Physics 171.201
Midterm Exam 2

November 16th, 2005

Answer all **four** problems. Be sure that you pace yourself so that you have enough time to work on each problem. Note that the problems do not have equal weight. Partial credit will be given, so be sure to **show your work** as clearly as possible**.** Good luck!

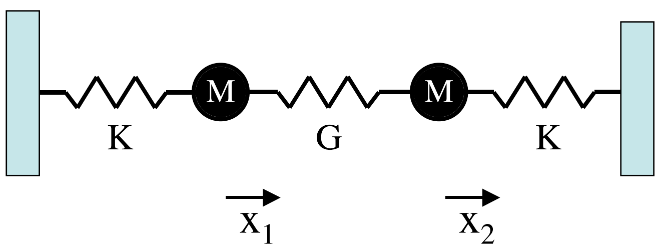
Problem 1 (30 points)

The picture below shows two undamped harmonic oscillators with mass M and spring constants K coupled together by a spring of spring constant G. Call the displacement of the mass on the left x_1 and that of the mass on the right x_2.

(a) Write down the coupled differential equations for the motion of each of the masses.

(b) Determine the normal mode frequencies for the system.

(c) Imagine that the mass on the left is pulled to a position $x_1 = x_0$ and held still while the mass on the right is held still at its equilibrium position. At time $t = 0$, the masses are released from rest from these positions. Assuming $G << K$, find the approximate time that the two masses return to precisely the same positions that they have at $t = 0$; that is, find the next time that $x_1 = x_0$, $x_2 = 0$.



Problem 2 (20 points)

Consider a continuous string of mass density ρ, and length L under a tension T. One end of the string is clamped down so that it cannot move. The other end is free. Find that wavelengths and frequencies of the first three normal modes of the string. Recall that the boundary condition at the free end corresponds to the condition that the first derivative of the displacement at the end must be zero.

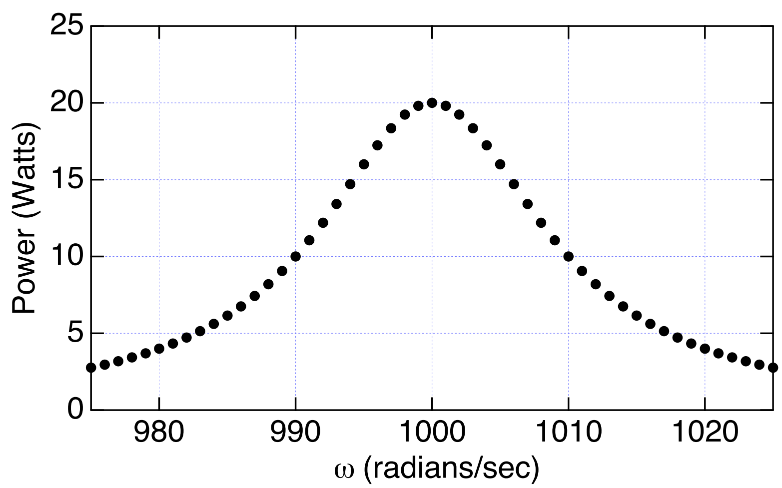
Problem 3 (20 points)

The graph below shows the mean power absorbed by a forced mechanical oscillator as a function of the frequency of the driving force.

(a) What is the quality factor Q of the system?

(b) At resonance, what is the amount of work done by the driving force averaged over a full period of oscillation?

(c) If the driving force is turned off, how many seconds will elapse as the energy in the oscillator decays from an initial value of E_0 to a value $E = E_0 e^{-1}$.



Problem 4 (30 points)

Consider a very long string of mass density ρ under a tension T. The end of the string is attached to a massless dashpot as shown in the picture below. The dashpot is applies a frictional drag proportional to its velocity, $F_{fric} = -C\dot{y}$, where y is the displacement of the dashpot and C is a constant. A traveling wave with amplitude A and frequency ω propagates on the string toward the dashpot.

(a) Imagine that the frictional drag coefficient C is tuned so that none of the wave incident on the dashpot is reflected. What is the value of C in terms of ρ and T?

(b) Imagine instead that the frictional drag coefficient C is exactly one half of the value you determined in part (a). What is the amplitude of the wave that is reflected back up the string from the dashpot in terms of A?

(c) For the case in part (b), determine the time-averaged power absorbed by the dashpot.