Physics 171.201
Final Exam

December 14th, 2004

Answer all **seven** problems. Be sure that you pace yourself so that you have enough time to work on each problem. Note that the problems do not have equal weight. Partial credit will be given, so be sure to **show your work** as clearly as possible**.** Good luck!

<u>List of potentially useful formulae</u>

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$E = mc^2$$

$$z' = z$$

$$\vec{p} = m\vec{v}$$

$$t' = \frac{t - (v/c^2)x}{\sqrt{1 - v^2/c^2}}$$

$$E^2 = m_0{}^2 c^4 + p^2 c^2$$

$$u_x{}' = \frac{u_x - v}{1 - u_x v/c^2}$$

$$E^2 - p^2c^2 = E'^2 - p'^2 c^2$$

$$u_y{}' = \frac{u_y \sqrt{1 - v^2/c^2}}{1 - u_x v/c^2}$$

Maxwell's Equations (in the absence of currents and free charges):

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \times \vec{E} = \frac{-1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

Poynting Vector: $\vec{S} = \frac{c}{4\pi}\frac{1}{\mu}\vec{E} \times \vec{B}$

EM Energy Density: $U = \frac{1}{8\pi}\left(\varepsilon \vec{E}^2 + \frac{1}{\mu}\vec{B}^2\right)$

Problem 1 (30 points)

Consider a string with mass density ρ and tension T. The ends of the string are held fixed at x = -L/2 and x = L/2 as shown in the picture below. The string is pulled in the transverse direction into the shape shown in the picture and held still in this position. The displacement of the string in this position is

$$y(x) = \begin{cases} 0 & x < -L/4 \\ -h & -L/4 < x < 0 \\ h & 0 < x < L/4 \\ 0 & L/4 < x \end{cases}$$

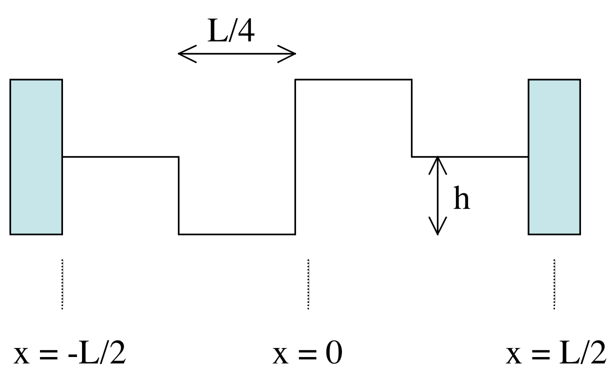
(a) As discussed in class, one can represent the displacement y(x) as a fourier series

$$y(x) = \sum_{n=1}^{\infty} A_n \cos(k_n x) + \sum_{n=1}^{\infty} B_n \sin(k_n x)$$

For the particular displacement given above, the ratio A_n/B_n is independent of n. What is this ratio?

(b) At time t = 0, the string is released. Calculate the amplitudes of the 4 longest wavelength normal modes that are excited.

(c) Once the string is released, will it ever return to the shape shown in the picture? If so, how often? (Neglect any damping.)



Problem 2 (30 points)

Consider a circularly polarized electromagnetic plane wave with wave vector k and frequency ω traveling in vacuum ($\varepsilon = \mu = 1$) along the positive z-direction. The time-average Poynting vector for the wave is given by:

$$\langle \vec{S} \rangle = \frac{c}{8\pi} E_0^2 \hat{z}$$

(a) Write down an expression for the electric field vector corresponding to this wave.

(b) Imagine a polarizer with easy axis along the x-direction is placed in the path of the wave. What is the electric field vector corresponding to the wave that is transmitted through the polarizer?

(c) Imagine now a second polarizer with easy axis along the y-direction is added after the first polarizer. What is the intensity of the wave that is transmitted through the two polarizers?

(d) Imagine now a third polarizer with easy axis making an angle θ with x-direction is inserted between the two polarizers. What is the total transmission intensity through the three polarizers?

