171.201, Fall 2004

Midterm #1 Solutions

Problem #1

(a) $L = L'\sqrt{1-v^2/c^2} = L'\sqrt{1-9/25} = 150 \cdot \frac{4}{5} = \boxed{120\,m}$

(b) $$u = \frac{u_x' + v}{1 + u_x' v/c^2} = \frac{\frac{1}{5}c + \frac{3}{5}c}{1 + \frac{3}{25}} = \frac{4/5\,c}{28/25} = \boxed{\frac{5}{7}c}$$

(c) $V_{arrow}\, t = (120 + V_{train}\, t)$

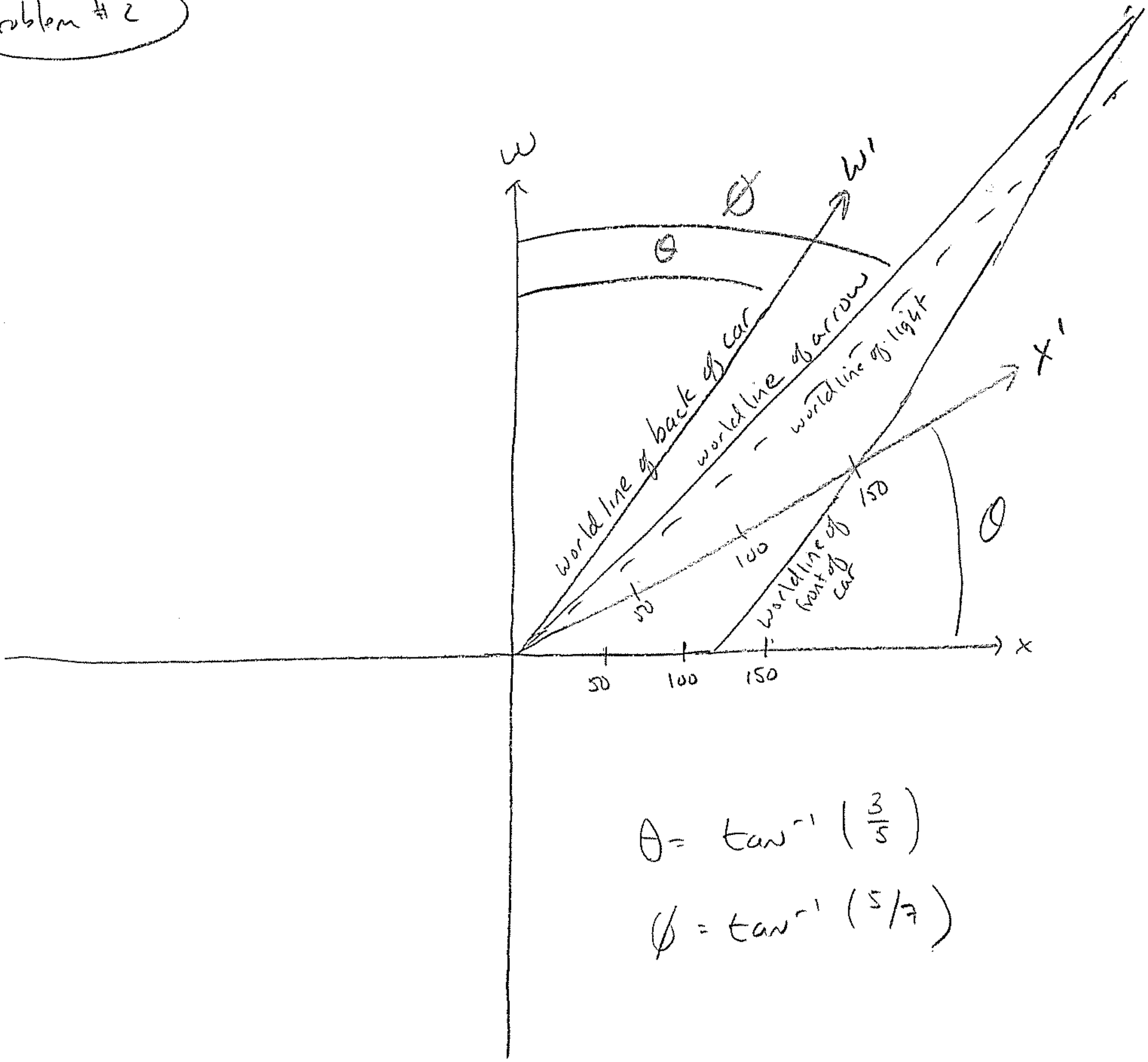
$$t = \frac{120}{V_{arrow} - V_{train}} = \frac{120}{\frac{5}{7}c - \frac{3}{5}c} = \frac{35 \cdot 120}{4 \times 3 \times 10^8} \text{ seconds}$$

