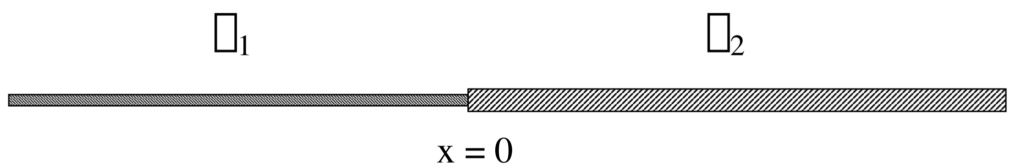
Physics 171.201
Midterm Exam 2

November 10th, 2004

Answer all **three** problems. Be sure that you pace yourself so that you have enough time to work on each problem. Note that the problems do not have equal weight. Partial credit will be given, so be sure to **show your work** as clearly as possible**.** Good luck!

Problem 1 (30 points)

Two very long segments of string are joined at $x = 0$ as shown. The segment for $x < 0$ has a mass per unit length of $\mu_1 = 0.16$ kilograms/meter, and the segment for $x > 0$ has $\mu_2 = 0.25$ kilograms/meter. The entire string is under a uniform tension $T_0 = 100$ N. Suppose a transverse traveling wave described by $y(x,t) = A\cos(\omega t - kx)$ is incident on the boundary ($x = 0$) from the left with amplitude $A = 1$ meter and frequency $\omega = 10$ radians/second.



(a) Calculate the amplitudes of the reflected and transmitted waves.

(b) What is the frequency and wavelength of the transmitted wave?

(c) What fraction of the energy of the incident wave gets transmitted?

Problem 2 (30 points)

A mass M is subject to a resistive force $\square b\dot{x}$ but *no* springlike restoring force.

(a) Show that the displacement of the mass as a function of time has the form $x(t) = C \square \frac{v_0}{\square} \exp(\square\square t)$, where C and v_0 are set by initial conditions. What is $\square$ in terms of M and b?

(b) At t = 0 the mass is at rest at x = 0. At this instant a driving force $F = F_0 \cos(\square t)$ is switched on. Find the values of A and $\square$ in the steady-state solution $x(t) = A \cos(\square t - \square)$.

> [HINT: Using complex notation is a convenient approach to solving this problem. However, if you use trigonometry, you might find the follow identities helpful:
>
> $$\cos(\square + \square) = \cos(\square)\cos(\square) - \sin(\square)\sin(\square)$$
> $$\sin(\square + \square) = \sin(\square)\cos(\square) - \cos(\square)\sin(\square)$$

(c) Write down the general solution for x(t) [as opposed to the steady-state solution] for the displacement in the presence of the driving force and find the values of C and v_0 for the initial conditions given in part (b). Sketch x as a function of t.

Problem 3 (40 points)

The picture below on the left shows a simple pendulum comprised of a light rigid rod of length ℓ supporting a mass M.

(a) Show that if the angular displacement □ of the pendulum is small, then the equation of motion for the mass can be written in terms of the lateral displacement x as: $\ddot{x} + \frac{g}{\ell}x = 0$.

Consider now two identical such pendulums coupled together by three springs of stiffness K as shown below on the right. Call the displacement of the mass on the left x_1 and that of the mass on the right x_2.

(b) Write down the coupled differential equations for the motion of each of the masses that describe small amplitude displacements from their equilibrium positions.

(c) Determine the normal mode frequencies for the system.

(d) Imagine that the two masses are initially at rest in equilibrium. At time t = 0, a bullet strikes the mass on the left imparting a sudden impulse that gives the mass a velocity v_0. The displacements and velocities of the masses at the instant after the bullet strikes are then:

$$x_1(0) = 0$$
$$\dot{x}_1(0) = v_0$$
$$x_2(0) = 0$$
$$\dot{x}_2(0) = 0$$

Find the ratio between the amplitudes of the two normal modes that describes the subsequent motion of the masses.