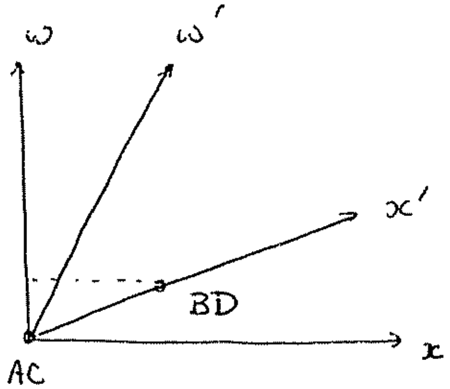
Soo Hyung Lee

Special Relativity & Waves
2005 Midterm Exam 1

Prob 1

a)



No, Alice sees the event AC first.

b) Invert the Lorentz transformation equation.

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

The distance measured by Alice is given by

$$x_2 - x_1 = \frac{x_2' + vt_2'}{\sqrt{1 - v^2/c^2}} - \frac{x_1' + vt_1'}{\sqrt{1 - v^2/c^2}} = \frac{x_2' - x_1' + vt_2' - vt_1'}{\sqrt{1 - v^2/c^2}}$$

Since $t_2' = t_1'$, we have

$$x_2 - x_1 = \frac{x_2' - x_1'}{\sqrt{1 - v^2/c^2}}$$

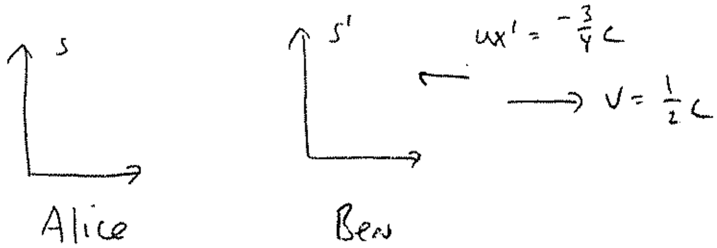
Therefore

$$L = \frac{L'}{\sqrt{1 - v^2/c^2}} = \frac{L'}{\sqrt{1 - (\frac{1}{2}c)^2/c^2}} = \frac{L'}{\sqrt{\frac{3}{4}}} = \boxed{\frac{2}{\sqrt{3}} L'}$$

Prob 1 continued

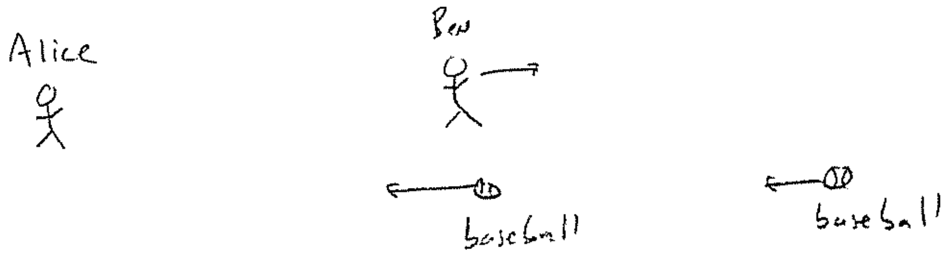
(c) Invert the velocity transformation

$$u_x = \frac{u_x' + v}{1 + u_x' v / c^2}$$



Plug in the values. $$u_x = \frac{-\frac{3}{4}c + \frac{1}{2}c}{1 - \frac{3}{4}\frac{1}{2}c^2/c^2} = \frac{-\frac{1}{4}c}{\frac{5}{8}} = \boxed{-\frac{2}{5}c}$$

(d)



The time between throws as measured by Alice is

$$\Delta t_t = \frac{\Delta t_t'}{\sqrt{1 - v^2/c^2}} = \frac{2}{\sqrt{3}}\Delta t_t' = \frac{2}{\sqrt{3}} \text{ seconds}$$

But, between throws Alice sees Ben move a distance $v\Delta t_t$

So each successive ball, in addition to being thrown later, must travel this extra distance. The time for the ball to travel this extra distance is

