A Domino Theory of Flavor
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and
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The Flavor Puzzle

Xing, Zhang & Zhou (2008)

Not randomly distributed, even on log scale

All Yukawas $\gtrsim 10^{-5}$

Equal hierarchy between successive generations

Pattern may be suggestive of a framework beyond just generating small numbers

mass (GeV)

$10^3$

$10^2$

$10^1$

$10^{-1}$

$10^{-2}$

$10^{-3}$

$10^{-4}$

$\Upsilon$  $t$  $\Upsilon$  $b$  $\tau$

$\Xi$  $c$  $\Upsilon$  $s$  $\mu$

$\Lambda$  $u$  $\Lambda$  $d$  $e$

ups  downs  leptons
The Flavor Puzzle

Xing, Zhang & Zhou (2008)

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All Yukawas \( \geq 10^{-5} \)

Equal hierarchy between successive generations

Pattern may be suggestive of a framework beyond just generating small numbers

Goal: Generate Pattern Democratically
Inspiration

Radiative fermion mass generation models have a long history

1. Georgi & Glashow (1973) - e from $\mu$

2. Babu, Balakrishna, & Mohapatra (1990) - in Pati-Salam


4. Fox & Dobrescu (2008) - up and lepton masses

5. and many more...
Outline

1. General Domino Framework

2. Yukawa Predictions

3. Experimental Signatures
General Domino Framework
Domino Mechanism

work in SUSY SU(5) GUT:

\[ \mathcal{W} \supset H_u \ 10_3 \ 10_3 + \lambda_{ij} \ \overline{\phi} \ 10_i \ \bar{5}_j \]

all allowed coefficients (e.g. \( \lambda_{ij} \)) are \( \mathcal{O}(1) \)

\( \overline{\phi} \) can be \( H_d \) so add no new fields to “MSSM”
Domino Mechanism

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tree-level top mass may appear to violate our flavor philosophy but can be written:

\[ \mathcal{W} \supset H_u \, (c_i \, 10_i) \, (c_j \, 10_j) + \lambda_{ij} \, \bar{\phi} \, 10_i \, \bar{5}_j \]
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tree-level top mass may appear to violate our flavor philosophy but can be written:

\[ \mathcal{W} \supset H_u \, (c_i \, 10_i) \, (c_j \, 10_j) + \lambda_{ij} \phi \, 10_i \, \bar{5}_j \]

two arbitrary flavor directions: \( c_i \) and \( \lambda_{ij} \)

these spurions break all flavor symmetries: \( U(3)_{10} \times U(3)_5 \)

These will generate all fermion masses (and mixings) in a hierarchical pattern
Top Yukawa - UV Completion

forbid all Yukawas and introduce two new fields: $\sigma$ and a (vector-like) $10_N$

\[
W = c_i \sigma \, 10_i \overline{10}_N + H_u \, 10_N \, 10_N + M_N \, 10_N \, \overline{10}_N
\]

\[
\begin{array}{c|c}
U(1)_{PQ} & \\
\hline
\sigma & +1 \\
10_i & -1 \\
H_u & 0 \\
H_d & 0 \\
\overline{5}_i & +1 \\
10_N & 0 \\
\end{array}
\]
Top Yukawa - UV Completion

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$$W = c_i \sigma 10_i \overline{10_N} + H_u 10_N 10_N + M_N 10_N \overline{10_N}$$

