

***D*-Dimensional Generalized Unitarity and Top Quark Physics**

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in collaboration with Kirill Melnikov



Introduction

- I will talk about next-to-leading order QCD correction
- At NLO final states are modelled more realistically
scale dependencies are less severe than for leading order
- Typically, NLO predictions are in excellent agreement with data
- LO predictions can be tuned by choosing appropriate scales
(but: What is "appropriate"?, How do cuts affect those predictions?)

Introduction

- Physics at the LHC: predictions for many-particle final states are of interest
(signal processes or background to BSM physics)
- NLO QCD predictions for many processes are still not available

NLO:

- one-loop virtual corrections
- real emission corrections

Outline

- **Introduction**
- **Virtual corrections**
 - ▶ Basic ideas of unitarity based methods
 - ▶ OPP algorithm, D -dimensional generalized unitarity
 - ▶ Massive fermions, top quark amplitudes
- **Real emission corrections**
 - ▶ Dipole formalism
- **Top quark phenomenology**
 - ▶ Top quark pair production and leptonic decay at NLO
 - ▶ Various distributions, spin observables for LHC & Tevatron

Virtual Corrections

Challenges

- large expressions for final states with more than 3 particles
(\Rightarrow numerical implementation)
- numerical stability
(\Rightarrow accuracy)
- fast computer implementation
(\Rightarrow efficiency)

Approaches

The *traditional* way to do a 1-loop calculation:

- generate all Feynman diagrams
- reduction of tensor integrals to scalar integrals
- reduction to minimal set of spin and color structures
- numerical evaluation

This talk: ***D-Dimensional Generalized Unitarity***

- completely orthogonal approach
 - basic ingredients are tree level amplitudes
 - better scaling with increasing number of external legs
 - method is ready for phenomenology
- application: **Top Quark Pair Production**

Basic ideas

***D*-Dimensional Generalized Unitarity**

Basic ideas

cuts in $D \neq 4$ dimensions

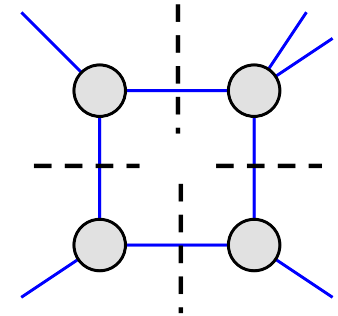
[van Neerven]
[Bern, Morgan]
[Giele, Kunszt, Melnikov]

D-Dimensional

Generalized

cut four propagators

[Britto, Cachazo, Feng]



Unitarity

[Bern, Dixon, Dunbar, Kosower]

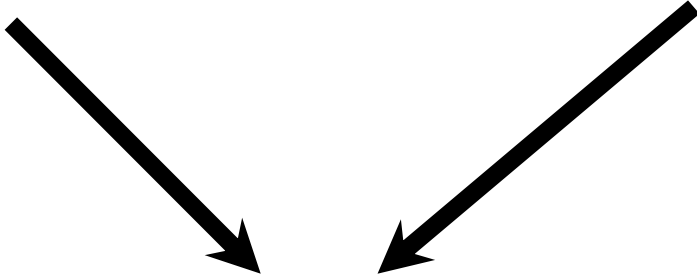
optical theorem

$$2\text{Im} \left(\text{Diagram with bubble} \right) = \int d\Pi \left| \text{Diagram with cut} \right|^2$$

Basic ideas

Master Integral Basis

Unitarity

$$A^{1\text{-loop}} = \sum_j c_j I_j$$
$$I = \int \frac{d^D \ell}{2\pi} \frac{1}{D_0 \dots D_N}$$
$$\text{Im} (A^{1\text{-loop}}) \sim \int d\Pi |A^{\text{tree}}|^2$$

$$\sum_j c_j \text{Im} (I_j) \sim \int d\Pi |A^{\text{tree}}|^2$$

we know how to get the coefficients c_j by calculating tree amplitudes

Organization of the calculation

Color ordering

[Bern, Dixon, Kosower]:

$$A^{1\text{-loop}} = \sum_i C_i A_i^{\text{prim.}}$$

color factor

color ordered primitive amplitude

primitive amplitude:

- fixed ordering of external legs
- color factors are stripped off
- gauge invariant
- minimal set of building blocks

Tensor integral reduction

traditional approach: (improved) Passarino-Veltman reduction

OPP algorithm: • meshes very well with the cut-based unitarity method

[Osso]a, Papadopoulos, Pittau]

• a tensor integral reduction at the integrand level

$$\mathcal{A}^{\text{prim}} = \int d^D \ell \frac{\text{Num}(\ell, \{p_i\})}{D_1 \dots D_N}$$

$$D_k = (\ell + p_k)^2 - m_k^2$$

for the moment:



OPP algorithm

1. partial fractioning:

master formula

$$\frac{\text{Num}(\ell, \{p_i\})}{D_1 \dots D_N} = \sum_{[i|j|k|l]} \frac{\bar{d}_{ijkl}(\ell)}{D_i D_j D_k D_l} + \sum_{[i|j|k]} \frac{\bar{c}_{ijk}(\ell)}{D_i D_j D_k} + \sum_{[i|j]} \frac{\bar{b}_{ij}(\ell)}{D_i D_j} + \sum_{[i]} \frac{\bar{a}_i(\ell)}{D_i}$$

2. choose vector basis that spans *physical space* $\{p_i\}$ + *transverse space* $\{n_i\}$ coefficients can be decomposed into *integral coefficients* and *spurious terms*

$$\bar{c}(\ell) = c + \tilde{c}_{1..r} \times \underbrace{(n_1^{\mu_1} \dots n_r^{\mu_r} \ell_{\mu_1} \dots \ell_{\mu_r})}_{\text{vanish after loop integration}}$$

$$n_i \cdot p_j = 0$$

$$\sum_j c_j I_j$$

vanish after loop integration

OPP algorithm

1. partial fractioning:

master formula

$$\frac{\text{Num}(\ell, \{p_i\})}{D_1 \dots D_N} = \sum_{[i|j|k|l]} \frac{\bar{d}_{ijkl}(\ell)}{D_i D_j D_k D_l} + \sum_{[i|j|k]} \frac{\bar{c}_{ijk}(\ell)}{D_i D_j D_k} \\ + \sum_{[i|j]} \frac{\bar{b}_{ij}(\ell)}{D_i D_j} + \sum_{[i]} \frac{\bar{a}_i(\ell)}{D_i}$$

3. assuming that we know the LHS, i.e. $\text{Num}(\ell, \{p_i\})$, we can evaluate this equation for different values of ℓ , so that the determination of the coefficients is a purely algebraic problem

OPP algorithm

1. partial fractioning:

master formula

$$\frac{\text{Num}(\ell, \{p_i\})}{D_1 \dots D_N} = \sum_{[i|j|k|l]} \frac{\bar{d}_{ijkl}(\ell)}{D_i D_j D_k D_l} + \sum_{[i|j|k]} \frac{\bar{c}_{ijk}(\ell)}{D_i D_j D_k} \\ + \sum_{[i|j]} \frac{\bar{b}_{ij}(\ell)}{D_i D_j} + \sum_{[i]} \frac{\bar{a}_i(\ell)}{D_i}$$

- 3.** assuming that we know the LHS, i.e. $\text{Num}(\ell, \{p_i\})$, we can evaluate this equation for different values of ℓ , so that the determination of the coefficients is a purely algebraic problem
- 4.** OPP tell us that we should choose ℓ such that some denominators vanish
 \Rightarrow this leads to an efficient recursive determination of the coefficients

OPP algorithm

Example: 4 particle process

master formula for N=4

$$\frac{\text{Num}(\ell, \{p_i\})}{D_1 \dots D_4} = \frac{\bar{d}_{1234}(\ell)}{D_1 D_2 D_3 D_4} + \sum_{1 < i < j < k < 4} \frac{\bar{c}_{ijk}(\ell)}{D_i D_j D_k} + \sum_{1 < i < j < 4} \frac{\bar{b}_{ij}(\ell)}{D_i D_j} + \sum_{1 < i < 4} \frac{\bar{a}_i(\ell)}{D_i}$$

Step 1: multiply by $D_1 D_2 D_3 D_4$ and choose ℓ such that $D_1 = D_2 = D_3 = D_4 = 0$

⇒ there are two complex solutions ℓ_{\pm}

$$\text{Num}(\ell_{\pm}, \{p_i\}) = \bar{d}_{1234}(\ell_{\pm})$$

OPP algorithm

Example: 4 particle process

master formula for N=4

$$\frac{\text{Num}(\ell, \{p_i\})}{D_1 \dots D_4} = \frac{\bar{d}_{1234}(\ell)}{D_1 D_2 D_3 D_4} + \sum_{1 < i < j < k < 4} \frac{\bar{c}_{ijk}(\ell)}{D_i D_j D_k} + \sum_{1 < i < j < 4} \frac{\bar{b}_{ij}(\ell)}{D_i D_j} + \sum_{1 < i < 4} \frac{\bar{a}_i(\ell)}{D_i}$$

