171.201, Final Exam Solutions, Fall 2005

Problem 1

a) Three dimensional wave equation

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 P}{\partial t^2}$$

Let $P = X(x)Y(y)Z(z)e^{i\omega t}$

$$\left(YZ\frac{\partial^2 X}{\partial x^2} + XZ\frac{\partial^2 Y}{\partial y^2} + XY\frac{\partial^2 Z}{\partial z^2}\right)\cancel{e^{i\omega t}} = -\frac{\omega^2}{v^2}XYZ\,\cancel{e^{i\omega t}}$$

Divide by XYZ to get

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = -\frac{\omega^2}{v^2}$$

Each term on the left hand side should equal a constant.

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -k_x^2 \longrightarrow \frac{\partial^2 X}{\partial x^2} = -k_x^2 X$$

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -k_y^2 \longrightarrow \frac{\partial^2 Y}{\partial y^2} = -k_y^2 Y$$

$$\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = -k_z^2 \longrightarrow \frac{\partial^2 Z}{\partial z^2} = -k_z^2 Z$$

and the constants should satisfy

$$-k_x^2 - k_y^2 - k_z^2 = -\frac{\omega^2}{v^2} \longrightarrow \boxed{k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{v^2}}$$

Each of one dimensional differential equation above has a solution that looks like

ex) for x, $\quad X(x) = A\cos k_x x + B\sin k_y y$

Using the boundary conditions

① $\left.\frac{\partial X}{\partial x}\right|_{x=0} = A k_x \sin k_x(0) + B k_x \cos k_x(0) = A = 0$

② $\left.\frac{\partial X}{\partial x}\right|_{x=L} = B k_x \cos k_x L = 0$

The second boundary condition gives

$$k_x L = n\pi$$

$$k_x = \frac{n\pi}{L}$$

Therefore, the solution is given by

$$\therefore \boxed{P(x,y,z,t) = \Delta \cos(k_x x)\cos(k_y y)\cos(k_z z)\, e^{i\omega t}}$$

where $k_x = k_y = k_z = \frac{n\pi}{L}$ where $n =$ integer

and $\Delta =$ amplitude.

b) The four lowest frequencies are given when the following condition is satisfied.

$$(n_x, n_y, n_z) = \begin{cases} (1 \quad 0 \quad 0) & \text{degenerate} \\ (1 \quad 1 \quad 0) & \text{degenerate} \\ (1 \quad 1 \quad 1) & \text{non-degenerate} \\ (2 \quad 0 \quad 0) & \text{degenerate} \end{cases}$$

Therefore.

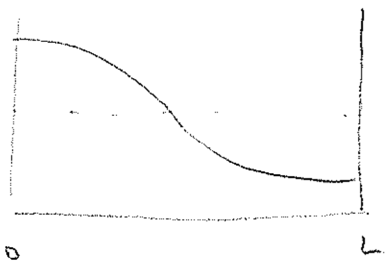
$$\omega_1^2 = v^2\left[\left(\frac{\pi}{L}\right)^2 + 0^2 + 0^2\right] = v^2\frac{\pi^2}{L^2}$$

$$\omega_2^2 = v^2\left[\left(\frac{\pi}{L}\right)^2 + \left(\frac{\pi}{L}\right)^2 + 0^2\right] = 2v^2\frac{\pi^2}{L^2}$$

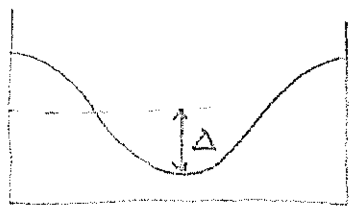
$$\omega_3^2 = v^2\left[\left(\frac{\pi}{L}\right)^2 + \left(\frac{\pi}{L}\right)^2 + \left(\frac{\pi}{L}\right)^2\right] = 3v^2\frac{\pi^2}{L^2}$$

$$\omega_4^2 = v^2\left[\left(\frac{2\pi}{L}\right)^2 + 0^2 + 0^2\right] = 4v^2\frac{\pi^2}{L^2}$$