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\begin{array}{c|c}
U(1)_{\text{PQ}} & \\
\hline
\sigma & +1 \\
10_i & -1 \\
H_u & 0 \\
H_d & 0 \\
\bar{5}_i & +1 \\
10_N & 0 \\
\end{array}
\]

generates top yukawa

$$W = \frac{\langle \sigma \rangle^2}{M_N^2} H_u 10_3 10_3$$
Up Yukawas

\[ \mathcal{W} \supset H_u (c_i 10_i) (c_j 10_j) + \bar{\phi} 10_i \lambda_{ij} \bar{5}_j \]

\[ c \otimes c = \text{top mass } \propto 1 \]

\[ H \ 10 \ y_u \ 10 \]

\[ y_u \sim \begin{pmatrix} \phantom{1} \\ \phantom{1} \\ 1 \end{pmatrix} \]
Up Yukawas

\[ \mathcal{W} \supset H_u (c_i 10_i) (c_j 10_j) + \bar{\phi} 10_i \lambda_{ij} \bar{5}_j \]

\[ c \otimes c = \text{top mass} \propto 1 \]

\[ (\lambda \lambda^\dagger) c \otimes (\lambda \lambda^\dagger) c = \text{charm mass} \propto \epsilon^2 \]

\[ y_u \sim \begin{pmatrix} \epsilon^2 \\ 1 \end{pmatrix} \]

H 10 y_u 10
Up Yukawas

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\[ (\lambda \lambda^\dagger) c \otimes (\lambda \lambda^\dagger) c = \text{charm mass} \propto \epsilon^2 \]

\[ (\lambda \lambda^\dagger)^2 c \otimes (\lambda \lambda^\dagger)^2 c = \text{up mass} \propto \epsilon^4 \]

\[ y_u \sim \begin{pmatrix} \epsilon^4 \\ \epsilon^2 \\ 1 \end{pmatrix} \]
Up Yukawas

\[ \mathcal{W} \supset H_u (c_i \, 10_i) \, (c_j \, 10_j) + \bar{\phi} \, 10_i \, \lambda_{ij} \, \bar{5}_j \]

\[ c \otimes c = \text{top mass } \propto 1 \]
\[ (\lambda \lambda^\dagger) \, c \otimes (\lambda \lambda^\dagger) \, c = \text{charm mass } \propto \epsilon^2 \]
\[ (\lambda \lambda^\dagger)^2 \, c \otimes (\lambda \lambda^\dagger)^2 \, c = \text{up mass } \propto \epsilon^4 \]
\[ c \otimes (\lambda \lambda^\dagger) \, c = \text{top-charm mixing } \propto \epsilon \]

\[ y_u \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \]

CKM mixing angles can arise at intermediate order between masses
Up Yukawa Diagrams

\[ \mathcal{W} \supset H_u (c_i 10_i) (c_j 10_j) + \phi 10_i \lambda_{ij} \bar{5}_j \]
Up Yukawa Diagrams

\[ \mathcal{W} \supset H_u (c_i 10_i) (c_j 10_j) + \overline{\phi} 10_i \lambda_{ij} \overline{5}_j \]

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\[ (\lambda \lambda^\dagger) c \otimes (\lambda \lambda^\dagger) c = \text{charm mass} \propto \epsilon^2 \]

\[ \frac{m_c}{m_t} \sim \frac{f_C}{(16\pi^2)^2} \lambda^4 \log \left( \frac{\Lambda^2}{m_\phi^2} \right) \sim \frac{1}{300} \]
Up Yukawa Diagrams

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\[ c \otimes (\lambda \lambda^\dagger) c = \text{top-charm mixing } \propto \epsilon \]
Mass Basis

We have freedom to choose a basis in which:

\[ U(2)_{10} \times U(3)_{\overline{5}} \implies \lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} & 0 \\ 0 & \lambda_{22} & \lambda_{23} \\ 0 & 0 & \lambda_{33} \end{pmatrix} \]
Mass Basis

We have freedom to choose a basis in which:

\[
U(2)_10 \times U(3)_{\overline{5}} \implies \lambda = \begin{pmatrix}
\lambda_{11} & \lambda_{12} & 0 \\
0 & \lambda_{22} & \lambda_{23} \\
0 & 0 & \lambda_{33}
\end{pmatrix}
\]

\[
(\lambda \lambda^\dagger) c \otimes (\lambda \lambda^\dagger) \ c = \text{charm mass} \propto \epsilon^2 \\
\ c \otimes (\lambda \lambda^\dagger) \ c = \text{top-charm mixing} \propto \epsilon
\]

