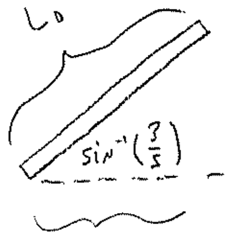
171.201, Fall '03 (written 10/05/05)

Midterm #1 solutions

(1) S'-frame:

(a)



$\Big\} \; L_y' = \frac{3}{5} L_0$

$L_x' = \frac{4}{5} L_0$

transform lengths to S-frame:

$L_y = L_y'$ (no length contraction perpendicular to motion)

$L_x = L_x' \sqrt{1 - v^2/c^2}$

If angle is 45° in S-frame, then $L_x = L_y$

$$\Rightarrow \quad L_y' = L_x' \sqrt{1 - v^2/c^2}$$

$$\frac{3}{5} \cancel{L_0} = \frac{4}{5} \cancel{L_0} \sqrt{1 - v^2/c^2}$$

$$\left(\frac{3}{4}\right)^2 = 1 - v^2/c^2 \quad \Rightarrow \quad \frac{7}{16} = \frac{v^2}{c^2}$$

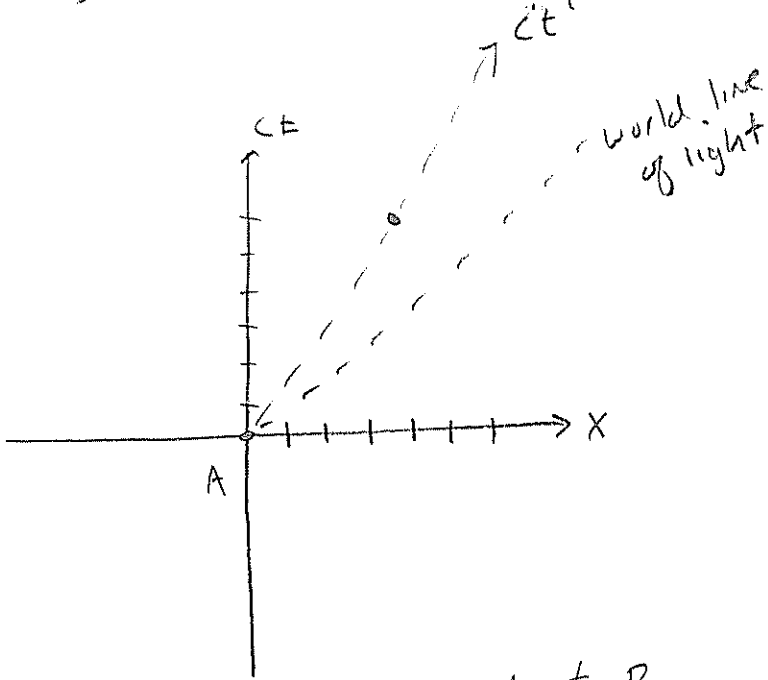
$$\boxed{v = \sqrt{7}/4 \; c}$$

(b) $L = \sqrt{L_x^2 + L_y^2} = \sqrt{2 L_y^2} = \sqrt{2} \; L_y'$

$$\boxed{L = \frac{3\sqrt{2}}{5} L_0}$$

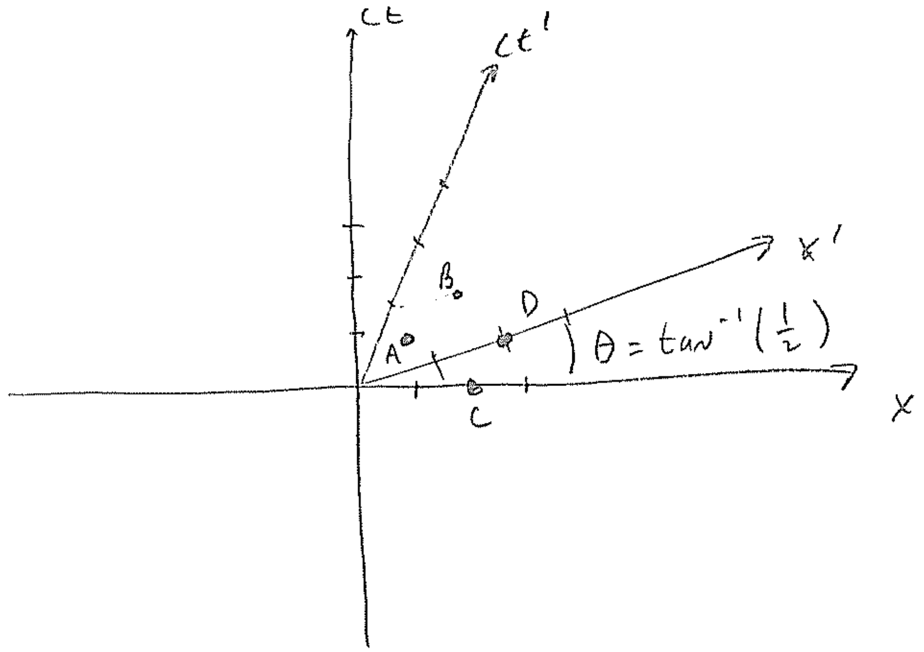
(2) A @ $ct = 0,\ x = 0$

B @ $ct = 6,\ x = 4$



Yes there is a reference frame where A & B occur at
the same place. Since B lies above the world line of
light, there is a reference frame in which the ct' axis
goes through B, implying both A & B occur at $x' = 0$.

3



Key features to get full credit.

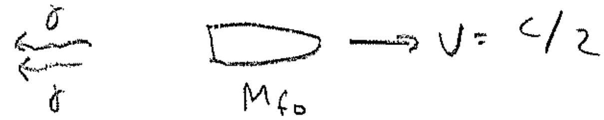
(1) slope of x' axis $= \frac{v}{c} = \frac{1}{2}$

(b) B is further from origin than A

(c) D is further to the right than C.

(4) before

after



(a) conservation of momentum: $\bar{P}_i = \bar{P}_f$

$$\bar{P}_i = 0$$

$$P_f = \frac{E_\gamma}{c} - M_f V = \frac{E_\gamma}{c} - \frac{M_{0f}}{\sqrt{1 - v^2/c^2}}\, v$$

$$\Rightarrow \quad \frac{M_{0f}}{\sqrt{1 - v^2/c^2}}\, v = E_\gamma / c$$

conservation of energy: $E_i = M_0 c^2$

$$E_f = E_\gamma + M_f c^2$$

$$\Rightarrow \quad \frac{M_{0f}}{\sqrt{1 - v^2/c^2}}\, c^2 + E_\gamma = M_0 c^2$$

$$\frac{M_{0f}}{\sqrt{1 - v^2/c^2}} \left(c^2 + vc \right) = M_0 c^2$$

$$M_{0f} = \frac{M_0 c^2 \sqrt{1 - v^2/c^2}}{c^2 + vc}$$

$$= M_0 \left(\frac{\sqrt{1 - 0.25}}{1 + 0.5} \right) = M_0 \frac{\sqrt{3}}{\cancel{2}} \, \frac{\cancel{2}}{3}$$

$$\boxed{M_{0f} = M_0 / \sqrt{3}}$$

(4) (b) revisit momentum equation:

$$\frac{E_\gamma}{c} = \frac{M_0 t}{\sqrt{1-v^2/c^2}}\, v = \frac{M_0/\sqrt{3}}{\sqrt{3}/2}\ \frac{1}{2} c$$

$$\boxed{E_\gamma = M_0 c^2/3}$$