Step 1: multiply by $D_1 D_2 D_3 D_4$ and choose ℓ such that $D_1 = D_2 = D_3 = D_4 = 0$

⇒ there are two complex solutions ℓ_{\pm}

$$\text{Num}(\ell_{\pm}, \{p_i\}) = \bar{d}_{1234}(\ell_{\pm}) = d_{1234} + \tilde{d}_{1234}(\ell) \ell.n_1$$

⇒ evaluate equation for two solutions ℓ_{\pm} and solve for $d_{1234}, \tilde{d}_{1234}$

OPP algorithm

Example: 4 particle process

Step 2:

$$\frac{\text{Num}(\ell, \{p_i\})}{D_1 \dots D_4} - \frac{\bar{d}_{1234}(\ell)}{D_1 D_2 D_3 D_4} = \sum_{1 < i < j < k < 4} \frac{\bar{c}_{ijk}(\ell)}{D_i D_j D_k} + \sum_{1 < i < j < 4} \frac{\bar{b}_{ij}(\ell)}{D_i D_j} + \sum_{1 < i < 4} \frac{\bar{a}_i(\ell)}{D_i}$$

multiply by $D_1 D_2 D_3$ and choose ℓ such that $D_1 = D_2 = D_3 = 0$

⇒ there are infinite complex solutions for ℓ

$$\frac{\text{Num}(\ell, \{p_i\})}{D_4} - \frac{\bar{d}_{1234}(\ell)}{D_4} = \bar{c}_{123}(\ell)$$

OPP algorithm

Example: 4 particle process

Step 2:

$$\frac{\text{Num}(\ell, \{p_i\})}{D_1 \dots D_4} - \frac{\bar{d}_{1234}(\ell)}{D_1 D_2 D_3 D_4} =$$

$$\sum_{1 < i < j < k < 4} \frac{\bar{c}_{ijk}(\ell)}{D_i D_j D_k} + \sum_{1 < i < j < 4} \frac{\bar{b}_{ij}(\ell)}{D_i D_j} + \sum_{1 < i < 4} \frac{\bar{a}_i(\ell)}{D_i}$$

multiply by $D_1 D_2 D_3$ and choose ℓ such that $D_1 = D_2 = D_3 = 0$

⇒ there are infinite complex solutions for ℓ

$$\frac{\text{Num}(\ell, \{p_i\})}{D_4} - \frac{\bar{d}_{1234}(\ell)}{D_4} = \bar{c}_{123}(\ell) = c_{123}$$

$$+ \tilde{c}_{123}^1(n_1 \cdot \ell) + \tilde{c}_{123}^2(n_2 \cdot \ell) + \tilde{c}_{123}^3(n_1 \cdot \ell)(n_2 \cdot \ell) + \tilde{c}_{123}^4(n_1 \cdot \ell)^2(n_2 \cdot \ell)$$

$$+ \tilde{c}_{123}^5(n_1 \cdot \ell)(n_2 \cdot \ell)^2 + \tilde{c}_{123}^6((n_1 \cdot \ell)^2 - (n_2 \cdot \ell)^2)$$

OPP algorithm

Step 3...: solve for all coefficients recursively

Finally: take the loop integral over $\frac{\text{Num}(\ell, \{p_i\})}{D_1 D_2 D_3 D_4}$

⇒ all spurious terms vanish

$$\mathcal{A}^{\text{prim}} = \int d^D \ell \frac{\text{Num}(\ell, \{p_i\})}{D_1 \dots D_4} = d_{1234} I_{1234}^4 + \sum_{1 < i < j < k < 4} c_{ijk} I_{ijk}^3 + \sum_{1 < i < j < 4} b_{ij} I_{ij}^2 + \sum_{1 < i < 4} a_i I_i^1$$

- coefficients $d_{ijkl}, c_{ijk}, b_{ij}, a_i$ have been calculated
- scalar integrals $I_{i_1 \dots i_n}^n$ can be taken from an integral library

QCD 1-loop package
[Ellis, Zanderighi]

OPP algorithm

Step 3....: solve for all coefficients

Finally:

QUESTION:

How do we evaluate

$\text{Num}(\ell, \{p_i\})$

in the reduction steps ?

$A^{\text{prim}} =$

$$\sum_{1 < i < j < k < 4} c_{ijk} I_{ijk}^3$$

$$\sum_{1 < i < j < 4} b_{ij} I_{ij}^2 + \sum_{1 < i < 4} a_i I_i^1$$

- coefficients $d_{ijkl}, c_{ijk}, b_{ij}, a_i$ have been calculated
- scalar integrals $I_{i_1 \dots i_n}^n$ can be taken from an integral library

QCD 1-loop package
[Ellis, Zanderighi]

Unitarity

remember: we extract the coefficients by considering only those loop momenta for which certain **sets of inverse propagators vanish**
⇒ virtual particles go on-shell

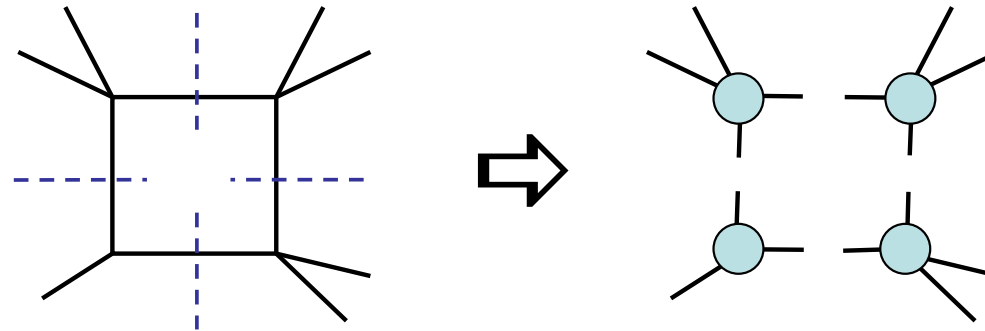
Unitarity

remember: we extract the coefficients by considering only those loop momenta for which certain **sets of inverse propagators vanish**

⇒ virtual particles go on-shell

⇒ equivalent to unitarity cuts

*this is where
OPP + Unitarity
mesh*



$$\text{Num}(\ell, \{p_i\}) \rightarrow \prod_k A_k^{\text{tree}}(\ell, \{p_i\})$$

⇒ **basic ingredients are on-shell tree level amplitudes**
with complex kinematics

Unitarity

remember: we extract the coefficients by considering only those

remember Step 1:

$$\text{Num}(\ell_{\pm}, \{p_i\}) = \bar{d}_{1234}(\ell_{\pm}) = d_{1234} + \tilde{d}_{1234}(\ell)$$



*this is w
OPP + U*

$$A_1^{\text{tree}}(\ell, \{p_i\}) \times A_2^{\text{tree}}(\ell, \{p_i\}) \times A_2^{\text{tree}}(\ell, \{p_i\}) \times A_4^{\text{tree}}(\ell, \{p_i\})$$

$$\text{Num}(\ell, \{p_i\}) \rightarrow \prod_k A_k^{\text{tree}}(\ell, \{p_i\})$$

⇒ **basic ingredients are on-shell tree level amplitudes**
with complex kinematics

D-dimensional unitarity



Rational terms

the full truth: $A^{1\text{-loop}} = \sum_j c_j I_j + \mathcal{R}$

rational terms (arrow pointing to \mathcal{R})

QCD needs regularization: $D = 4 - 2\varepsilon$ (Dim. Reg.)

rational terms: originate from ε -dependent terms in integral coefficients

Dim. Reg. requires ε to be a complex parameter

⇒ hard for numerical implementation

solution:

Using our knowledge about the particular structure of D -dependence of a one loop amplitude, we can construct copies of QCD in integer-dimensional ($D > 4$) spaces to interpolate the result in $D = 4 - 2\varepsilon$