This basis is near mass basis for the Yukawa couplings
$\mathcal{W} \supset H_u (c_i 10_i) (c_j 10_j) + H_d 10_i \lambda_{ij} \bar{5}_j$

\[ c \otimes \lambda^\dagger c = b, \tau \text{ mass } \propto \delta \]

\[ y_d \sim \delta \begin{pmatrix} \phantom{1} \\ 1 \end{pmatrix} \]
$\mathcal{W} \supset H_u (c_i 10_i) (c_j 10_j) + H_d 10_i \lambda_{ij} \bar{5}_j$

\[
\begin{align*}
    c \otimes \lambda^\dagger c &= b, \tau \text{ mass } \propto \delta \\
    (\lambda^\dagger \lambda) c \otimes (\lambda^\dagger \lambda) \lambda^\dagger c &= s, \mu \text{ mass } \propto \delta \epsilon^2 \\
\end{align*}
\]

$y_d \sim \delta 
\begin{pmatrix}
\epsilon^4 & \epsilon^3 & \epsilon^2 \\
\epsilon^3 & \epsilon^2 & \epsilon \\
\epsilon^2 & \epsilon & 1
\end{pmatrix}$
\[ \mathcal{W} \supset H_u (c_i \ 10_i) \ (c_j \ 10_j) + H_d \ 10_i \ \lambda_{ij} \ \bar{5}_j \]  

add SUSY breaking  
\[ \mathcal{L} \supset B \mu \ H_u^{(3)} H_d^{(3)} \]

\[ c \otimes \lambda^\dagger c = b, \tau \text{ mass } \propto \delta \]

\[ (\lambda^\dagger \lambda) \ c \otimes (\lambda^\dagger \lambda) \ \lambda^\dagger c = s, \mu \text{ mass } \propto \delta \epsilon^2 \]

\[ y_d \sim \delta \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \]

\[ \text{generates } H_u^{\dagger} \ 10 \ y_d \ \bar{5} \]

\[ b, \tau \text{ mass} \]
\[ \mathcal{W} \supset H_u (c_i \ 10_i) \ (c_j \ 10_j) + H_d \ 10_i \ \lambda_{ij} \ 5_j \] 
add SUSY breaking \( \mathcal{L} \supset B \mu \ H_u^{(3)} \ H_d^{(3)} \)

\[ c \otimes \lambda^\dagger c = b, \ \tau \text{ mass } \propto \delta \]

\[ (\lambda^\dagger \lambda) \ c \otimes (\lambda^\dagger \lambda) \ \lambda^\dagger c = s, \ \mu \text{ mass } \propto \delta \epsilon^2 \]

\[ \vdots \]

\[ y_d \sim \delta \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \]

generates \( H_u^\dagger \ 10 \ y_d \ 5 \)

\[ b, \ \tau \text{ mass } \]

\[ s, \ \mu \text{ mass } \]
\[ \mathcal{W} \supset H_u (c_i \, 10_i) (c_j \, 10_j) + H_d \, 10_i \, \lambda_{ij} \, 5_j \] 

add SUSY breaking \( \mathcal{L} \supset B \mu \, H_u^{(3)} \, H_d^{(3)} \)

\[ c \otimes \lambda^\dagger \, c = b, \tau \text{ mass} \propto \delta \]

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generates \( H_u^\dagger \, 10 \, y_d \, \overline{5} \)

b, \tau \text{ mass}

s, \mu \text{ mass}

this simple structure works with only 2 spurions, so requires unification
Split SUSY

Both $5$'s and $45$'s have proton decay causing components through $\bar{\phi} 10 \bar{5}$

so $\phi$ must get mass at GUT scale (could project out components, spoils unification)

SM flavor structure is generated near the GUT scale
Split SUSY

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so $\phi$ must get mass at GUT scale (could project out components, spoils unification)