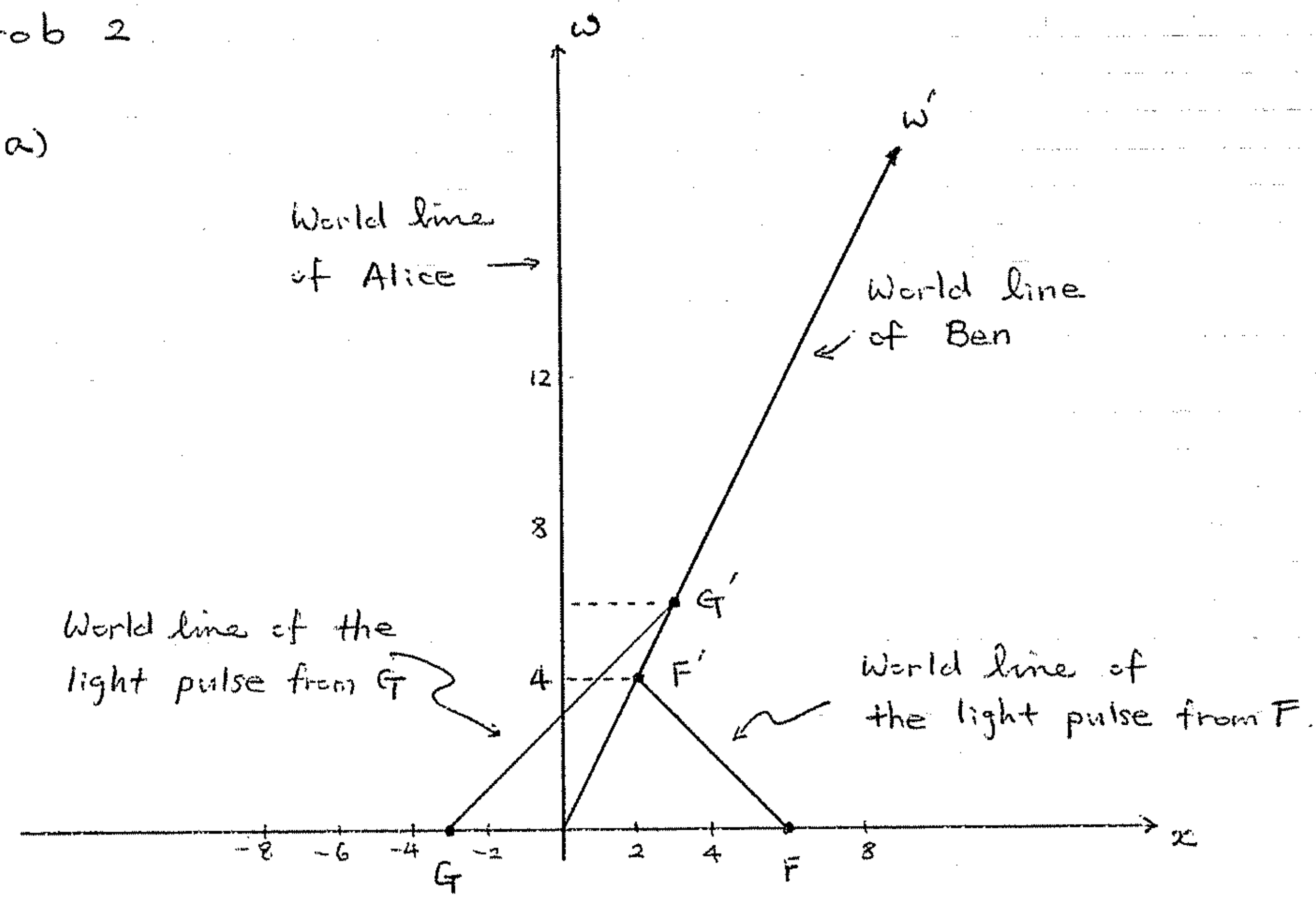
$$\Delta t_{fly} = \frac{v\Delta t_t}{u_x} = \frac{\frac{1}{2}c\,\frac{2}{\sqrt{3}}s}{2/5\,c} = \frac{5}{2\sqrt{3}} \text{ seconds}$$