c) For any $n = 1$, the antinode falls at the center of the room



For $n = 2$, the pressure looks like



Therefore

For ω_1 : $P_{max} = P_0$

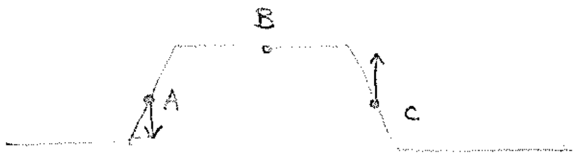
ω_2 : $P_{max} = P_0$

ω_3 : $P_{max} = P_0$

ω_4 : $P_{max} = P_0 + \Delta$

Problem 2

a) The phase velocity of the string is given by $v = \sqrt{T/\rho}$.



Slope at A is given by

$$m = \frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = v_y \cdot \frac{1}{v_x}$$

Solving for v_y, we get

$$v_y = m v_x = \frac{h}{d} v = \frac{h}{d}\sqrt{\frac{T}{\rho}}$$

Therefore

$$\therefore \boxed{\begin{aligned} v_A &= -\frac{h}{d}\sqrt{\frac{T}{\rho}} \\ v_B &= 0 \\ v_c &= \frac{h}{d}\sqrt{\frac{T}{\rho}} \end{aligned}}$$

b) The reflection amplitude is given by

$$R = \frac{Z_1 - Z_2}{Z_1 + Z_2} = -\frac{1}{3}$$

$$Z_1 = \sqrt{T\rho_1} = \sqrt{T\rho}$$
$$Z_2 = \sqrt{T\rho_2}$$

$$\frac{\sqrt{T\rho} - \sqrt{T\rho_2}}{\sqrt{T\rho} + \sqrt{T\rho_2}} = -\frac{1}{3}$$

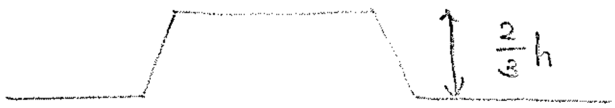
$$\sqrt{\cancel{T}\rho} - \sqrt{\cancel{T}\rho_2} = -\frac{1}{3}\left(\sqrt{\cancel{T}\rho} + \sqrt{\cancel{T}\rho_2}\right)$$

$$\frac{\cancel{4}^{2}}{\cancel{3}}\sqrt{\rho} = \frac{\cancel{2}}{\cancel{3}}\sqrt{\rho_2}$$

$$2\sqrt{\rho} = \sqrt{\rho_2} \longrightarrow \boxed{\rho_2 = 4\rho}$$

c) $$T = \frac{2Z_1}{Z_1+Z_2} = \frac{2\sqrt{T\rho}}{\sqrt{T\rho}+\sqrt{T4\rho}} = \frac{2\sqrt{\rho}}{\sqrt{\rho}+2\sqrt{\rho}} = \frac{2\sqrt{\rho}}{3\sqrt{\rho}} = \frac{2}{3}$$

The transmitted pulse looks like



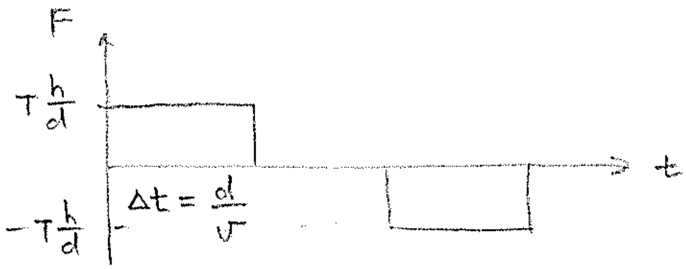
d) $$K = \frac{1}{2}\int dm\, v_y^2 = \frac{1}{2}\int_0^{2d} \cancel{\rho}\frac{h^2}{d^2}\frac{T}{\cancel{\rho}}\, dx$$

$$= \frac{Th^2}{\cancel{2}d^{\cancel{2}}}\cdot \cancel{2d} = \boxed{\frac{Th^2}{d}}$$