SM flavor structure is generated near the GUT scale

SUSY breaking in $\phi$ sector also at GUT scale so flavor diagrams unsuppressed

$B_\mu \sim \langle \sigma \rangle \sim M_N^2 \sim M_{GUT}^2$

SUSY breaking in $\phi$ sector feeds down to SM sector through loops

so must work in Split SUSY with scalars at (or 1-loop below) GUT scale
Domino Mechanism

\[ W = H_u 10_3 10_3 + H_d 10_i \lambda_{ij} \bar{5}_j \]  
add SUSY breaking \[ L \ni B \mu H_u^{(3)} H_d^{(3)} \]
Domino Mechanism

\[ \mathcal{W} = H_u 10_3 10_3 + H_d 10_i \lambda_{ij} \bar{\tilde{5}}_j \quad \text{add SUSY breaking} \quad \mathcal{L} \ni B \mu H_u^{(3)} H_d^{(3)} \]
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Predictions for Yukawa Couplings
Parameters and Planck Slope

\[ W = H_u 10^3 10^3 + \phi 10^3 \lambda^5 \]

\[ \mathcal{L} \ni B \mu \phi \phi \]

\[ U(2)^{10} \times U(3)^{5} \implies \lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} & 0 \\ 0 & \lambda_{22} & \lambda_{23} \\ 0 & 0 & \lambda_{33} \end{pmatrix} \]
Parameters and Planck Slope

\[ W = H_u \begin{pmatrix} 10 \ 10 \ 3 \ 3 \end{pmatrix} + \phi \begin{pmatrix} 10 \ & \lambda \ & 5 \end{pmatrix} \quad \mathcal{L} \equiv B \mu \phi \phi \]

\[ U(2)_1 \times U(3)_5 \implies \lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} & 0 \\ 0 & \lambda_{22} & \lambda_{23} \\ 0 & 0 & \lambda_{33} \end{pmatrix} \]

7 parameters vs. 6 masses, 3 mixings, and 1 phase in quark sector
Parameters and Planck Slope

\[ W = H_u \begin{pmatrix} 10 & 10 \end{pmatrix} + \phi \begin{pmatrix} 10 & \lambda & 5 \end{pmatrix} \]

\[ \mathcal{L} \equiv B \mu \phi \phi \]

\[ U(2)_{10} \times U(3)_{\bar{5}} \implies \lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} & 0 \\ 0 & \lambda_{22} & \lambda_{23} \\ 0 & 0 & \lambda_{33} \end{pmatrix} \]

7 parameters vs. 6 masses, 3 mixings, and 1 phase in quark sector

Higher dim Planck suppressed ops:

\[ \frac{\langle \sigma \rangle^2}{M_p^2} H_u \begin{pmatrix} 10 \\ 10 \end{pmatrix} \]

\[ \frac{\langle \sigma \rangle^2}{M_p^2} H_d \begin{pmatrix} 10 & \bar{5} \end{pmatrix} \]

\[ \frac{\Sigma^\dagger \Sigma}{M_p^2} H_u^\dagger \phi \]

\[ \implies \left( \frac{M_{\text{GUT}}}{M_p} \right)^2 \sim 10^{-5} \]

contributes to 1st generation masses and CP phase, gives \[ J \sim 3 \times 10^{-5} \]

Only exact predictions are between downs and leptons, but all 13 Yukawas predicted at O(1)
Up Yukawas

\[ m_t \implies \frac{\langle \sigma \rangle}{M_N} \approx 1 \]

\[ \sim \frac{f_C}{(16\pi^2)^2} \lambda^4 \log \left( \frac{\Lambda^2}{m^2_\phi} \right) \approx 4 \]

\[ (\phi, \bar{\phi}) = (5, \bar{5}) \quad (45, \overline{45}) \]

\[ \frac{m_c}{m_t} \implies \lambda_{33}^2 \lambda_{32}^2 \approx 4 \\
\lambda_{22}^2 \lambda_{21}^2 \approx 3 \]