So, all told the time between Alice's receiving successive baseballs is

$$\Delta t_{catch} = \Delta t_t + \Delta t_{fly} = \frac{2}{\sqrt{3}} + \frac{5}{2\sqrt{3}} = \boxed{\frac{9}{2\sqrt{3}} \text{ seconds}}$$

Prob 2

a)



Alice sees the light pulse from F reaches Ben first.

b) $$\omega'^2 - x'^2 = \omega^2 - x^2$$

$$\omega'^2 - 0 = [(4)^2 - (2)^2] \times 10^{16}\ m^2$$

$$\omega'^2 = (16 - 4) \times 10^{16}\ m^2$$

$$\omega' = \sqrt{12} \times 10^8\ m$$

To find time

$$t' = \frac{\omega'}{c} = \frac{\sqrt{12}\ \cancel{\times 10^8}\ m}{3 \times \cancel{10^8}\ m/s} = \boxed{\frac{2}{\sqrt{3}}\ \text{seconds}}$$

Prob 3

a)

$$E = M_{e^-}c^2 + M_{e^+}c^2 = \frac{M_o c^2}{\sqrt{1 - v_{e^-}^2/c^2}} + \frac{M_o c^2}{\sqrt{1 - v_{e^+}^2/c^2}}$$

$$= \frac{2 M_o c^2}{\sqrt{1 - (\frac{3}{5}\cancel{c})^2/\cancel{c^2}}} = \frac{2 M_o c^2}{\sqrt{1 - 9/25}} = \frac{2 M_o c^2}{4/5}$$

$$= \boxed{\frac{5}{2} M_o c^2}$$

$$\vec{p} = \vec{p}_{e^-} + \vec{p}_{e^+} = M_{e^-} v \,\hat{x} + M_{e^+} v \,\hat{y}$$

$$= \frac{M_o}{\sqrt{1 - v_{e^-}^2/c^2}} \frac{3}{5} c \,\hat{x} + \frac{M_o}{\sqrt{1 - v_{e^+}^2/c^2}} \cdot \frac{3}{5} c \,\hat{y}$$

$$= \frac{\cancel{5}}{4} M_o \cdot \frac{3}{\cancel{5}} c \,\hat{x} + \frac{\cancel{5}}{4} M_o \frac{3}{\cancel{5}} c \,\hat{y} = \frac{3}{4} M_o c \,(\hat{x} + \hat{y})$$

$$p^2 = p_x^2 + p_y^2 = \frac{9}{16} M_o^2 c^2 + \frac{9}{16} M_o^2 c^2 = \frac{9}{8} M_o^2 c^2$$

$$\boxed{|\vec{p}| = \frac{3}{2\sqrt{2}} M_o c}$$

b) Lab C.O.M.

$$E^2 - p^2c^2 = E'^2 - \cancelto{0}{p'^2c^2} = M_{ox}^2 c^4$$

$$\left(\frac{25}{4} - \frac{9}{8}\right) M_o^2 \cancel{c^4} = M_{ox}^2 \cancel{c^4}$$

$$M_{ox} = \sqrt{\frac{41}{8}} M_o = \boxed{\frac{\sqrt{41}}{2\sqrt{2}} M_o}$$

Prob 3, continued....

c) $$E_x = M_{o_x} c^2 + K$$

$$K = E_x - M_{ox} c^2$$

Since energy is conserved, $E_x = E$, which we found in part (a).

$$K = \frac{5}{2} M_o c^2 - \sqrt{\frac{41}{8}} M_o c^2$$

$$= \boxed{\left(\frac{5}{2} - \sqrt{\frac{41}{8}}\right) M_o c^2}$$

d) In order to conserve momentum, photon traveling in x-direction should have

$$P_{\gamma_x} = P_x = \frac{3}{4} M_o c$$

The same is true for the photon traveling in y-direction.

$$P_{\gamma_y} = P_y = \frac{3}{4} M_o C$$

The total energy is then

$$E_{\gamma_t} = P_{\gamma_x} C + P_{\gamma_y} C = \frac{3}{2} M_o C^2 \neq E = \frac{5}{2} M_o C^2$$

$\therefore$ $\boxed{\text{It is NOT possible}}$