D -dimensional unitarity

there are two sources of D -dependence

dimensionality of
loop momentum

number of spin and
polarization states

⇒ D -dep. from loop integration

⇒ D -dep. from contraction of metric tensors or gamma matrices

extra dimensions in ℓ
enter isotropically:

$$\mathcal{A}(\ell) = \mathcal{A}(\ell_4, \tilde{\ell}^2)$$

$$\tilde{\ell}^2 = \sum_{i=5}^D \ell_i^2$$

dependence on D is linear:

$$\mathcal{A}(D_s) = \mathcal{A}_0 + (D_s - 4)\mathcal{A}_1$$

⇒ loop momentum can be restricted
to 5-dimensional subspace

⇒ evaluate at $D_s = 6$ and $D_s = 8$
to obtain the full D_s -dependence

D-dimensional unitarity

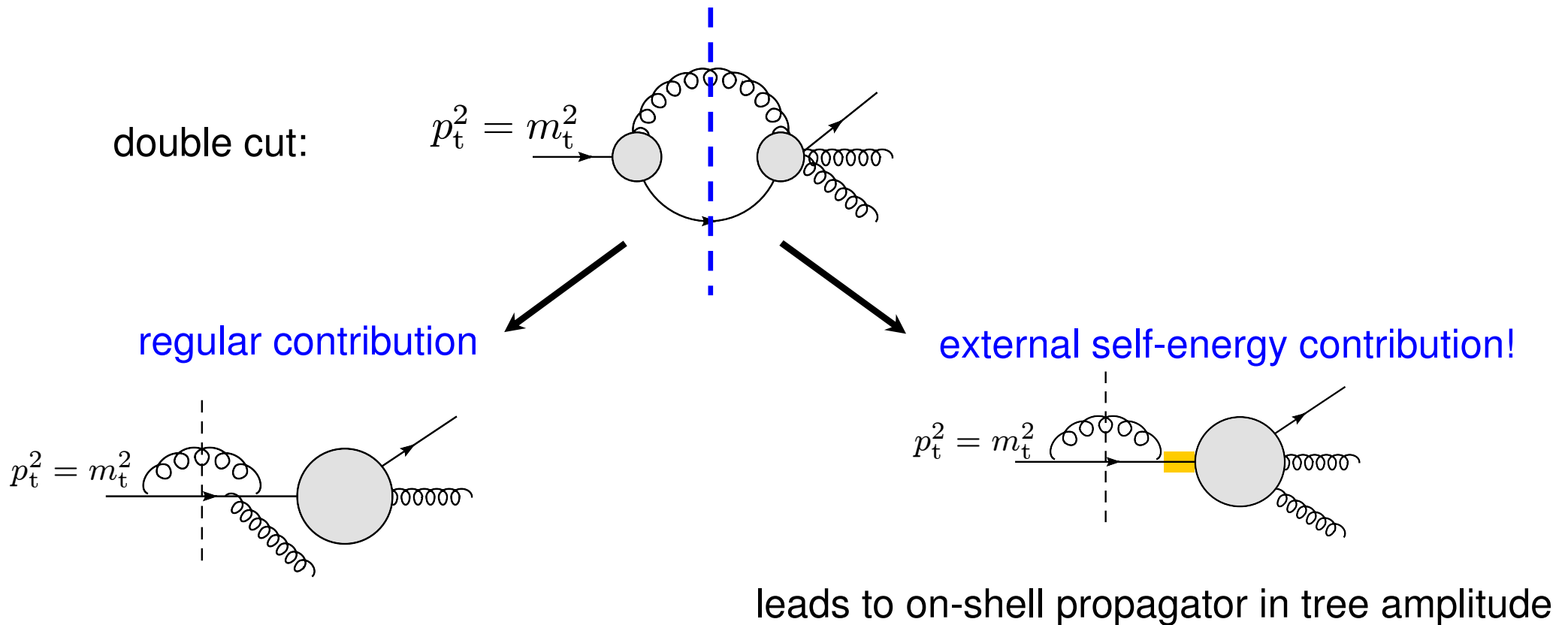
recap: **Shopping list for a 1-loop QCD amplitude**
(*D*-dimensional generalized unitarity)

all we need: tree level amplitudes for complex on-shell momenta with

- spinors, gamma matrices and polarization vectors in $D_s = 6, 8$ dimensions
- loop momenta restricted to 5-dimensional subspace
- external momenta restricted to 4-dimensional subspace

Massive Fermions

Unitarity and self-energy corrections on massive quarks lines:
 (general issue for unitarity based methods!)



- solution:**
- discard terms that lead to those cuts
 (truncate BG recurrence relations)
 - add wave function renormalization constants δZ_t later

Massive Fermions

Implementation:

- *Rocket-like* Fortran90 program:

$$0 \rightarrow t\bar{t} + N \text{ gluons}$$

$$0 \rightarrow t\bar{t} + q\bar{q} + N \text{ gluons}$$

at NLO QCD

(including N_f -terms with massive top quark loop)

- fully numerical implementation:
helicity amplitudes via Berends-Giele recurrence relations
- all one needs for: $pp \rightarrow t\bar{t} + X$,
 $pp \rightarrow t\bar{t} + \text{jet} + X$

Real Corrections

Difficulties

- extraction and cancellation of singularities against virtual corrections
- numerical stable evaluation
- efficient phase space integration

Difficulties

Treatment of singularities

- singularities from virtual corrections:

$$\sigma_N^{\text{virt}} \sim \int d\text{PS}_N 2\text{Re}(\mathcal{A}_{\text{tree}}^* \mathcal{A}_{\text{virt}}) = \int d\text{PS}_N \left\{ \left(\frac{a_{-2}}{\varepsilon_{\text{IR}}^2} + \frac{a_{-1}}{\varepsilon_{\text{IR}}} \right) |\mathcal{A}_{\text{tree}}|^2 + \text{finite} \right\}$$

- singularities from real corrections:

$$\sigma_{N+1}^{\text{real}} \sim \int d\text{PS}_{N+1} |\mathcal{A}_{\text{real}}|^2 \begin{cases} \text{unresolved} \\ \text{PS} \end{cases} \left\{ \begin{aligned} & - \int d\text{PS}_N \left(\frac{a_{-2}}{\varepsilon_{\text{IR}}^2} + \frac{a_{-1}}{\varepsilon_{\text{IR}}} \right) |\mathcal{A}_{\text{tree}}|^2 \\ & + \int_0^1 dx \int d\text{PS}_N(x) P_{ij}(x) \frac{1}{\varepsilon} |\mathcal{A}_{\text{tree}}|^2 \\ & + \int d\text{PS}_{N+1} \text{finite} \end{aligned} \right\}$$

this separation cannot be done explicitly

Dipole Formalism

[Catani, Dittmaier, Seymour, Trocsanyi]

solution: introduce *local* counter terms, *Dipoles* \mathcal{D}

$$\sigma_{N+1}^{\text{real}} \sim \int d\text{PS}_{N+1} \{ |\mathcal{A}_{\text{real}}|^2 - \mathcal{D} \} + \int d\text{PS}_N \int d\text{PS}_1 \mathcal{D}$$

- \mathcal{D} contains all singularities of $|\mathcal{A}_{\text{real}}|^2$ in the $N+1$ particle phase space ($D=4$)
 $\Rightarrow \{ |\mathcal{A}_{\text{real}}|^2 - \mathcal{D} \}$ is free of singularities and can be evaluated numerically
- \mathcal{D} is simple enough so that $\int d\text{PS}_1 \mathcal{D}$ can be calculated analytically in $D = 4 - 2\varepsilon$
 \Rightarrow singularities can be cancelled analytically against virt. corr. + pdfs

Due to universality of soft and collinear factorization, a general algorithm to construct dipoles exists.

Dipole Formalism

What is a dipole ?

$$\mathcal{D} = \sum_{\text{dipoles}} D_{ij,k}(p_i, p_j, p_k)$$

$$D_{ij,k} : \text{PS}_{N+1} \rightarrow \text{PS}_N$$

$$D_{ij,k} = \frac{1}{p_i \cdot p_j} \text{tree} \langle \dots, \tilde{i}j, \dots, k, \dots \rangle \left[\frac{T_{\tilde{i}j} T_k}{T_{\tilde{i}j}^2} V_{\lambda, \lambda'} \right] \dots \tilde{i}j, \dots, k, \dots \rangle_{\text{tree}}$$

universal

tree amplitude, vector in color and helicity space

color correlator

spin correlator

Dipole Formalism

Challenges

- Many dipoles for complicated processes, non-trivial structure due to colour and spin correlations
⇒ Need reliable library to generate them
- Every dipole involves calculation of a LO amplitude
⇒ Crucial to have fast procedures to calculate LO
- Dipole method relies on numerical subtraction of large numbers
⇒ Make sure that matrix elements are precise enough in the singular regions
- An efficient phase space integration is needed
(integrand $|\mathcal{A}_{\text{real}}|^2 - \mathcal{D}$ has not the peak structure of a tree level amplitude)

Top Quark Phenomenology

Top Quark Pheno

Top quark phenomenology is rich:

- large cross section
- top quark mass, spin, charge, branching fractions
- spin correlations, forward-backward asymmetry
- sensitive to new physics

Top Quark Pheno

$t\bar{t}$ production beyond leading order QCD:

first analytic calculation: [Nason, Dawson, Ellis, 1990] total $t\bar{t}$ production cross section
(on-shell top production) [Bernreuther, Brandenburg, Si, Uwer, 2001] production+decay
+ various threshold corrections, electroweak corrections

programs: **MCFM**: NLO $t\bar{t}$ production, no top decay
(LO $t\bar{t}$ production, LO top decay)

PowHeg, MC@NLO: NLO $t\bar{t}$ production, LO top decay

our program: NLO $t\bar{t}$ production and NLO leptonic decay

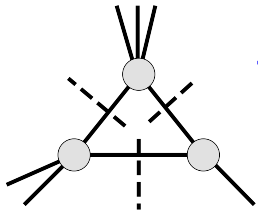
JHEP 0908:049, 2009
arXiv:0907.3090 [hep-ph]

flexible MC program:

- accounts for all spin correlations
- allows for arbitrary cuts

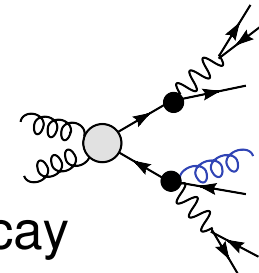
Top Quark Pheno

NLO $t\bar{t}$ production and NLO leptonic decay



- top production:**
- virtual corrections using D -dimensional generalized unitarity
 - real corrections using dipole subtraction method

- top decay:**
- on-shell approximation, error $\mathcal{O}\left(\frac{\Gamma_t}{m_t}\right)$
 - retain all spin correlations
 - include virtual and real corrections to decay
 - neglect non-factorizable contributions



Results:

- Predictions for:
- Tevatron
 - LHC @ 10TeV

Realistic final states: di-lepton final state
require two b-jets (k_T -clustering with $R = 0.4$)

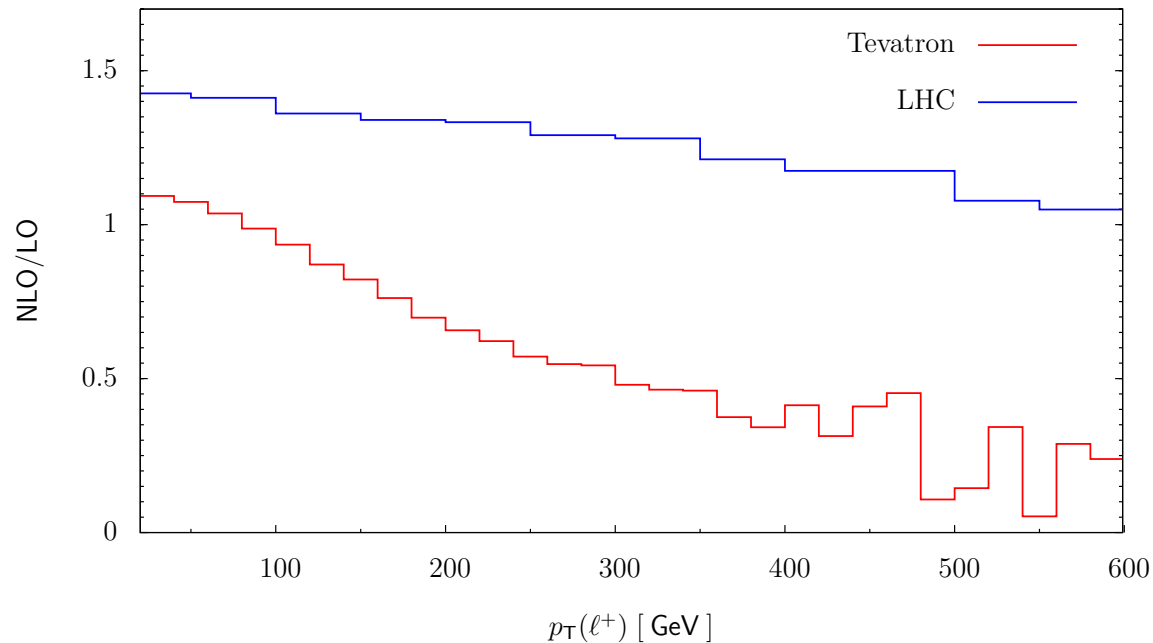
Cuts: $p_T^{\text{b-jet}} > 20 \text{ GeV}$

$$p_T^{\text{lep}} > 20 \text{ GeV}$$

$$p_T^{\text{miss}} > 40 \text{ GeV}$$

Results:

K - factor



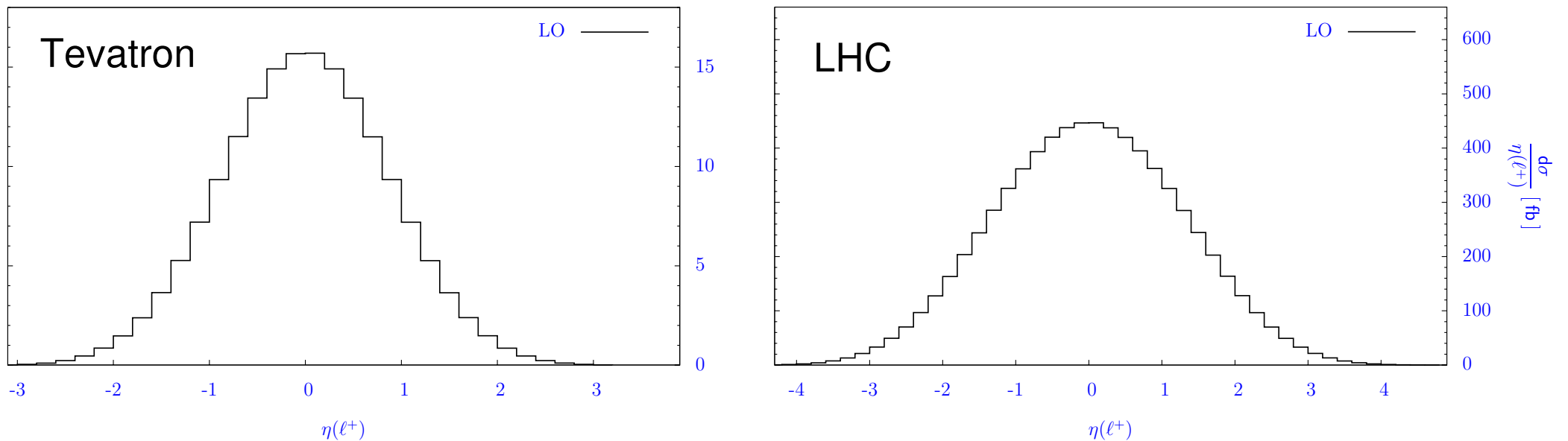
K-factors are not constant

Tevatron: NLO change p_T -distribution significantly

LHC: smaller change but non-negligible

Results:

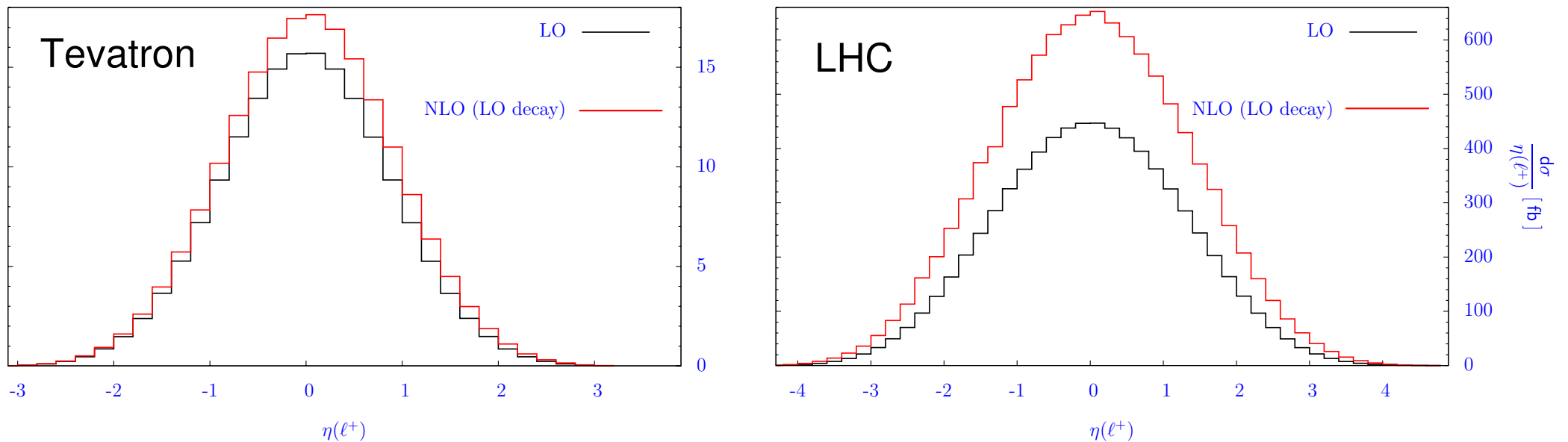
lepton rapidity distribution



- NLO corrections to rapidity distribution are important
- NLO correction to decay shifts rapidity distributions significantly

Results:

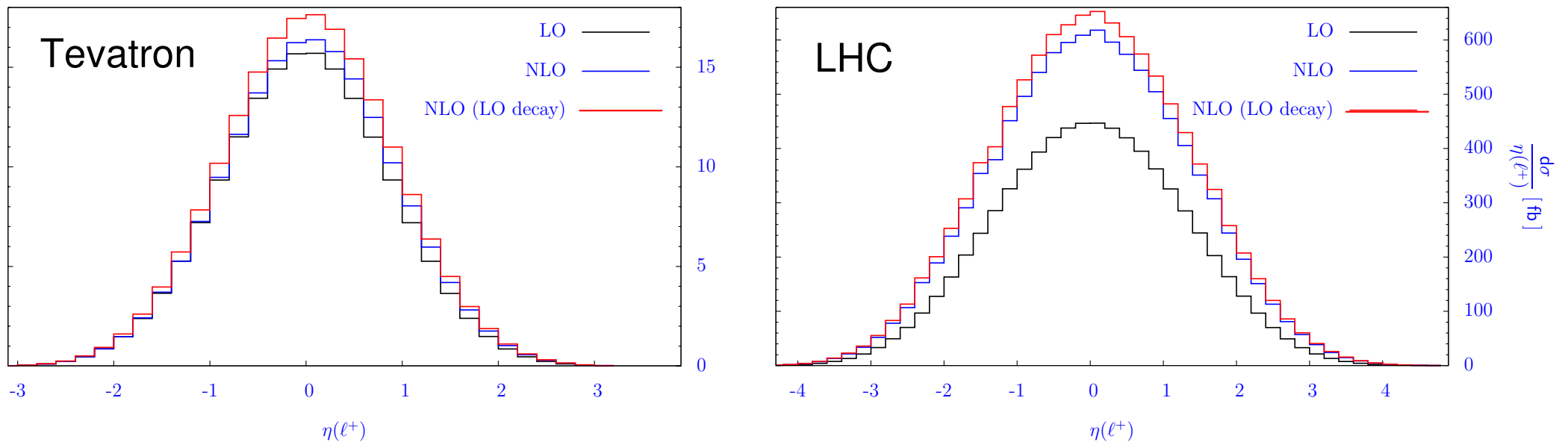
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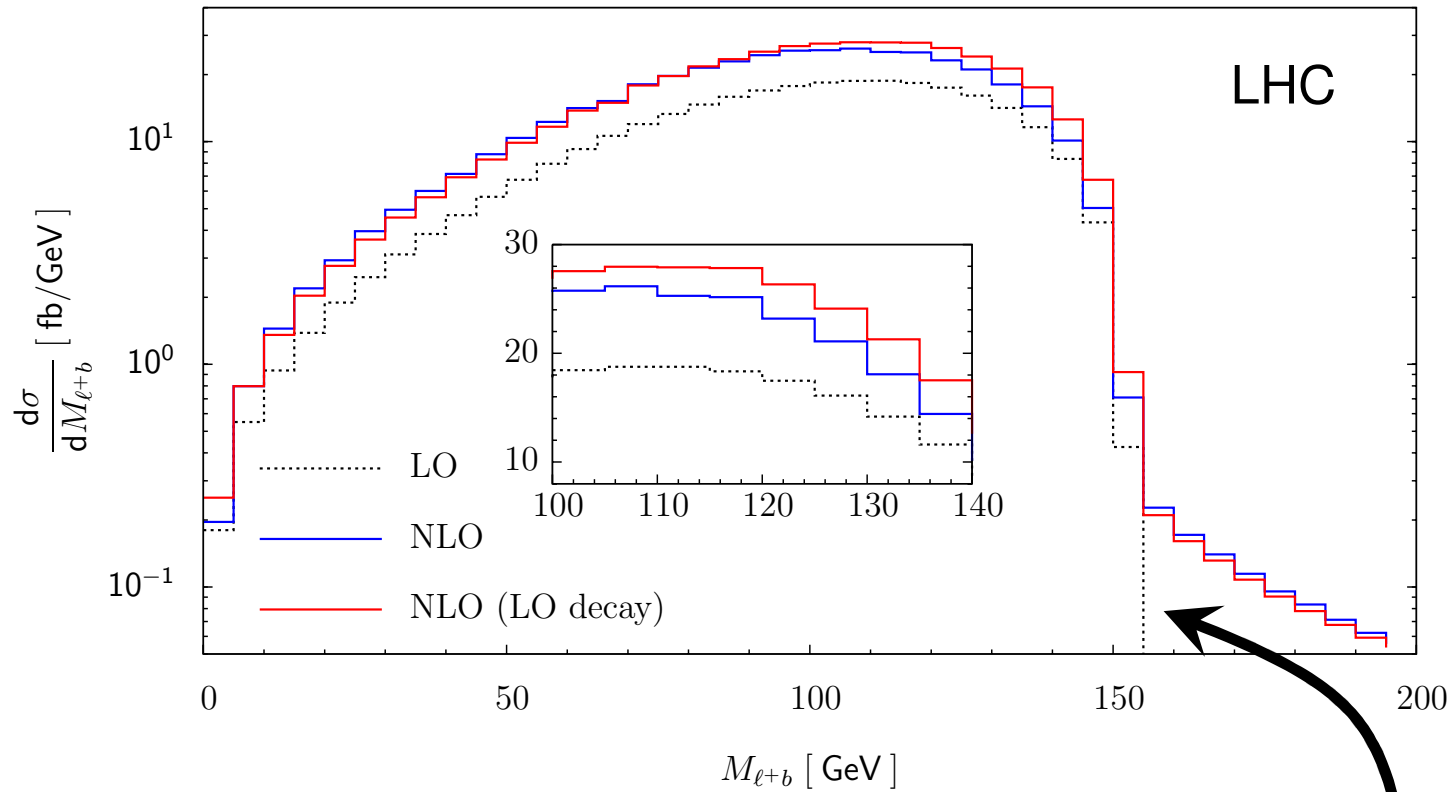
lepton rapidity distribution



- NLO corrections to rapidity distribution are important
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Results:

invariant mass of lepton and b-jet $M_{\ell+b}^2 = (p(\ell^+) + p(\text{b-jet}))^2$

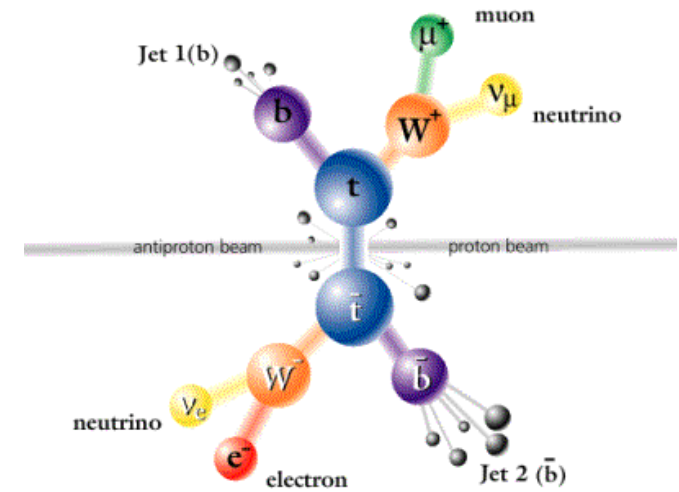


- boundary is top mass dependent
- spin studies for BSM particles
- NLO induces a tail

$$\max(M_{\ell+b}^2) = m_{\text{top}}^2 - m_W^2$$

Results:

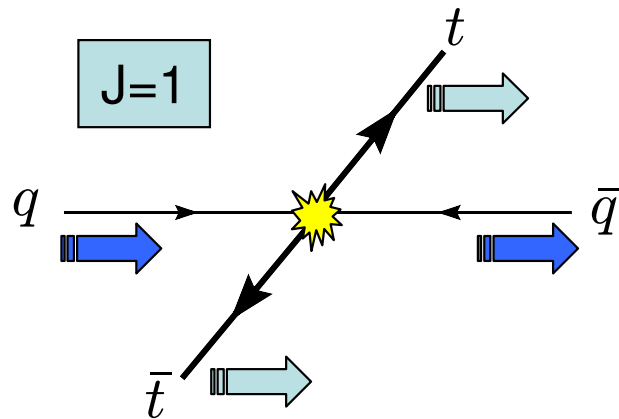
spin correlations



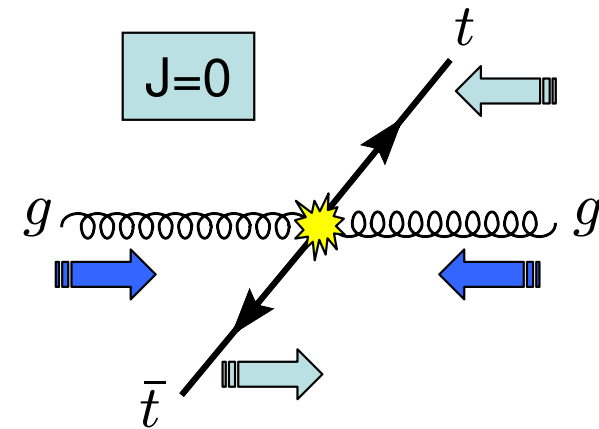
- large mass and short life time prevent hadronization effects to wash out spin information
 - ⇒ top spin correlations induced by production process are conserved
- spin correlations are strongest close to threshold
- spin correlations of top quarks are passed to decay products
 - ⇒ leptons prefer to fly parallel or anti-parallel wrt. each other

