, B, and C.



b) A detector placed behind a narrow slit of width S measures the diffraction patterns A and B made by two plane waves of light, one red and one blue, as shown in the picture. Which pattern corresponds to which color?



Problem 6, continued:

c) You are given three polarizers oriented in the $\hat{x}-\hat{y}$ plane. Polarizer A has its polarization axis parallel to $\hat{y}$; Polarizer B has its polarization axis at 45° to $\hat{y}$; and Polarizer C has its polarization axis at 30° to $\hat{y}$. An unpolarized plane electromagnetic wave is incident along the $\hat{z}$ direction. In what order would you place the polarizers to minimize the transmission through the three? In what order would you place them to maximize the transmission?

d) Which home experiment that you performed during the semester was your favorite? Which was your least favorite? Overall, do you think the home experiments were worthwhile? (Hint: There is no wrong answer to this question.)