$$U = \frac{1}{2}\int_0^{2d} T\left(\frac{dy}{dx}\right)^2 dx = \frac{1}{2}\int_0^{2d} T\left(\frac{h}{d}\right)^2 dx = \frac{Th^2}{\cancel{2}d^{\cancel{2}}}\cdot \cancel{2d}$$

$$= \boxed{\frac{Th^2}{d}}$$

e) $$F = T\frac{dy}{dx} = T\frac{h}{d}$$



$$F(t) = T\frac{h}{d}\left[\theta(t) - \theta\left(t-\frac{d}{v}\right)\right] - T\frac{h}{d}\left[\theta\left(t-\frac{2d}{v}\right) - \theta\left(t-\frac{3d}{v}\right)\right]$$

Problem #3

(a) The difference in path of l is

$$\Delta l = \sqrt{\left(\frac{\delta}{2}\right)^2 + r^2 + 2\cdot\frac{\delta}{2}\cdot r\sin\theta} - \sqrt{\left(\frac{\delta}{2}\right)^2 + r^2 - 2\cdot\frac{\delta}{2}\cdot r\sin\theta}$$

$$\simeq r\left(1 + \frac{1}{2}\frac{\delta}{r}\sin\theta + \frac{1}{4}\left(\frac{\delta}{r}\right)^2\right)$$

$$- r\left(1 - \frac{1}{2}\frac{\delta}{r}\sin\theta + \frac{1}{4}\left(\frac{\delta}{r}\right)^2\right)$$

$$= \delta\sin\theta$$

(b) The intensity measured by the detector is

$$I(\theta) = \left| A\cos\left(\frac{2\pi r_+}{\lambda} + \phi\right) + A\cos\left(\frac{2\pi r_-}{\lambda} + \phi\right)\right|^2$$

where $r_+ = \left[\left(\frac{\delta}{2}\right)^2 + r^2 + 2\cdot\frac{\delta}{2}\cdot r\sin\theta\right]^{1/2}$

$r_- = \left[\left(\frac{\delta}{2}\right)^2 + r^2 - 2\cdot\frac{\delta}{2}\cdot r\sin\theta\right]^{1/2}$

$$\Rightarrow I(\theta) = 4A^2\cos^2\left(\frac{\pi(r_+ + r_-)}{\lambda} + \phi\right)\cos^2\left(\frac{\pi(r_+ - r_-)}{\lambda}\right)$$

Note that $r_+ + r_- \simeq 2r$

$r_+ - r_- \simeq \delta\sin\theta$

$$\Rightarrow I(\theta) = 4A^2\cos^2\left(\frac{2\pi r}{\lambda} + \phi\right)\cos^2\left(\frac{\pi\delta\sin\theta}{\lambda}\right)$$

If θ is also small, then r is almost a constant.

Define $I_{max} \equiv 4A^2\cos^2\left(\frac{2\pi r}{\lambda} + \phi\right)$

$$\Rightarrow I(\theta) = I_{max}\cos^2\left(\frac{\pi\delta\sin\theta}{\lambda}\right)$$

(c) In this case, we have

$$I(\theta) = \left| A\cos\left(\frac{2\pi r_+}{\lambda} + \phi + \pi\right) + A\cos\left(\frac{2\pi r_-}{\lambda} + \phi\right)\right|^2$$

$$= 4A^2\cos^2\left(\frac{\pi(r_+ + r_-)}{\lambda} + \phi + \frac{\pi}{2}\right)\cos^2\left(\frac{\pi(r_+ - r_-)}{\lambda} + \frac{\pi}{2}\right)$$

$$\simeq 4A^2\cos^2\left(\frac{2\pi r}{\lambda} + \phi + \frac{\pi}{2}\right)\cos^2\left(\frac{\pi\delta\sin\theta}{\lambda} + \frac{\pi}{2}\right)$$

$$= 4A^2\sin^2\left(\frac{2\pi r}{\lambda} + \phi + \frac{\pi}{2}\right)\sin^2\left(\frac{\pi\delta\sin\theta}{\lambda}\right)$$

