Missing Energy ($E_T$) at the LHC: The Dark matter Connection

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(Collaborators: Ian-Woo Kim, J.H. Song)
Outline

Missing Energy Events
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Summary
Missing Energy Events

Pauli’s “Neutron”, Fermi’s “Neutrino”
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In $\beta$ decay, the electron energy spectrum is continuous:

$$^3H \rightarrow ^3He^+ + e^- + \nu_e$$ (hep-ex/0109033).

*KATRIN experiment: $^3H \rightarrow ^3He^+ + e^- + \nu_e$ (hep-ex/0109033).
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For a 3-body decay, $M \rightarrow abc$, the kinetic energy of $a$:

$$0 \leq K_a \leq \frac{(M - m_\alpha)^2 - (m_b + m_c)^2}{2M}.$$

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⇒ The non-detectable nature introduced the 1\(^{st}\) “dark matter”.

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† Cowan-Reines in 1956: $\bar{\nu}_e + p \rightarrow e^+ + n$.
† Lederman-Schwartz-Steinberger in 1962 (BNL): $\nu_\mu + Al \rightarrow \mu + X$.
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“Dark matter direct detection”.
\( W^\pm \) and Missing Energy

- The discovery of \( W^\pm \rightarrow \ell \nu_\ell \) (UA1/UA2 in 1983):

**Experimental Observation of Isolated Large Transverse Energy Electrons with Associated Missing Energy at \( \sqrt{s} = 540 \text{ GeV} \)**

UA1 Collaboration, CERN, Geneva, Switzerland

![Graph showing missing transverse energy and number of events](image)
At the Tevatron Run II:

**Missing E$_T$ - W Candidate**

![Graph showing Missing E$_T$ - W Candidate](image)

- Data
- PMCS+QCD
- QCD bkg

D0 Run II Preliminary
At the Tevatron Run II:

The transverse momentum of $\nu$ or $e$ has a Jacobian peak:

$$p_T = E \sin \theta,$$

$$\frac{d\hat{\sigma}}{dm_{\nu e}^2 dp_{eT}^2} \propto \frac{\Gamma_W M_W}{(m_{\nu e}^2 - M_W^2)^2 + \Gamma_W^2 M_W^2} \frac{1}{m_{\nu e}^2 \sqrt{1 - 4p_{eT}^2/m_{\nu e}^2}}.$$
Transverse mass variable $W \rightarrow e\nu$:

$$m_{e\nu}^T = (E_{eT} + E_{\nu T})^2 - (p_{eT} + p_{\nu T})^2 = 2E_{eT}E_{T}^{\text{miss}}(1 - \cos \phi) \leq m_{e\nu}^2.$$
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⇒ If $p_T(W) = 0$, then: $m_{e\nu} T = 2E_{eT} = 2E_{T}^{miss}$. 

Transverse Mass - W Candidate

Missing E$_T$ - W Candidate
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⇒ If $p_T(W) = 0$, then: $m_{e\nu}^2_T = 2E_{eT} = 2E_{T}^{\text{miss}}$.

⇒ If $p_T(W) \neq 0$ (some transverse motion $\delta P_V$), then:

\[
p'_{eT} \sim p_{eT} \left[1 + \delta P_V/M_V\right],
\]

\[
m_{e\nu}^2_T \sim m_{e\nu}^2_T \left[1 - (\delta P_V/M_V)^2\right],
\]

\[
m_{e\nu}^2 = m_{e\nu}^2.
\]
Large(r) missing energy events at the Tevatron:

SM prediction:
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First SUSY bound: CDF with $25.3 \text{ nb}^{-1}$ (!) (1989)
No events found with $E_T > 40 \text{ GeV} \Rightarrow \sigma_{\text{MSSM}} < 0.1 \text{ nb}$
$\Rightarrow m_{\tilde{g}}, m_{\tilde{q}} > 80 \text{ GeV}.$
Large(r) missing energy events at the Tevatron:

**SM prediction:**

![Graph showing Missing E_T for Low Kinematic Region]

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$\Rightarrow m_{\tilde{g}}, m_{\tilde{q}} > 80 \text{ GeV}.$

Current SUSY bound: CDF with $2 \text{ fb}^{-1}$

$\Rightarrow \sigma_{MSSM} < 0.1 \text{ pb}$

$\Rightarrow m_{\tilde{g}} > 320 \text{ GeV}, m_{\tilde{q}} > 390 \text{ GeV}.$
Missing energy events in $e^+e^-$ collisions

At LEP I (L3):
Neutrino counting:
$$e^+e^- \rightarrow \gamma + \nu_i\bar{\nu}_i$$
$N_\nu \approx 3$. 