Problem 3 (30 points)

Alice is standing on a train platform. Bored of waiting for the train to arrive, she takes out two firecrackers, one she calls firecracker A and one she calls firecracker B. She lights the fuse of A and sets it down on the platform. She then measures off 50 meters along the platform, sets down firecracker B, and lights its fuse. After the two firecrackers explode, Alice determines that A exploded $1x10^{-7}$ seconds before B. Ben meanwhile is traveling in a train car running along platform in the direction from A to B. According to his measurements the two firecrackers explode simultaneously.

(a) What is the velocity of the train? (Recall that $c = 3x10^{8}$ meters/second.)

(b) What is the distance between the two firecrackers according to Ben?

(c) If firecracker A explodes just as Ben is passing by it, how much time does he say transpires before the light from the explosion of B reaches him?

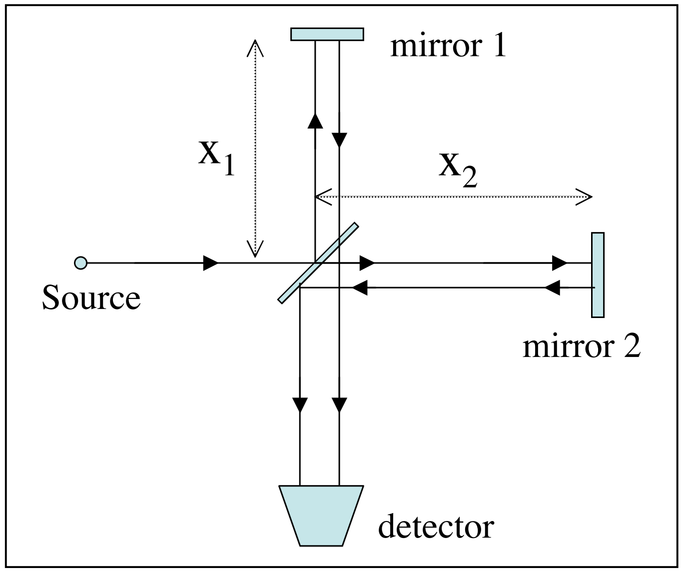
Problem 4 (30 points)

The Michelson interferometer is an instrument that played a key role in experiments leading up to Einstein's development of special relativity. A schematic diagram of a Michelson interferometer is shown on the next page. A plane wave of light with wavelength λ is emitted from a source. The light is incident on a "half-mirror" oriented at 45 degrees with respect to the direction of incident light. Half of the light intensity is transmitted through the half-mirror and half is reflected. The transmitted half is then reflected from mirror 2. Half the light reflected from mirror 2 then gets reflected from the half-mirror into a detector. Meanwhile, the half of the initial light that was reflected by the half-mirror is reflected from mirror 1. Half the light reflected from mirror 1 then gets transmitted through the half-mirror into the detector.

(a) Assuming all the mirrors work perfectly, so that the half-mirror reflects exactly half the intensity of incident light and the mirrors 1 & 2 reflect fully, and assuming all the mirrors are perfectly aligned, derive an expression for the intensity measured at the detector in terms of the lengths x_1 and x_2.

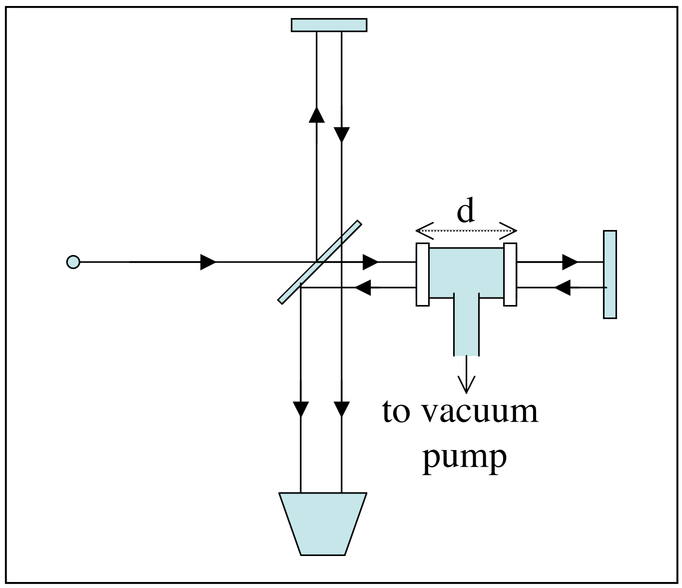
(b) Consider now the experiment shown in the second picture on the next page. A vacuum chamber of length d with windows is placed in the path between the half-mirror and mirror 2. At the start of the experiment the chamber is filled with air. As the chamber is evacuated, the intensity at the detector is monitored. The resulting data is shown on the next page. What is the index of refraction of the air?