Define $I_{max} \equiv 4A^2\sin^2\left(\frac{2\pi r}{\lambda} + \phi + \frac{\pi}{2}\right)$

$$\Rightarrow I(\theta) = I_{max}\sin^2\left(\frac{\pi\delta\sin\theta}{\lambda}\right)$$

(d) If no material, then the path length is just T for the spatial region that is going to be occupied by the material. Now after the material is placed, the wavelength of the EM wave when it travels through the region is just λ/n. So the phase

The phase factor induced by this path length is just T/λ.

factor induced by the path length (still T) changes to $T/\left(\frac{\lambda}{n}\right) = nT/\lambda$. The point is the phase changes by an additional amount as

of the EM wave $\quad \frac{1}{2\pi}\Delta\phi = \frac{nT}{\lambda} - \frac{T}{\lambda} = (n-1)\frac{T}{\lambda}$

Since two antennae emit in phase, but the interterfence pattern observed shows that it looks like they emit precisely out of phase. This is induced by the presence of the material, so we have

$$\frac{1}{2\pi}\Delta\phi = (n-1)\,\frac{T}{\lambda} = \pi(2K+1)$$

where K is an integer non-negative

$$\Rightarrow \quad n = \frac{\lambda\pi}{2\pi T}(2K+1)+1$$

~~For K=0~~ $\cancel{\Rightarrow n = 1 + \frac{\lambda\pi}{T}}$

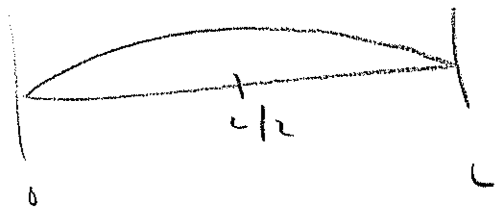
$$= \frac{\lambda}{2T}(2K+1)+1$$

For $K=0 \Rightarrow n = 1 + \frac{\lambda}{2T}$

Problem # 4

(a) $$y(x,t) = \sum_{n=1}^{\infty} A_n \sin(k_n x)\cos(\omega_n t + \phi_n) \qquad \text{where } k_n = \frac{n\pi}{L}$$

By symmetry only terms that are <u>odd</u> about $x = \frac{L}{2}$ will contribute. $n = 1$ term looks like:



which is even about $x = \frac{L}{2}$. Therefore $n = 1$ is not excited. $\boxed{\lambda_1 = 2L}$

(b) We want A_2.

$$\dot{y}(x,0) = \sum_{n=1}^{\infty} A_n \sin(k_n x)\cos\phi_n = 0 \quad \Rightarrow \quad \phi_n = \pi/2$$

$$\frac{dy}{dt}(x,0) = \sum_{n=1}^{\infty} -\omega_n A_n \sin(k_n x)\overbrace{\sin\left(\frac{\pi}{2}\right)}^{1} = \begin{cases} V_0 & |x - \frac{L}{4}| < \frac{A}{2} \\ -V_0 & |x - \frac{3L}{4}| < \frac{A}{2} \\ 0 & \text{otherwise} \end{cases}$$

This is just a fourier series. Use expression for finding fourier coefficients:

$$-\omega_n A_n = \frac{2}{L}\int_0^L f(x)\sin(k_n x)\,dx = \int_{\frac{L}{4}-\frac{A}{2}}^{\frac{L}{4}+\frac{A}{2}} \frac{2V_0}{L}\sin(k_n x)\,dx$$

$$- \int_{\frac{3L}{4}-\frac{A}{2}}^{\frac{3L}{4}+A/2} \frac{2V_0}{L}\sin(k_n x)\,dx$$

$$+\omega_n A_n = \frac{2V_0}{L} K_n^{-1} \left\{ \cos\left[\frac{n\pi}{L}\left(\frac{L}{4}+\frac{A}{2}\right)\right] - \cos\left[\frac{n\pi}{L}\left(\frac{L}{4}-\frac{A}{2}\right)\right] \right.$$