Up Yukawas good with O(1) numbers
\[ \sim \frac{f_C}{16\pi^2} \frac{y_t \lambda}{2} \log \left( \frac{m_\phi^2 - B\mu}{m_\phi^2 + B\mu} \right) \approx \frac{f_C}{16\pi^2} y_t \lambda \frac{B\mu}{m_\phi^2} \]
Down Yukawas

\[ \frac{f_C}{16\pi^2} \frac{y_t \lambda}{2} \log \left( \frac{m^2 \phi - B\mu}{m^2 \phi + B\mu} \right) \approx \frac{f_C}{16\pi^2} y_t \lambda \frac{B\mu}{m^2 \phi} \]

naturally generates \[ \frac{m_b}{m_t} \approx 10^{-2} \] at 1-loop

even though \[ \frac{m_c}{m_t} \approx 3 \times 10^{-3} \] at 2-loop
Down Yukawas

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\( (\phi, \bar{\phi}) = (5, 5) \quad (45, 45) \)

\[ \begin{align*}
\lambda_{33} & \approx 2 & 1 \\
\lambda_{32} & \approx 4 & 2 \\
\lambda_{22} & \approx 1/2 & 1/3
\end{align*} \]
Down Yukawas

\[ \sim \frac{f_C}{16\pi^2} \frac{y_t \lambda}{2} \log \left( \frac{m^2_\phi - B\mu}{m^2_\phi + B\mu} \right) \approx \frac{f_C}{16\pi^2} y_t \lambda \frac{B\mu}{m^2_\phi} \]

naturally generates \( \frac{m_b}{m_t} \approx 10^{-2} \) at 1-loop

even though \( \frac{m_c}{m_t} \approx 3 \times 10^{-3} \) at 2-loop

\( (\phi, \overline{\phi}) = (5, \overline{5}) \quad (45, \overline{45}) \)

\[ \lambda_{33} \approx \begin{cases} 2 & \text{Planck slop generates all 1st generation} \\ 1 & \text{masses at the same level} \end{cases} \]

\[ \lambda_{32} \approx \begin{cases} 4 & \text{Planck slop generates all 1st generation} \\ 2 & \text{masses at the same level} \end{cases} \]

\[ \lambda_{22} \approx \begin{cases} 1/2 & \text{Planck slop generates all 1st generation} \\ 1/3 & \text{masses at the same level} \end{cases} \]
SU(5) Breaking Effects

in minimal model:

bottom mass
tau mass
SU(5) Breaking Effects

in minimal model:

\[
\frac{m_\tau}{m_b} = \frac{3}{2}
\]

from ratio of color factors

this is often preferred to 1

Antusch & Spinrath (2009), Raby et. al. (2008)
SU(5) Breaking Effects

in minimal model:

\[ \begin{array}{c}
Q \quad H_u^{(3)} \quad H_d^{(3)} \quad D \\
Q \quad U \\
H_u \\
\end{array} \]

\[ \begin{array}{c}
E \quad H_u^{(3)} \quad H_d^{(3)} \quad L \\
U \quad Q \\
H_u \\
\end{array} \]

\[ \Rightarrow \quad \frac{m_\tau}{m_b} = \frac{3}{2} \]

from ratio of color factors
this is often preferred to 1
Antusch & Spinrath (2009), Raby et. al. (2008)

\[ \begin{array}{c}
10_2 \quad \bar{5}_2 \\
10_3 \quad \bar{5}_3 \\
\end{array} \]

\[ \begin{array}{c}
10_2 \quad \bar{5}_2 \\
\end{array} \]

\[ \Rightarrow \quad \frac{m_\mu}{m_s} \gtrsim \frac{3}{2} \]