Results:

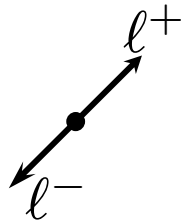
spin correlations



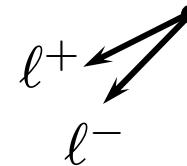
close to threshold:
S-wave production
($L=0$)



⇒ leptons preferably **anti-parallel**



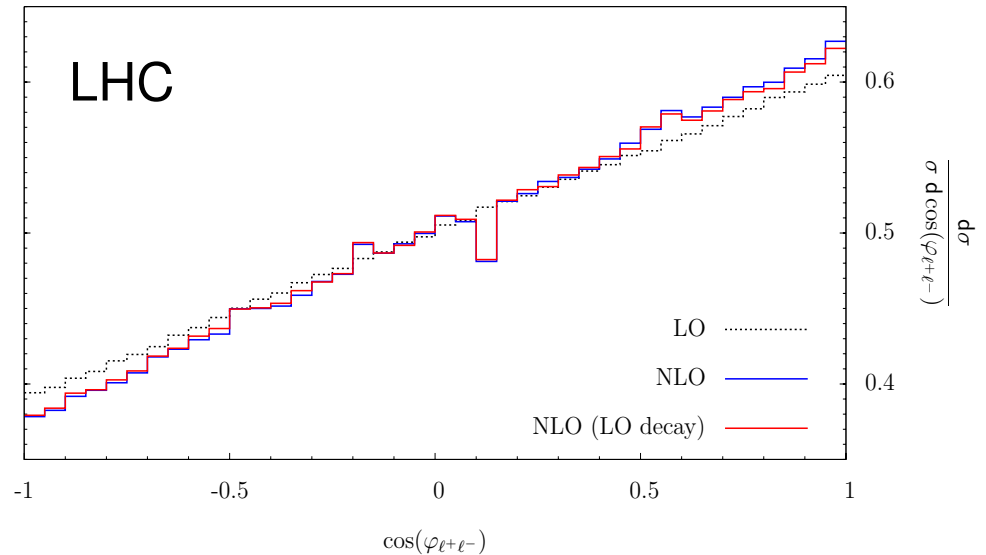
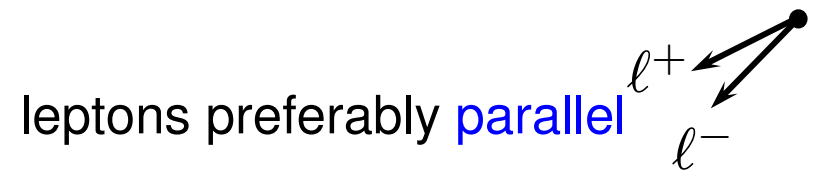
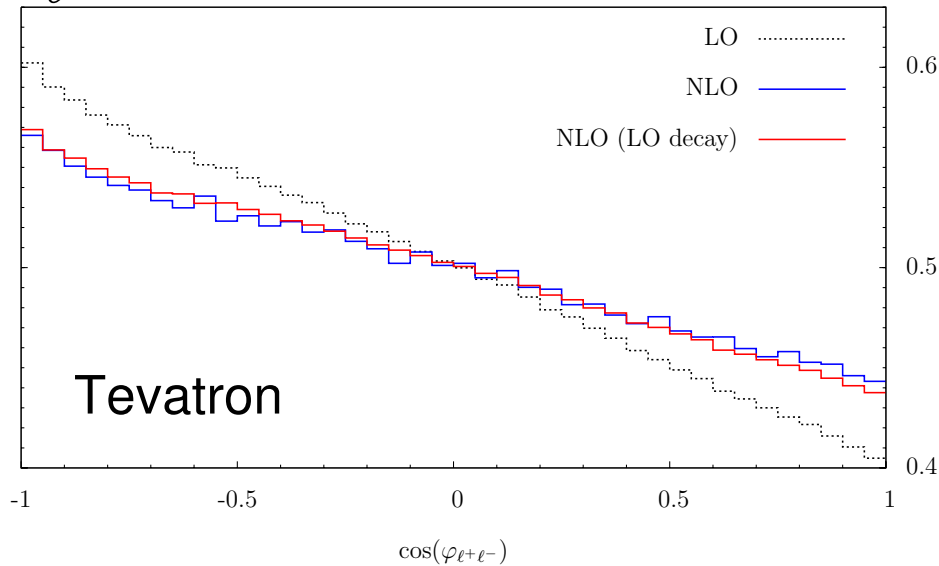
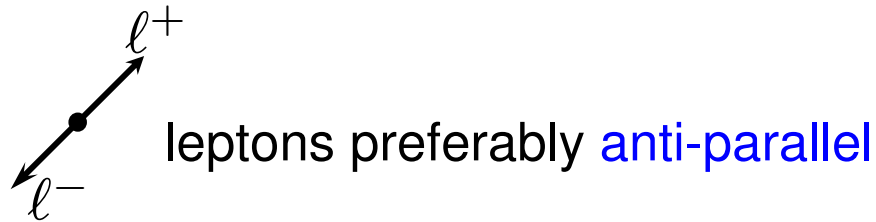
⇒ leptons preferably **parallel**



typical observable:

$$\frac{1}{\sigma} \frac{d\sigma}{d \cos(\varphi_{\ell^+ \ell^-})}$$

$\varphi_{\ell^+ \ell^-}$: angle between the directions of flight of leptons in the corresponding **top rest frame**

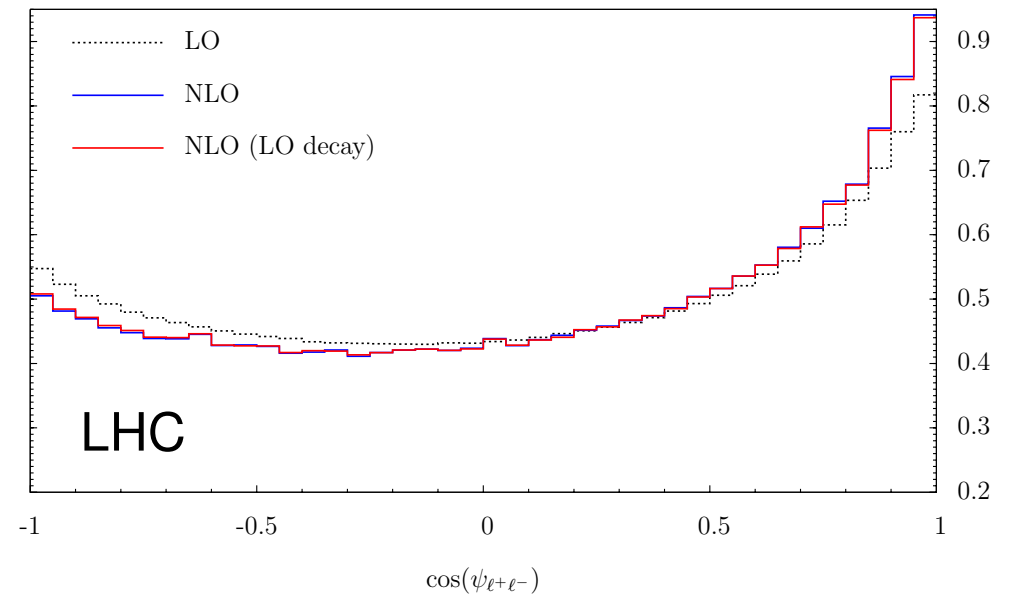
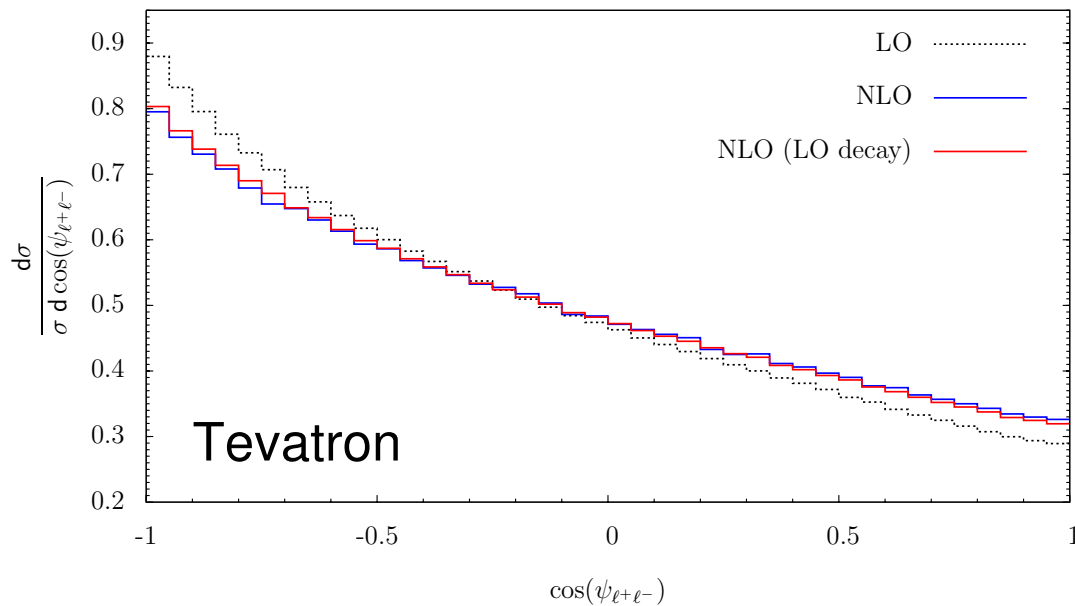