$$= \boxed{3.5 \times 10^{-6} \text{ seconds}}$$

(d) $d = \frac{5}{7}c \times 3.5 \times 10^{-6}$

$$= \frac{5 \times 3 \times 3.5}{7} \times 10^2 = \boxed{750 \text{ meters}}$$

Problem # 2



$$\theta = \tan^{-1}\left(\frac{3}{5}\right)$$

$$\phi = \tan^{-1}\left(5/7\right)$$

Problem # 3

(a) like $s^2 = c^2t^2 - x^2 - y^2 - z^2$

& $\quad \pi^2 = E^2 - p_x^2c^2 - p_y^2c^2 - p_z^2c^2$

are Lorentz Invariants

$\boxed{c^2\rho^2 - j_x^2 - j_y^2 - j_z^2}$ is a Lorentz invariant

(b) 4-vectors all transform in the same way. So, we use the transformation equations we know from $\bar{x}, t$ to write down equivalent equations for J, ρ:

$$\rho' = \frac{\rho - \left(\frac{v}{c^2}\right) j_x}{\sqrt{1 - v^2/c^2}}$$

$$j_x' = \frac{j_x - v\rho}{\sqrt{1 - v^2/c^2}}$$

$$j_y' = j_y$$

$$j_z' = j_z$$

so, if $\quad c\rho = 2, \; j_x = 2, \; j_y = 2, \; j_z = 2 \quad ; \; v = \sqrt{\frac{3}{4}}\, c$

then

$$\boxed{\begin{aligned} \rho' &= \frac{2/c - \frac{\sqrt{3}}{2} \cdot 2/c}{\sqrt{1 - 3/4}} = (4 - 2\sqrt{3})/c \quad ; \quad j_y' = j_y \\ j_x' &= \frac{2 - \sqrt{\frac{3}{4}}\, 2/c}{1/2} = (4 - 2\sqrt{3}) \qquad\qquad J_z' = j_z \end{aligned}}$$

Problem # 4

(a) momentum is conserved.

$$\bar{P}_i = \bar{P}_f$$

$$M_A \bar{v}_A + M_B \bar{v}_B = \bar{P}_D$$

$$\frac{6M_0}{\sqrt{1-16/25}} \cdot \frac{4}{5}c - \frac{3M_0}{\sqrt{1-16/25}} \cdot \frac{4}{5}c = \bar{P}_D$$

$$\frac{5}{3} \cdot 3 \cdot \frac{4}{5} M_0 c = \bar{P}_D$$

$$\boxed{P_D = 4 M_0 c}$$

(b) energy is conserved

$$E_i = E_f$$

$$M_A c^2 + M_B c^2 = M_c c^2 + E_D$$

$$\frac{5}{3} \cdot 6 M_0 c^2 + \frac{5}{3} \cdot 3 M_0 c^2 = 10 M_0 c^2 + E_D$$

$$\boxed{5 M_0 c^2 = E_D}$$

(c)

$$E^2 = p^2 c^2 + M_0^2 c^4$$

$$(5 M_0 c^2)^2 + (4 M_0 c)^2 = M_{0D}^2 c^4$$

$$(25 - 16) M_0^2 \cancel{c^4} = M_{0D}^2 \cancel{c^4}$$

$$\boxed{M_{0D} = 3 M_0}$$

(d) If $M_c = 11 M_0$, then $E_D = 4 M_0 c^2 = P_D c$

$\Rightarrow E_D = P_D c$ which is case for massless particle

$\Rightarrow \boxed{V = c}$