O(1) mass splittings in components of \( \phi \) then easily give \( \frac{m_\mu}{m_s} \approx 3 \)
Experimental Signatures
Proton Decay

\[ \mathcal{W} \supset \lambda_{ij} 10_i \bar{5}_j \phi \supset h_d^{(3)} Q L + h_d^{(3)} U D \]

\( h_d \) gives dim 6 proton decay

easily the dominant decay mode since not Yukawa suppressed because \( \lambda \) is \( O(1) \)

\[ \Gamma \sim \frac{1}{8\pi} \lambda^4 \frac{m_p^5}{M_h^4} \approx \frac{1}{10^{35} \text{ yr}} \lambda^4 \left( \frac{2 \times 10^{16} \text{ GeV}}{M_h} \right)^4 \]

potentially observable at next generation experiments (DUSEL, Hyper-K)
Proton Decay Predictions

near mass basis: \[ \lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} & 0 \\ 0 & \lambda_{22} & \lambda_{23} \\ 0 & 0 & \lambda_{33} \end{pmatrix} \]

potentially many observable predictions:

\[
\begin{align*}
\Gamma (p \to e^+ \pi^0) & \propto \lambda_{11}^4 \\
\Gamma (p \to \mu^+ \pi^0) & \propto \lambda_{11}^2 \lambda_{12}^2 \\
\Gamma (p \to e^+ K^0) & \propto \lambda_{11}^2 \lambda_{12}^2 \\
\Gamma (p \to \mu^+ K^0) & \propto \lambda_{12}^4 \\
\Gamma (n \to e^+ \pi^-) & \propto \lambda_{11}^4 \\
\Gamma (n \to \mu^+ \pi^-) & \propto \lambda_{11}^2 \lambda_{12}^2 \\
\Gamma (n \to e^+ K^-) & \propto 0 \\
\Gamma (n \to \mu^+ K^-) & \propto 0 \\
\Gamma (n \to \nu_e \pi^0) & \propto \lambda_{11}^4 \\
\Gamma (n \to \nu_\mu \pi^0) & \propto \lambda_{11}^2 \lambda_{12}^2 \\
\Gamma (n \to \nu_e K^0) & \propto \lambda_{11}^2 \lambda_{12}^2 \\
\Gamma (n \to \nu_\mu K^0) & \propto (\lambda_{12}^2 + \lambda_{11} \lambda_{22})^2
\end{align*}
\]

observe flavor mechanism of SM in proton branching ratios
The QCD Axion + Strong CP

Our Yukawas are forbidden by a $U(1)_{\text{PQ}}$ and generated when it is broken:

\[
\begin{array}{c|c}
U(1)_{\text{PQ}} & \\
\hline
\sigma & +1 \\
10_i & -1 \\
H_u & 0 \\
H_d & 0 \\
\bar{5}_i & +1 \\
10_N & 0 \\
\end{array}
\]

$U(1)_{\text{PQ}}$ has a mixed anomaly with $SU(3)_C$ and thus have a QCD axion (mix of KSVZ and DFSZ) with $f \sim M_{\text{GUT}}$

This flavor mechanism thus necessarily solves the strong CP problem.
Long-Lived Particles

the axino is often the LSP, so all superpartners decay to it through dim 5, GUT-suppressed operators

\[ \mathcal{W} \propto \frac{\alpha_s}{4\pi} \frac{S}{f} G_\alpha G^\alpha + \frac{\alpha_{EM}}{4\pi} \frac{S}{f} F_\alpha F'^\alpha \]

in particular, the gluino: \( \tilde{G} \rightarrow G + \tilde{a} \) with \( \tau \sim 2 \times 10^4 \text{s} \left( \frac{\text{TeV}}{m_{\tilde{G}}} \right)^3 \left( \frac{f}{10^{16} \text{GeV}} \right)^2 \)
Long-Lived Particles