part (a):

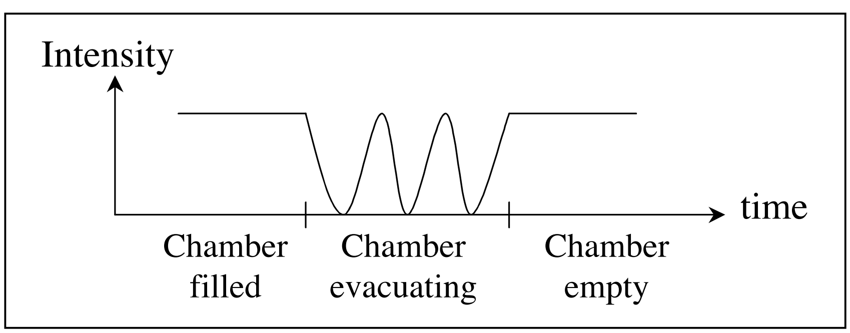


mirror 1
x_1
x_2
Source
mirror 2
detector

part (b):



d
to vacuum
pump



Intensity
time
Chamber
filled
Chamber
evacuating
Chamber
empty

Problem 5 (30 points)

Consider the electrical circuit below that consists of two identical LC circuits coupled by a common capacitance C with the directions of current flow I_a and I_b indicated by the arrows. The equations for the voltages across the inductors are:

$$V_1 - V_2 = L\frac{dI_a}{dt} \qquad\qquad V_2 - V_3 = L\frac{dI_b}{dt}$$

The equations for the currents in terms of the charges on the capacitors are:

$$\frac{dq_1}{dt} = -I_a \qquad\qquad \frac{dq_2}{dt} = I_a - I_b \qquad\qquad \frac{dq_3}{dt} = I_b$$

where,

$$q_1 = CV_1 \qquad q_2 = CV_2 \qquad q_3 = CV_3$$

(a) Construct two coupled differential equations involving I_a and I_b.

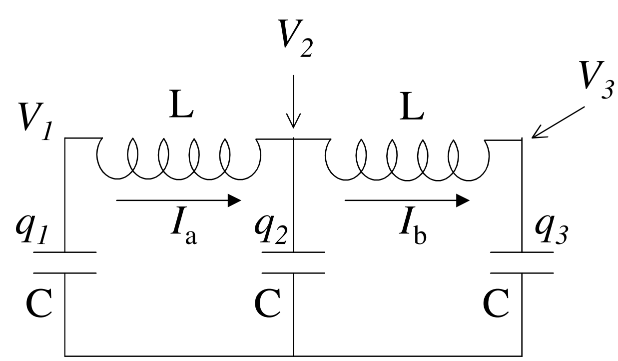
(b) Find the frequencies for the normal modes of current oscillation in the circuit.

(c) What are the relationships between I_a and I_b for to each normal mode?

(d) Consider the following initial conditions for the current:

$$I_a(t=0) = I_0 \qquad \frac{dI_a}{dt} = 0$$
$$I_b(t=0) = 0 \qquad \frac{dI_b}{dt} = 0$$

At what times will $I_b(t) = I_0$? What is the value of I_a at these times?



Problem 6 (25 points)

Consider a tube of length L and radius R.

(a) If both ends of the tube are open, derive an expression for the allowed wavelengths of standing sound waves inside the tube. Neglect end effects.