![Graph showing neutrino events](image-url)
New Physics Expectation in $E_T$:

$\dagger$ M. Mangano, arXiv:0809.1567 [hep-ph].
Missing Energy and New Physics at LHC

New Physics Expectation in $E_T$:

- Setting a bound for mass scale may not be too hard.
- Establishing $E_T$ signal would be challenging,
  \Rightarrow that would be a revolutionary discovery for BSM physics!

\[ M. \text{ Mangano, arXiv:0809.1567 [hep-ph].} \]
It has been shown quite promising (mSUGRA at ATLAS$^\dagger$)

Dark matter connection: LHC vs. Cosmology

Steps to follow:

- Discover missing-energy events at a collider and estimate the mass of the WIMP.
- Observe dark matter particles in direct detection experiments and determine whether their mass is the same as that observed in collider experiments.

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Cosmic relic density:

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma v \rangle} \sim \frac{m_\chi^2}{\alpha^2}.$$ 

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After that,

- Determine the qualitative physics model that leads to missing-energy events.
- Determine the parameters of this model that predict the relic density.
- Determine the parameters of this model that predict the direct and indirect detection cross sections.
- Measure products of cross sections and densities from astrophysical observations to reconstruct the density distribution of dark matter.

\[\text{§Baltz, Battaglia, Peskin and Wizansky, hep-ph/0602187.}\]
Optimistic conclusions were obtained for mSUGRA and for MSSM parameter-determinations:

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For most general cases, situations may be much more complex:

The “LHC inverse problem”: Data \( \Rightarrow \) many possible solutions!


Determining the Dark Matter Mass

- Model-independent approaches at colliders
Determining the Dark Matter Mass

– Model-independent approaches at colliders

The difficulties:

- Two missing particles in each event;
- Unknown parton frame leads to less constrained kinematics.
Edges, End-points etc.

- Simple decay chain:††

\[
\begin{align*}
\text{In general, } m_{\ell\ell}^{\text{max}} &= M_Z - M_X \quad \text{(gives mass difference).} \\
\text{If } Y \text{ is also on-shell, } m_{\ell\ell}^{\text{max}} &= \sqrt{(M_Z^2 - M_Y^2)(M_Y^2 - M_X^2)}/M_Y.
\end{align*}
\]

Longer decay chain‡‡

Similarly, \( m_{q\ell\ell}^{\text{max}} = \sqrt{(M_q^2 - M_{\tilde{\chi}_2}^2)(M_{\tilde{\chi}_2}^2 - M_{\tilde{\chi}_1}^2)} / M_{\tilde{\chi}_2}. \)

Longer decay chain

\[ m_{q\ell\ell}^{\text{max}} = \sqrt{(M_{\tilde{q}}^2 - M_{\tilde{\chi}_2}^2)(M_{\tilde{\chi}_2}^2 - M_{\tilde{\chi}_1}^2)}/M_{\tilde{\chi}_2}. \]

† Only probe mass differences.
† May encounter combinatoric ambiguities.

Fully Constructable Kinematics

Kinematical on-shell conditions

Assume:

- **n signal events**: particles 3, 5, 7; 4, 6, 8 observed; 1, 2 missing.
- **Unknowns**: masses \( N, X, Y, Z \) (4); 4-momenta of 1, 2 (8n) \( \Rightarrow 4 + 8n \).

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Let constraints $\geq$ unknowns ⇒ $n \geq 2$.

† With many events ($n$), it’s an over-constrained system.
† If only 3 on-shell particles in each chain,
  there will be fewer constraints than unknowns.

Simulated results.

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Remarks:
- Very selective channels.
- Very restrictive kinematics.
- Realistic experimental conditions will further dilute the solutions.

Transverse Mass Variables $M_{T2}$

In the attempt to determine the absolute masses (parent and missing one), without fully reconstructing the events, $M_{T2}$ was introduced.

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Recall the invariant mass/transverse mass of $ab$ (or $e\nu$):

$$m_{ab}^2 = m_a^2 + m_b^2 + 2(E_T^a E_T^b \cosh \Delta \eta - \vec{p}_T^a \cdot \vec{p}_T^b) \geq m_T^2.$$ 

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Consider a pair production/decay $D_1 \rightarrow a_1 \ b_1$, $D_2 \rightarrow a_2 \ b_2$:

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m_D^2 \geq \max(m_{TD1}^2, \ m_{TD2}^2).
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$$m_D^2 \geq \max(m_{TD_1}^2, m_{TD_2}^2).$$

Only knowing $|\vec{p}_{Tb_1} + \vec{p}_{Tb_2}| = E_T$, one defines:

$$M_{T2}^2(m_{a1}, m_{a2}; m_b) = \min_{|\vec{p}_{Tb_1} + \vec{p}_{Tb_2}| = E_T} [\max(m_{T1}^2, m_{T2}^2)].$$

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This is a “functional”:†

* For each event ($E_T$), run through trial $\vec{p}_{Tb1}$ and $\vec{p}_{Tb2} = \vec{E}_T - \vec{p}_{Tb1}$:
  → It is smaller than the true $\max(m_{TD1}^2, m_{TD2}^2)$;
  → With many events, it still doesn’t go over it.

