Physics 171.201
Midterm Exam 1

October 5th, 2004

Answer all **four** problems. Be sure that you pace yourself so that you have enough time to work on each problem. Note that the problems do not have equal weight. Partial credit will be given, so be sure to **show your work** as clearly as possible**.** Good luck!

List of potentially useful formulae

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - (v/c^2)x}{\sqrt{1 - v^2/c^2}}$$

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2}$$

$$u_y' = \frac{u_y\sqrt{1 - v^2/c^2}}{1 - u_x v/c^2}$$

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$E = mc^2$$

$$\vec{p} = m\vec{v}$$

$$E^2 = m_0{}^2c^4 + p^2c^2$$

$$E^2 - p^2c^2 = E'^2 - p'^2c^2$$

Problem 1 (20 points)

Ben, standing at the rear end of a railroad car, shoots an arrow toward the front end of the car. The velocity of the arrow as measured by Ben is $\frac{1}{5}c$. The length of the car as measured by Ben is 150 meters. Alice, standing on the station platform observes all of this as the train passes by her with a velocity of $\frac{3}{5}c$. (Recall the speed of light c = $3x10^8$ m/s.) What values does Alice measure for the following quantities:

(a) The length of the railroad car?

(b) The velocity of the arrow?

(c) The length of the time the arrow is in the air?

(d) The distance that the arrow travels?

Problem 2 (15 points)

Draw a Minkowski diagram for the situation described in Problem 1 from Alice's perspective; that is, such that Alice's coordinates axes are orthogonal. Show on the diagram the worldlines of the front end and back end of railroad car and of the arrow. Also show the coordinate axes of Ben's reference frame. Be sure to specify the angles between various lines (including the axes) on the diagram and to include a couple of tick marks (with numbers) on each axis.

Problem 3 (25 points)

In class, we discussed two 4-vectors in special relativity, s = (ct, x, y, z) and π = (E, p_x, p_y, p_z). Another 4-vector can be composed from charge density ρ and current density $\vec{j}$ as follows:

$$\begin{pmatrix} c\rho \\ j_x \\ j_y \\ j_z \end{pmatrix}$$

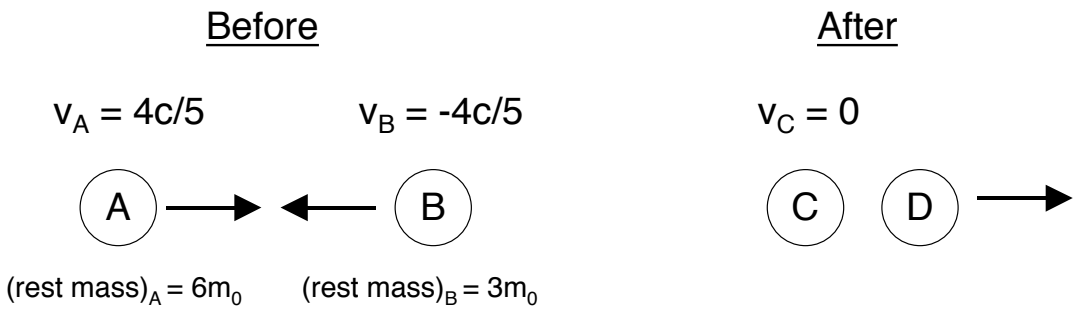
where j_x, j_y, and j_z are the components of $\vec{j}$ along x, y and z, respectively.

(a) Construct a quantity from a combination of ρ, j_x, j_y, and j_z that is a Lorentz invariant.

(b) Imagine you are in a reference frame in which ρ=2/c, j_x=2, j_y=2, and j_z=2. Determine the charge density and the three components of the current density observed by someone moving at a velocity $v=\sqrt{3/4}c$ along the x-direction with respect to your reference frame. To solve the problem, use the properties of 4-vectors that we discussed in class; i.e., don't try to solve the problem using E&M!

Problem 4 (40 points)

Consider a collision between two particles, A and B, in which the particles change into two new particles, C and D, as in the picture below. The rest mass of particle A is $6m_0$, and the rest mass of particle B is $3m_0$. In the laboratory frame, where the collision is viewed, A and B travel in opposite directions with the same magnitude of velocity, $\frac{4}{5}c$. After the collision, particle C is observed to be at rest.



If the rest mass of particle C is $10m_0$, then find in the lab frame

(a) the momentum of particle D.

(b) the total energy of particle D.

(c) the rest mass of particle D.

If the rest mass of particle C is $11m_0$, then find

(d) the velocity of particle D.