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$$\mathcal{W} \propto \frac{\alpha_s}{4\pi} \frac{S}{f} G_\alpha G^\alpha + \frac{\alpha_{EM}}{4\pi} \frac{S}{f} F_\alpha F^\alpha$$

in particular, the gluino:

$$\tilde{G} \rightarrow G + \tilde{a} \quad \text{with} \quad \tau \sim 2 \times 10^4 \text{ s} \left( \frac{\text{TeV}}{m_{\tilde{G}}} \right)^3 \left( \frac{f}{10^{16} \text{ GeV}} \right)^2$$

solves the cosmological long-lived gluino problem of Split SUSY, can solve the primordial Lithium problems of BBN

 gluinos stop in LHC detectors, observable through out of time decays to monojets

measurement of mass and lifetime points to the GUT scale
Higgs Mass

Higgs quartic determined at GUT scale by SUSY relation, RG evolve to low scale

Higgs mass is sharply predicted (insensitive to exact value of scalar soft masses), most uncertainty arises from top mass and $\alpha_s$
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Higgs mass is sharply predicted (insensitive to exact value of scalar soft masses), most uncertainty arises from top mass and $\alpha_s$.

Search for the Higgs Particle

Status as of March 2009

<table>
<thead>
<tr>
<th>Excluded by LEP Experiments 95% confidence level</th>
<th>minimal model</th>
<th>Excluded by Tevatron Experiments</th>
<th>Excluded by Indirect Measurements 95% confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>114</td>
<td>140 160 170 180 185 200 GeV/c²</td>
<td>90% confidence level 95% confidence level</td>
</tr>
</tbody>
</table>

$Higgs mass values$
Summary

• All three generations treated identically by fundamental theory

• Arbitrary $O(1)$ couplings naturally generate the hierarchical pattern (not just the small sizes) of masses for all quarks and leptons

• Framework may be more general than just radiative generation.

• Though flavor is generated at GUT scale, many observable predictions:
  
  • Novel source of SU(5) breaking effects
    
    can change $b$-$\tau$ unification

  • Predicts QCD axion solves strong CP, novel proton decay, long-lived particles at BBN and LHC, Higgs mass
Color Factors

\[
(\phi, \overline{\phi}) = (5, \overline{5}) \quad (45, \overline{45})
\]

\[
\begin{array}{ccc}
10^\dagger 10 & 2 & \frac{9}{2} \\
\overline{5}^\dagger \overline{5} & 4 & 9 \\
\end{array}
\]

\[
H_u 1010 & 4 & \frac{21}{4} \\
H_u^\dagger 10\overline{5} & 2 & \frac{9}{8} \\
\end{array}
\]

\[
H_u^\dagger 10\overline{5} & 3 & \frac{3}{2}
\]

wavefunction renormalizations bigger than vertex, but cancel for masses (not for mixing angles)
Color Factors

\( (\phi, \overline{\phi}) = (5, \overline{5}) \quad (45, \overline{45}) \)

\[ H_u^{\dagger} 10 \bar{5} \]

6 \[ 30 \frac{3}{4} \]

2-3 mixing
(also mass)

\[ H_u^{\dagger} 10 \bar{5} \]

6 \[ 15 \frac{3}{16} \]

s, \( \mu \) mass

tend to give smaller hierarchies between downs/leptons than ups
Model with 45’s

\[ 10 \times \overline{5} = 5 + 45 \implies \overline{\phi} 10 \overline{5} \]

all radiative generation works except b, tau at one-loop from top

\[ 10 \times 10 = \overline{5}_s + 45_a + 50_s \implies \phi 10 \overline{3} 10 \overline{3} = 0 \]
Model with 45’s

\[ 10 \times \bar{5} = 5 + 45 \implies \bar{\phi} 10 \bar{5} \]

all radiative generation works except b, tau at one-loop from top

\[ 10 \times 10 = \bar{5}_s + \bar{45}_a + \bar{50}_s \implies \phi 10_3 10_3 = 0 \]

add two vector-like multiplets (instead of one): \( 10_{N_1} 10_{N_2} \)