- substantial angular correlations, even at NLO
- NLO effects at Tevatron are significant

simpler observable:

$$\frac{1}{\sigma} \frac{d\sigma}{d \cos(\psi_{\ell^+ \ell^-})}$$

$\psi_{\ell^+ \ell^-}$: opening angle of the leptons in the **laboratory frame**



- top quark rest frames need not to be reconstructed
- angular correlations remain, stronger NLO effects at LHC

Summary

***D*-dimensional generalized unitarity...**

- ... is a **robust** and **transparent** method to calculate 1-loop corrections
- ... basic ingredients are on-shell **tree amplitudes**
- ... is ready for **phenomenology**

Top quark pair production

- flexible MC program for NLO $t\bar{t}$ production and NLO leptonic decay
 - ... accounts for all spin correlations
 - ... interesting distributions sensitive to spin correlations and NLO QCD corrections

Extras

dimensional space time, the number of spin eigenstates changes. For example, massless spin-one particles in D_s dimensions have $D_s - 2$ spin eigenstates while spinors in D_s dimensions have $2^{(D_s-2)/2}$ spin eigenstates. In the latter case, D_s should be even.

The spin density matrix for a massless spin-one particle with momentum l and polarization vectors $e_\mu^{(i)}$ is given by

$$\sum_{i=1}^{D_s-2} e_\mu^{(i)}(l) e_\nu^{(i)}(l) = -g_{\mu\nu}^{(D_s)} + \frac{l_\mu b_\nu + b_\mu l_\nu}{l \cdot b}, \quad (5)$$

where b_μ is an arbitrary light-cone gauge vector associated with a particular choice of polarization vectors. Similarly, the spin density matrix for a fermion with momentum l and mass m is given by

$$\sum_{i=1}^{2^{(D_s-2)/2}} u^{(i)}(l) \bar{u}^{(i)}(l) = \not{l} + m = \sum_{\mu=1}^D l_\mu \gamma^\mu + m. \quad (6)$$

While, as we see from these examples, the number of spin eigenstates depends explicitly on the space-time dimensionality, the loop-momentum l itself has implicit D -dependence. We can define the loop momentum as a D -dimensional vector, with the requirement $D \leq D_s$ [35]. We now extend the notion of dimensional dependence of the one-loop scattering amplitude in Eq. (1) by taking the sources of all unobserved particles in D_s -dimensional space-time

$$\mathcal{A}_{(D,D_s)}(\{p_i\}, \{J_i\}) = \int \frac{d^D l}{i(\pi)^{D/2}} \frac{\mathcal{N}^{(D_s)}(\{p_i\}, \{J_i\}; l)}{d_1 d_2 \cdots d_N}. \quad (7)$$

The numerator function $\mathcal{N}^{(D_s)}(\{p_i\}, \{J_i\}; l)$ depends explicitly on D_s through the number of spin eigenstates of virtual particles. However, the dependence of the numerator function on the loop momentum dimensionality D emerges in a peculiar way. Since external particles are kept in four dimensions, the dependence of the numerator function on $D - 4$ components of the loop momentum l appears only through its dependence on l^2 . Specifically

$$l^2 = \bar{l}^2 - \tilde{l}^2 = l_1^2 - l_2^2 - l_3^2 - l_4^2 - \sum_{i=5}^D l_i^2, \quad (8)$$

where \bar{l} and \tilde{l} denote four- and $(D - 4)$ -dimensional components of the vector l . It is apparent from Eq. (8) that there is no preferred direction in the $(D - 4)$ -dimensional subspace of the D -dimensional loop momentum space.

A simple, but important observation is that in one-loop calculations, the dependence of scattering amplitudes on D_s is *linear*. This happens because, for such dependence to appear,

we need to have a closed loop of contracted metric tensors and/or Dirac matrices coming from vertices and propagators. Since only a single loop can appear in one-loop calculations, we find

$$\mathcal{N}^{(D_s)}(l) = \mathcal{N}_0(l) + (D_s - 4)\mathcal{N}_1(l). \quad (9)$$

We emphasize that there is no explicit dependence on either D_s or D in functions $\mathcal{N}_{0,1}$.

For numerical calculations we need to separate the two functions $\mathcal{N}_{0,1}$. To do so, we compute the left hand side of Eq. (9) for $D_s = D_1$ and $D_s = D_2$ and, after taking appropriate linear combinations, obtain

$$\begin{aligned} \mathcal{N}_0(l) &= \frac{(D_2 - 4)\mathcal{N}^{(D_1)}(l) - (D_1 - 4)\mathcal{N}^{(D_2)}(l)}{D_2 - D_1}, \\ \mathcal{N}_1(l) &= \frac{\mathcal{N}^{(D_1)}(l) - \mathcal{N}^{(D_2)}(l)}{D_2 - D_1}. \end{aligned} \quad (10)$$

Because both D_1 and D_2 are integers, amplitudes are numerically well-defined. We will comment more on possible choices of $D_{1,2}$ in the forthcoming sections; here suffice it to say that if fermions are present in the loop, we have to choose *even* D_1 and D_2 .

Having established the D_s -dependence of the amplitude, we discuss analytic continuation for sources of unobserved particles. We can interpolate D_s either to $D_s \rightarrow 4 - 2\epsilon$ (the t'Hooft-Veltman (HV) scheme) [34] or to $D_s \rightarrow 4$ (the four-dimensional helicity (FDH) scheme) [35]. The latter scheme is of particular interest in supersymmetric (SUSY) calculations since all SUSY Ward identities are preserved. We see from Eq. (9) that the difference between the two schemes is simply $-2\epsilon\mathcal{N}_1$.

We now substitute Eq.(10) into Eq. (7). Upon doing so, we obtain explicit expressions for one-loop amplitudes in HV and FDH schemes. We derive

$$\begin{aligned} \mathcal{A}^{\text{FDH}} &= \left(\frac{D_2 - 4}{D_2 - D_1} \right) \mathcal{A}_{(D, D_s=D_1)} - \left(\frac{D_1 - 4}{D_2 - D_1} \right) \mathcal{A}_{(D, D_s=D_2)}, \\ \mathcal{A}^{\text{HV}} &= \mathcal{A}^{\text{FDH}} - \left(\frac{2\epsilon}{D_2 - D_1} \right) (\mathcal{A}_{(D, D_s=D_1)} - \mathcal{A}_{(D, D_s=D_2)}). \end{aligned} \quad (11)$$

We emphasize that $D_s = D_{1,2}$ amplitudes on the r.h.s. of Eq. (11) are conventional one-loop scattering amplitudes whose numerator functions are computed in higher-dimensional space-time, i.e. all internal metric tensors and Dirac gamma matrices are in integer $D_s = D_{1,2}$ dimensions. The loop integration is in $D \leq D_s$ dimensions. It is important that explicit dependence on the regularization parameter $\epsilon = (4 - D)/2$ is not present in these amplitudes.

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function. Hence the integrand of the N -particle amplitude in Eq. (1) can be parameterized as

$$\begin{aligned} \frac{\mathcal{N}^{(D_s)}(l)}{d_1 d_2 \cdots d_N} = & \sum_{[i_1|i_5]} \frac{\bar{e}_{i_1 i_2 i_3 i_4 i_5}^{(D_s)}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} + \sum_{[i_1|i_4]} \frac{\bar{d}_{i_1 i_2 i_3 i_4}^{(D_s)}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} \\ & + \sum_{[i_1|i_3]} \frac{\bar{c}_{i_1 i_2 i_3}^{(D_s)}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_2]} \frac{\bar{b}_{i_1 i_2}^{(D_s)}(l)}{d_{i_1} d_{i_2}} + \sum_{[i_1|i_1]} \frac{\bar{a}_{i_1}^{(D_s)}(l)}{d_{i_1}}. \end{aligned} \quad (13)$$

where the dependence on the external momenta and sources are suppressed. From four-dimensional unitarity we know that computation of each cut of the scattering amplitude is simplified if convenient parameterization of the residue is chosen. We now discuss how these parameterizations change when D_s -dimensional unitarity cuts are considered.