$$\left. - \cos\left[\frac{n\pi}{L}\left(\frac{3L}{4}-\frac{A}{2}\right)\right] + \cos\left[\frac{n\pi}{L}\left(\frac{3L}{4}-\frac{A}{2}\right)\right] \right\}$$

$$A_n = \frac{2V_0}{L} \frac{1}{\underbrace{W_n K_n}_{\sqrt{\frac{\rho}{T}}\frac{L^2}{(n^2\pi^2)}}} \left\{ \cos\left(\frac{n\pi}{4} + \frac{n\pi A}{2L}\right) - \cos\left(\frac{n\pi}{4} - \frac{n\pi A}{2L}\right) \right.$$

$$\left. - \cos\left(\frac{3n\pi}{4} + \frac{n\pi A}{2L}\right) + \cos\left(\frac{3n\pi}{4} - \frac{n\pi A}{2L}\right) \right\}$$

$$\Rightarrow A_2 = \frac{2V_0 L\sqrt{\rho}}{4\pi^2\sqrt{T}} \left\{ \cos\left(\frac{\pi}{2} + \frac{\pi A}{L}\right) - \cos\left(\frac{\pi}{2} - \frac{\pi A}{L}\right) \right.$$

$$\left. - \cos\left(\frac{3\pi}{2} + \frac{\pi A}{L}\right) + \cos\left(\frac{3\pi}{2} - \frac{\pi A}{L}\right) \right\}$$

but $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$$A_2 = \frac{V_0 L\sqrt{\rho}}{2\pi^2\sqrt{T}} \left\{ -\sin\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi A}{L}\right) - \sin\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi A}{L}\right) \right.$$

$$\left. + \sin\left(\frac{3\pi}{2}\right)\sin\left(\frac{\pi A}{L}\right) + \sin\left(\frac{3\pi}{2}\right)\sin\left(\frac{\pi A}{L}\right) \right\}$$

$$\boxed{A_2 = -\frac{2V_0 L\sqrt{\rho}}{\pi^2\sqrt{T}} \sin\left(\frac{\pi A}{L}\right)}$$

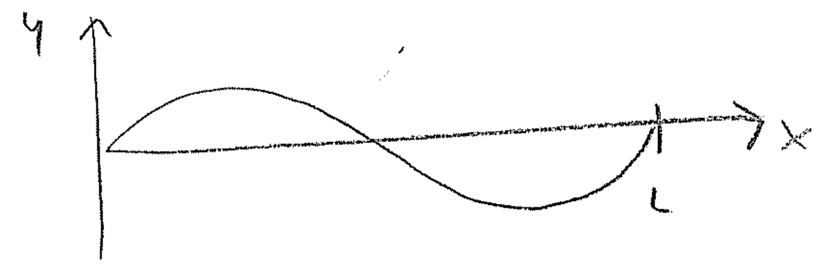
(c) All even harmonics ($n = 2, 4, 6, 8 \ldots$) are excited. These modes have periods equal to

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{v K_n} = \sqrt{\frac{\rho}{T}} \frac{2\pi}{n\pi/L} = \sqrt{\frac{\rho}{T}} \frac{2L}{n}$$

so, $T_4 = \frac{1}{2} T_2$, $T_6 = \frac{1}{3} T_2$, $T_8 = \frac{1}{4} T_2$, etc and each mode will pass through 0 whenever $n=2$ does. Each mode passes through 0 twice per cycle.

$$\Longrightarrow t_{min} = \frac{T_2}{2} = \boxed{\sqrt{\frac{\rho}{T}} \frac{L}{2}}$$

(d) at $\frac{t_{min}}{2} = \frac{T_2}{4}$ all the modes will be passing through zero except $n=2$, which will be at a maximum:



Problem # 5

(a) $\quad e_1 \xrightarrow{\frac{4}{5}c} \quad e_2$ (lab frame)

$$E_{tot} = E_1 + E_2 = M_1 c^2 + M_2 c^2 = \frac{M_0}{\sqrt{1 - v^2/c^2}} c^2 + M_0 c^2$$

$$= \frac{5}{3} M_0 c^2 + M_0 c^2 = \boxed{\frac{8}{3} M_0 c^2}$$

(b) $\quad e_1 \xrightarrow{v'} \quad \xleftarrow{-v'} e_2$ ((c.o.m. frame))

$$\underline{\text{lab}} \qquad\qquad \underline{\text{c.o.m}}$$

$$E^2 - p^2 c^2 = E'^2 - p'^2 c^2$$

in lab frame: $\bar{p} = \bar{p}_1 + \bar{p}_2 = MV + 0 = \frac{5}{3} M_0 \frac{4}{5} c = \frac{4}{3} M_0 c$

in c.o.m. frame $\bar{p}' = \bar{p}_1' + \bar{p}_2' = 0$

$$\Rightarrow \left(\frac{8}{3} M_0 c^2\right)^2 - \left(\frac{4}{3} M_0 c\right)^2 c^2 = E'^2 - 0$$

$$E'^2 = \left(\frac{64}{9} - \frac{16}{9}\right) M_0^2 c^4 = \frac{48}{9} M_0^2 c^4$$

$$\boxed{E' = \frac{4}{\sqrt{3}} M_0 c^2}$$

(c) In the center of mass frame, the final particle will have no velocity. Therefore, all of the energy is rest energy.

$$\boxed{M_{0f} = \frac{4}{\sqrt{3}} M_0}$$

(d) now we have,

$$(e_1) \xrightarrow{v = ?} \qquad \xleftarrow{v = ?} (e_2)$$

$$E_{tot} = E_1 + E_2 = M_1 c^2 + M_2 c^2 = 2 \frac{M_0}{\sqrt{1 - v^2/c^2}} c^2$$

$$\frac{8}{3} \cancel{M_0 c^2} = \frac{2}{\sqrt{1 - v^2/c^2}} \cancel{M_0 c^2}$$

$$(1 - v^2/c^2)^{1/2} = 6/8 = 3/4$$

$$1 - v^2/c^2 = 9/16$$

$$\boxed{v = \frac{\sqrt{7}}{4} c}$$

(e) Now, since the lab frame is the center of mass frame we can say that all the (lab) energy is converted into rest energy of the final particle.

$$E_{tot} = 2Mc^2 = 2 \frac{M_0}{3/4} c^2 = \frac{8}{3} M_0 c^2$$

$$\Rightarrow \boxed{M_{0f} = \frac{8}{3} M_0}$$

This result is larger than that in part (c), which makes this result preferrable.