Thus, one defines:

\[
M_{T2}^{\text{max}}(m_b) = \max_{\text{all events}} M_{T2}(m_{a1}, m_{a2}; m_b).
\]

a function of the trial missing mass \( m_b \).

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The kink structure:  

When varying the trial missing mass below to above the true value of \( m_b \), the curve \( M_{T2}^{max}(m_b) \) (for multi-body decay) changes the slope:

\[ \text{Gluino transverse mass (max)} \]

\[ M_x \] (GeV)

\[ 700 \quad 750 \quad 800 \quad 850 \quad 900 \]

\[ 0 \quad 50 \quad 100 \quad 150 \quad 200 \quad 250 \]

\[ \text{Heavy squark} \]


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\[ M_{x} \text{ (GeV)} \]

\[ \text{Gluino transverse mass (max)} \]

- For simple 2-body decay, no clear kink;
- For multi-body decays, combinatorics dilute the kink.

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$D$, a SM-like particles; $B, X$ carry a new quantum number.

“Antler Decay” Kinematics

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Advantages:

- More constrained kinematics: $M_D$ is known from other SM modes.

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The “Antler decay”

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Advantages:

- More constrained kinematics: \(M_D\) is known from other SM modes.
- Many channels:

  - **MSSM:** \(H \to \tilde{\chi}_2^0 + \tilde{\chi}_2^0 \to Z\tilde{\chi}_1^0 + Z\tilde{\chi}_1^0;\)
  
  - **Z’ SUSY:** \(Z’ \to \tilde{\ell}^+ + \ell^- \to \ell^- \tilde{\chi}_1^0 + \ell^+ \tilde{\chi}_1^0;\)
  
  - **UED:** \(Z^{(2)} \to L^{(1)} + L^{(1)} \to \ell^+ \gamma^{(1)} + \ell^- \gamma^{(1)};\)
  
  - **LHT:** \(H \to t^- + \bar{t}^- \to tA_H + \bar{t}A_H.\)
  
  - **ILC:** \(e^+e^- \to B_1 + \bar{B}_2 \to a_1X_1 + a_2X_2.\)

A new kinematical feature: cuspy structures!

\[ m \propto d \Gamma \frac{1}{d m} \text{[GeV]} \]

\[ p_T \propto d \Gamma \frac{1}{d p_T} \text{[GeV]} \]
A new kinematical feature: cuspy structures!

Pronounced “peaks” appear, suitable for observation!
Origin of the cusps:
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Limiting cases (at the corners)

\[ a_2 X_2 \leftarrow B_2 \leftarrow D \Rightarrow B_1 \rightarrow a_1 X_1 \]

- Back-to-back: \( (\cos \theta_1, \cos \theta_2) = (+1, -1) \quad \Leftarrow + \Rightarrow \)
  Maximum \( M_{aa} \) configuration.

- Head-on: \( (\cos \theta_1, \cos \theta_2) = (-1, +1) \quad \Rightarrow + \Leftarrow \)
  Medium \( M_{aa} \) configuration.

- Parallel: \( (\cos \theta_1, \cos \theta_2) = (\pm 1, \pm 1) \quad \Rightarrow + \Rightarrow, \quad \Leftarrow + \Leftarrow \)
  Zero \( M_{aa} \) configurations.
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  Zero \(M_{aa}\) configurations.
- Upon variable projection (losing info), singularities may be developed.
- It is purely kinematical, and new (rigorous singularity theorems in math).
The rapidities $\eta$ and $\zeta$ in the parent-rest frame:
\[
cosh \eta = \frac{m_D}{2m_B} \equiv c_\eta, \quad \cosh \zeta = \frac{m_B^2 - m_X^2 + m_a^2}{2m_a m_B} \equiv c_\zeta,
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thus: $\eta, \zeta$ (plus $m_D$) $\implies m_B, m_a$. 
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- **Cusp and Edge: ($M_a = 0$ case)**