A. Pentuple residue

To calculate the pentuple residue, we choose momentum l such that five inverse propagators in Eq. (13) vanish. We define

$$\bar{e}_{ijkmn}^{(D_s)}(l_{ijkmn}) = \text{Res}_{ijkmn} \left(\frac{\mathcal{N}^{(D_s)}(l)}{d_1 \cdots d_N} \right). \quad (14)$$

The momentum l_{ijkmn} satisfies the following set of equations $d_i(l_{ijkmn}) = \cdots = d_n(l_{ijkmn}) = 0$. The solution is given by

$$l_{ijkmn}^\mu = V_5^\mu + \sqrt{\frac{-V_5^2 + m_n^2}{\alpha_5^2 + \cdots + \alpha_D^2}} \left(\sum_{h=5}^D \alpha_h n_h^\mu \right), \quad (15)$$

where m_n is the mass in the propagator d_n which is chosen to be as $d_n = l^2 - m_n^2$ by adjusting the reference vector q_0 . The parameters α_n can be chosen freely. The four-dimensional vector V_5^μ depends only on external momenta and propagator masses. It is explicitly constructed using the Vermaseren-van Neerven basis as outlined in Ref. [24]. The $D - 4$ components of the vector l_{ijkmn} are necessarily non-vanishing; for simplicity we may choose l_{ijkmn} to be five-dimensional, independent of D_s . We will see below that this is sufficient to determine pentuple residue.

To restrict the functional form of the pentuple residue $\bar{e}_{ijkmn}(l)$ we apply the same reasoning as in four-dimensional unitarity case, supplemented with the requirement that $\bar{e}_{ijkmn}^{(D_s)}(l)$

depends only on even powers of s_e ; this requirement is a necessary consequence of the discussion around Eq. (8). These considerations lead to the conclusion that the pentuple residue is independent of the loop momentum

$$\overline{e}_{ijkmn}^{(D_s)}(l) = e_{ijkmn}^{(D_s, (0))}. \quad (16)$$

To calculate $e^{(0)}$ in the FDH scheme, we employ Eq. (10) and obtain

$$e_{ijkmn}^{(0), \text{FDH}} = \left(\frac{D_2 - 4}{D_2 - D_1} \right) \text{Res}_{ijkmn} \left(\frac{\mathcal{N}^{(D_1)}(l)}{d_1 \cdots d_N} \right) - \left(\frac{D_1 - 4}{D_2 - D_1} \right) \text{Res}_{ijkmn} \left(\frac{\mathcal{N}^{(D_2)}(l)}{d_1 \cdots d_N} \right). \quad (17)$$

The calculation of the residues of the amplitude on the r.h.s. of Eq. (17), is simplified by their factorization into products of tree amplitudes

$$\begin{aligned} \text{Res}_{ijkmn} \left(\frac{\mathcal{N}^{(D_s)}(l)}{d_1 \cdots d_N} \right) &= \sum \mathcal{M}(l_i; p_{i+1}, \dots, p_j, -l_j) \times \mathcal{M}(l_j; p_{j+1}, \dots, p_k; -l_k) \\ &\times \mathcal{M}(l_k; p_{k+1}, \dots, p_m; -l_m) \times \mathcal{M}(l_m; p_{m+1}, \dots, p_n; -l_n) \times \mathcal{M}(l_n; p_{n+1}, \dots, p_i; -l_i). \end{aligned} \quad (18)$$

Here, the summation is over all different quantum numbers of the cut lines. In particular, we have to sum over polarization vectors of the cut lines. This generates explicit D_s dependence of the residue, as described in the previous section. Note that the complex momenta $l_h^\mu = l^\mu + q_h^\mu$ are on-shell due to the unitarity constraint $d_h = 0$.

B. Quadrupole residue

The construction of the quadrupole residue follows the discussion of the previous subsection and generalizes the four-dimensional case studied in [23, 24]. We define

$$\overline{d}_{ijkn}^{(D_s)}(l) = \text{Res}_{ijkn} \left(\frac{\mathcal{N}^{(D_s)}(l)}{d_1 \cdots d_N} - \sum_{[i_1|i_5]} \frac{e_{i_1 i_2 i_3 i_4 i_5}^{(D_s, (0))}}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} \right), \quad (19)$$

where the last term in the r.h.s. is the necessary subtraction of the pentuple cut contribution. We now specialize to the FDH scheme. In this case, the most general parameterization of the quadrupole cut is given by

$$\overline{d}_{ijkn}^{\text{FDH}}(l) = d_{ijkn}^{(0)} + d_{ijkn}^{(1)} s_1 + (d_{ijkn}^{(2)} + d_{ijkn}^{(3)} s_1) s_e^2 + d_{ijkn}^{(4)} s_e^4, \quad (20)$$

where $s_1 = l \cdot n_1$. We used the fact that, in renormalizable quantum field theories, the highest rank of a tensor integral that may contribute to a quadrupole residue is four and

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find

$$\begin{aligned}
\int \frac{d^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} &= -\frac{D-4}{2} I_{i_1 i_2 i_3 i_4}^{D+2}, \\
\int \frac{d^D l}{(i\pi)^{D/2}} \frac{s_e^4}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} &= \frac{(D-2)(D-4)}{4} I_{i_1 i_2 i_3 i_4}^{D+4}, \\
\int \frac{d^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2} d_{i_3}} &= -\frac{(D-4)}{2} I_{i_1 i_2 i_3}^{D+2}, \\
\int \frac{d^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2}} &= -\frac{(D-4)}{2} I_{i_1 i_2}^{D+2}.
\end{aligned} \tag{26}$$

$$s_e^2 = (\tilde{l} \cdot n_5)^2$$

Using Eq. (26), we arrive at the following representation of the scattering amplitude

$$\begin{aligned}
\mathcal{A}_{(D)} &= \sum_{[i_1|i_5]} e_{i_1 i_2 i_3 i_4 i_5}^{(0)} I_{i_1 i_2 i_3 i_4 i_5}^{(D)} \\
&+ \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4}^{(0)} I_{i_1 i_2 i_3 i_4}^{(D)} - \frac{D-4}{2} d_{i_1 i_2 i_3 i_4}^{(2)} I_{i_1 i_2 i_3 i_4}^{(D+2)} + \frac{(D-4)(D-2)}{4} d_{i_1 i_2 i_3 i_4}^{(4)} I_{i_1 i_2 i_3 i_4}^{(D+4)} \right) \\
&+ \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3}^{(0)} I_{i_1 i_2 i_3}^{(D)} - \frac{D-4}{2} c_{i_1 i_2 i_3}^{(9)} I_{i_1 i_2 i_3}^{(D+2)} \right) \\
&+ \sum_{[i_1|i_2]} \left(b_{i_1 i_2}^{(0)} I_{i_1 i_2}^{(D)} - \frac{D-4}{2} b_{i_1 i_2}^{(9)} I_{i_1 i_2}^{(D+2)} \right) + \sum_{i_1=1}^N a_{i_1}^{(0)} I_{i_1}^{(D)}. \tag{27}
\end{aligned}$$

We emphasize that the explicit D -dependence on the r.h.s. of Eq. (27) is the consequence of our choice of the basis for master integrals in Eq. (26).

We note that the above decomposition is valid for any value of D . We can now interpolate the loop integration dimension D to $D \rightarrow 4 - 2\epsilon$. The extended basis of master integrals that we employ provides a clear separation between cut-constructible and rational parts of the amplitude. The cut-constructible part is given by the integrals in D -dimensions in Eq. (27), while the rational part is given by the integrals in $D+2$ and $D+4$ dimensions. However, it is possible to use smaller basis of master integrals by rewriting integrals $\{I_{i_1 i_2 i_3 i_4}^{(D+4)}, I_{i_1 i_2 i_3 i_4}^{(D+2)}, I_{i_1 i_2 i_3}^{(D+2)}, I_{i_1 i_2}^{(D+2)}\}$ in terms of $\{I_{i_1 i_2 i_3 i_4}^{(D)}, I_{i_1 i_2 i_3}^{(D)}, I_{i_1 i_2}^{(D)}\}$ using the integration-by-parts techniques.

Since we are interested in NLO computations, we only need to consider the limit $\epsilon \rightarrow 0$ in Eq. (27) and neglect contributions of order ϵ . This leads to certain simplifications. First, in this limit, we can re-write the scalar 5-point master integral as a linear combination of four-point master integrals up to $\mathcal{O}(\epsilon)$ terms. If we employ this fact in Eq. (27), we obtain

$$\lim_{D \rightarrow 4-2\epsilon} \left(\sum_{[i_1|i_5]} e_{i_1 \dots i_5}^{(0)} I_{i_1 \dots i_5}^{(D)} + \sum_{[i_1|i_4]} d_{i_1 \dots i_4}^{(0)} I_{i_1 \dots i_4}^{(D)} \right) = \sum_{[i_1|i_4]} \tilde{d}_{i_1 \dots i_4}^{(0)} I_{i_1 \dots i_4}^{(4-2\epsilon)} + \mathcal{O}(\epsilon). \tag{28}$$