Problem #6

(a) The coordinate of the center of mass is

$$X = \frac{X_1+X_2}{2}$$

The equation of motion of the C.O.M is

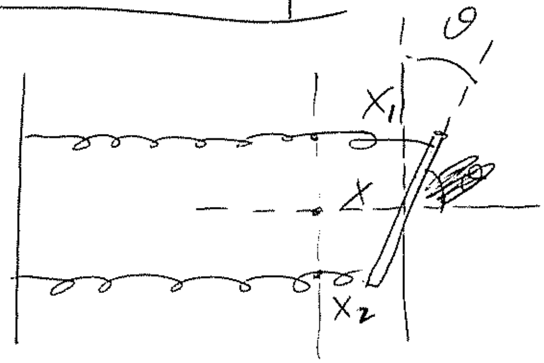
$$M\ddot{X} = -KX_1 - KX_2 = -2KX$$

$$\Rightarrow M\cdot\frac{1}{2}(\ddot{X}_1+\ddot{X}_2) = -K(X_1+X_2)$$

$$\Rightarrow \boxed{\ddot{X}_1+\ddot{X}_2+\frac{2K}{M}(X_1+X_2)=0}$$

or $$\boxed{\ddot{X}+\frac{2K}{M}X=0}$$

Note that



$$\tan\theta = \frac{X_1-X}{D/2} = \frac{2X_1-2X}{D} = \frac{X_1-X_2}{D}$$

$$\Rightarrow \theta = \operatorname{Arctan}\left(\frac{X_1-X_2}{D}\right)$$

where $-\pi/2 < \theta < \pi/2$

By $I\ddot{\theta} = \Sigma$ torques

$$= \left(KX_2\cdot\frac{D}{2} - KX_1\cdot\frac{D}{2}\right)\cos\theta$$

Suppose θ is mall, so $\tan\theta \simeq \theta$, $\cos\theta \simeq 1$

$$\theta \simeq \frac{X_1-X_2}{D}$$

$$\Rightarrow I\ddot{\theta} = \frac{I}{D}(\ddot{x}_1 - \ddot{x}_2)$$

$$\Rightarrow \frac{I}{D}(\ddot{x}_1 - \ddot{x}_2) = \frac{KD}{2}(x_2 - x_1)$$

$$\Rightarrow \ddot{x}_1 - \ddot{x}_2 + \frac{KD^2}{2I}(x_2 - x_1) = 0$$

or $$\ddot{y} + \frac{KD^2}{2I} y = 0$$

where $y = x_1 - x_2$

(b) The normal modes frequencies are

$$\sqrt{\frac{2K}{M}} \text{ and } \sqrt{\frac{KD^2}{2I}}$$

If the rod is thin then

$$I = \frac{1}{12} MD^2$$

$$\Rightarrow \sqrt{\frac{KD^2}{2I}} = \cancel{\sqrt{\frac{6M}{K}}} \sqrt{\frac{6K}{M}}$$

Problem 7

(a) phase velocity: $V_p = \frac{\omega}{K} = A + Bk^2$

group velocity: $V_g = \frac{d\omega}{dk} = A + 3Bk^2$

if $A, B > 0$ then $\boxed{V_g > V_p}$

(b) The power spectrum will peak approximately at the natural frequency of the oscillator.

Period of oscillator = 20 sec

$\Rightarrow \quad \omega_{peak} \approx \frac{2\pi}{20} = \boxed{\frac{\pi}{10} \text{ radians/sec}}$

If amplitude decays as $\sim e^{-\frac{\Gamma}{2m}t}$, then FWHM = $\frac{\Gamma}{m}$

$\frac{1}{e} \approx 0.37 \Rightarrow \frac{\Gamma}{m} \approx \frac{1}{40 \text{ seconds}}$

$\Rightarrow \boxed{\text{FWHM} \simeq 0.025 \text{ s}^{-1}}$

(c) circularly polarized light:

$\bar{E} = E_0 \cos(\omega t - kz)\hat{x} + E_0 \sin(\omega t - kz)\hat{y}$

$I_0 = \langle S \rangle \propto \langle \bar{E}^2 \rangle = \langle E_0^2 \cos^2(\omega t - kz) + E_0^2 \sin^2(\omega t - kz) \rangle$

$= E_0^2$

After 1st polarizer:

$\bar{E} = E_0 \sin(\omega t - kz)\hat{y}$

After 2nd polarizer:

$\bar{E} = E_0 \cos\theta \sin(\omega t - kz)$

After 3rd polarizer:

$\bar{E} = E_0 \cos\theta \cos(90 - \theta) \sin(\omega t - kz)\hat{x}$

After 4^{th} polarizer:

$$\bar{E} = E_0 \cos\theta \sin\theta \cos\theta \sin(\omega t - kz)$$

$$\therefore I_f = \langle \bar{S} \rangle \propto \langle \bar{E}^2 \rangle = \langle E_0^2 \cos^4\theta \sin^2\theta \sin(\omega t - kz) \rangle$$

$$I_f = \frac{1}{2} E_0^2 \cos^4\theta \sin^2\theta$$

$$\Rightarrow \boxed{I_f = \frac{\cos^4\theta \sin^2\theta}{2} I_0}$$

To get $\emptyset$ transmitted intensity, reorder the polarizers so that two orthogonal polarizers are adjacent. For example,

$$\boxed{P_1 P_3 P_2 P_4}$$

(d) Happy Holidays!