The end-point, instead of being $M_{aa}^{\text{max}} = m_D - 2m_X$, becomes

$$M_{aa}^{\text{max}} = m_B \left(1 - \frac{m_X^2}{m_B^2}\right) e^{\eta},$$

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Thus,
\[
\frac{M_{aa}^{\text{max}}}{M_{aa}^{\text{cusp}}} = e^{2\eta}, \quad (D \to B)
\]
\[
M_{aa}^{\text{max}} M_{aa}^{\text{cusp}} = m_B^2 \left(1 - \frac{m_X^2}{m_B^2}\right)^2. \quad (B \to X)
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Algebraically/graphically,

\[ \frac{d\Gamma}{dM_{aa}} \propto \begin{cases} 
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<thead>
<tr>
<th>Mass</th>
<th>$m_D$ (GeV)</th>
<th>$m_B$ (GeV)</th>
<th>$m_a$ (GeV)</th>
<th>$m_X$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass I</td>
<td>1250</td>
<td>600</td>
<td>0</td>
<td>550</td>
</tr>
<tr>
<td>Mass II</td>
<td>1000</td>
<td>440</td>
<td>0</td>
<td>300</td>
</tr>
<tr>
<td>Mass III</td>
<td>1000</td>
<td>350</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>Mass IV</td>
<td>600</td>
<td>250</td>
<td>$m_Z$</td>
<td>100</td>
</tr>
</tbody>
</table>

Mass I: “near threshold case” ($Z^{(2)}$ decay in the UED model).
Mass II: “boundary case” ($m_B \approx 0.44m_D$).
Mass III: “large mass gap case”.
Mass IV: “massive case” ($Z, t, ...$ in the final state).
Massive SM final state: \((M_a \neq 0)\)

For a massive case \(a = Z, t, \ldots, d\), \(d\Gamma/dM_{aa}\) may develop two cusps:
Cusp in Angular Distribution: \( (M_a = 0) \)

\( \Theta \) is the angle of a visible particle \((a_1)\) in the \(a_1a_2\) c.m. frame with respect to the c.m. moving direction. Then

\[
\frac{d\Gamma}{d \cos \Theta} \propto \begin{cases} 
\sin^{-3} \Theta, & \text{if } |\cos \Theta| \leq \tanh \eta, \\
0, & \text{otherwise}.
\end{cases}
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\[|\cos \Theta|_{\text{max}} = \tanh \eta = \sqrt{1 - \frac{4m_B^2}{m_D^2}}.\]
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\[\text{Complementarity: Large-mass gap worse for } M_{aa}, \text{ better for } \cos\Theta.\]
“Robustness” of the proposal

(a). Back to the lab-frame: Lorentz boost
\[ \Rightarrow M_{aa} \text{ not effected, } \cos \Theta \text{ peaks diluted:} \]

(b). Dynamical effects: matrix elements, spin-correlations etc.
\[ \Rightarrow M_{aa}, \cos \Theta \text{ not appreciably effected,} \]
(c). Off-shell decays: finite width effects

\[ m_B = 600 \, \text{GeV} \]

\[ \Rightarrow \Gamma_B \approx 10\% \text{ not good anymore.} \]
On-going studies: †

- Reconstruct the antler kinematics:

\[ D, a \text{ SM-like particles; } B \text{ (on-shell) and } X \text{ (missing)}. \]

†TH, I.-W. Kim and J. Song, in progress.
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![Diagram of antler kinematics]

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MSSM: \( H \to \tilde{\chi}^0_2 + \tilde{\chi}^0_2 \to Z\tilde{\chi}^0_1 + Z\tilde{\chi}^0_1; \)

Z' SUSY: \( Z' \to \ell^+ + \ell^- \to \ell^-\tilde{\chi}^0_1 + \ell^+\tilde{\chi}^0_1; \)

UED: \( Z^{(2)} \to L^{(1)} + L^{(1)} \to \ell^+\gamma^{(1)} + \ell^-\gamma^{(1)}; \)

LHT: \( H \to t + \bar{t} \to tA_H + \bar{t}A_H. \)

ILC: \( e^+e^- \to B_1 + \bar{B}_2 \to a_1X_1 + a_2X_2. \)

\[^\dagger\] TH, I.-W. Kim and J. Song, in progress.
On-going studies:

- Reconstruct the antler kinematics:

  \[ D, a \text{ SM-like particles; } B \text{ (on-shell) and } X \text{ (missing)} \].

  \[
  \begin{align*}
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  \text{ILC:} & \quad e^+e^- \rightarrow B_1 + \bar{B}_2 \rightarrow a_1X_1 + a_2X_2.
  \end{align*}
  \]

- Other channels with cusps:

  \[ \dagger \text{ Decay chain kinematics: cusps as well.} \]
  \[ \dagger \text{ Multi-particle final states: some dilution.} \]

\[ \dagger \text{TH, I.-W. Kim and J. Song, in progress.} \]
\[ \dagger \text{A. Agashe, M. Toharia et al.; P. Osland, Miller et al.} \]
Determining the missing particle mass of fundamental importance.

  e.g.: Ever since the neutrino was proposed and observed, its mass measurement is still actively pursued.
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We are all eagerly waiting for the excitement from the LHC!