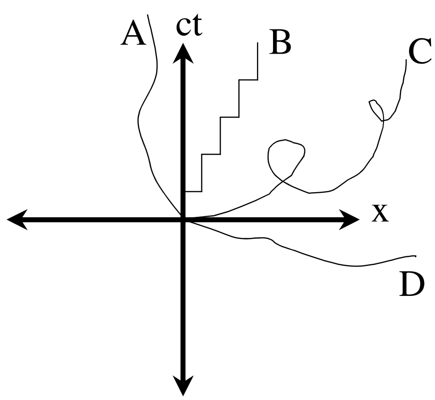
(b) Derive an expression for the allowed wavelengths for the case when one end of the tube is open and the other end is closed.

(c) Recall the home experiment involving the PVC tubes and the tuning fork. Explain briefly how one can use the tubes and the tuning fork to determine the speed of sound (which is essentially what you did in the experiment).

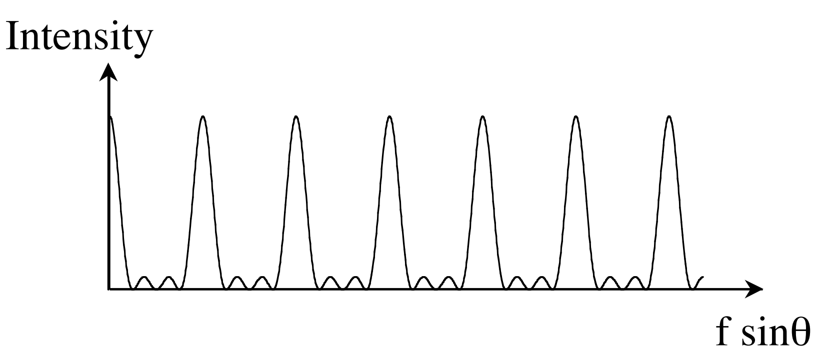
Probem 7 (25 points)

In each of the following short problems, be sure to explain your answer in order to earn full credit.

(a) (6 points) Consider the Minkowski Diagram below. Which of the lines A, B, C, or D is a possible worldline for an object with mass > 0?



(b) (6 points) Monochromatic light is incident on an opaque screen with a set of narrow slits that are spaced equally a distance f apart. The graph below shows the intensity measured at a detector a large distance behind the slits as a function of angle θ with respect to the direction normal to the screen. How many silts are there?



Problem 7 (continued)

(c) (7 points) The dispersion relation for waves in a solid material with atomic spacing a, which we derived in class, is shown below. Consider two wave pulses traveling through such a solid. Pulse A is comprised mostly of high frequency waves and travels at velocity v_A; Pulse B is comprised mostly of low frequency waves and travels at velocity v_B. Which of the following statements is most accurate?

A. $v_A = v_B$, and the shapes of the pulses change with time.

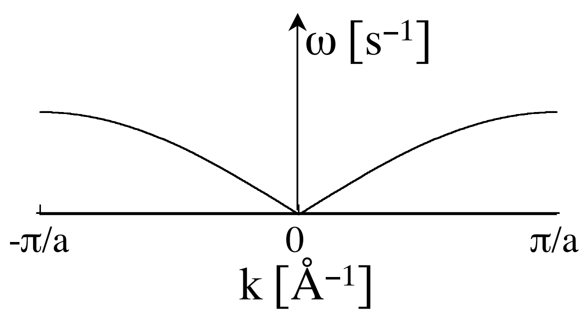
B. $v_A = v_B$, and the shapes of the pulses do not change with time.

C. $v_A < v_B$, and the shapes of the pulses change with time.

D. $v_A < v_B$, and the shapes of the pulses do not change with time.

E. $v_A > v_B$, and the shapes of the pulses change with time.

F. $v_A > v_B$, and the shapes of the pulses do not change with time.



(d) (6 points) A photon can not spontaneously convert into an electron – positron pair without the involvement of another particle because the photon is massless and hence:

A mass cannot be conserved.
B energy cannot be conserved.
C momentum cannot be conserved.
D causality is violated.