The emergence of scaling in QCD jets

Erik Gerwick

II Physikalisches Institut: Georg-August-Universität Göttingen

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Follow-up: can this fact be made useful?
The emergence of scaling in QCD jets

Outline

- Introduction: Multi-jet final states
- Jet scaling in data/theory
- The origins of jet scaling
- Jet vetos and other applications
- Conclusions/Speculation

Based on:
EG, Schumann; To appear
EG, Plehn, Schumann; PRL 108.032003, 2012; hep-ph/1108.3335
Englert, EG, Plehn, Schichtel, Schumann; hep-ph/1110.1043

Also:
Englert, Plehn, Schichtel, Schumann; hep-ph/1108.547
Motivation for understanding multi-jet final states

LHC as a QCD machine

- The goal of the LHC is to explore new physics in and beyond the SM
- Every analysis at the LHC requires understanding jets [especially events without jets!]
- High-multiplicities crucial for SM measurements [top properties...]
- Multi-jets are background for BSM searches [SUSY cascades, new colored states, black holes, ...]

QCD as a messy environment for new physics

- Initial and final states are non-perturbative objects (hadrons)
  [CTEQ, NNPDF; Fragmentation functions]
- Hard process and subsequent parton shower understandable in terms of perturbative QCD
- but...even making good predictions here can be a challenge
The challenge for theoretical predictions at the LHC

Calculational techniques for multi-jets

- Leading order calculations suffer from scale uncertainty
- Difference between two (equally good) scale choices $\mu$ and $\bar{\mu}$

$$\sigma_{n-\text{jets}}^{\text{LO}}(\bar{\mu}) - \sigma_{n-\text{jets}}^{\text{LO}}(\mu) \sim \alpha_s^n \left( n b_0 \alpha_s \ln \frac{\bar{\mu}^2}{\mu^2} \right)$$

- NLO calculation less scale sensitive but not always available [pure jets $n \leq 4$ for leading color, $n \leq 3$ at full NLO] [Hoeche et al. Blackhat]

- Also, there are selections/processes/observables with large logarithms

$$\alpha_s \log \left( \frac{Q}{Q_0} \right) \sim 1$$

- Analytic techniques to resum large logarithms [QCD resummation, SCET, but limitations]
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The challenge for theoretical predictions at the LHC

Examples of theoretical tools pushed to their limit

- MonteCarlo Matrix-Element/Parton-Shower methods flexible tools including some LO, NLO and logarithmic effects
- Good general agreement between early data and MonteCarlo methods
- But...inherent limitations on PS evolution is the largest uncertainty in many analysis. [i.e. signal modeling MC@NLO vs. POWHEG]

Jet scaling

- New tools for understanding multi-jet events always welcome.
- The idea of jet scaling may be one of them but...in what observable do we look
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Jet ratios as a handle on scaling and multi-jet rates

\[ \sigma_n \] is the exclusive \( n \) jet cross section (in addition to core process jets)

\[ R_{n+1/n} \equiv \frac{\sigma_{n+1}}{\sigma_n} \]

Why are jet ratios a convenient observable for study?

- Experimentally: systematics tend to cancel.
- Theoretical: scale uncertainties also tends to be weaker
- Visually: easy to interpret and much easier to see patterns [see next slide]
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Observed Scaling Patterns

Staircase [Steve Ellis, Kleiss, Stirling (1985); Berends (1989)]

- Ratios are constant
  \[ \sigma_n^{exclv} \sim R_0^n \equiv e^{-bn} \]
  \[ \Rightarrow \frac{\sigma_{n+1}}{\sigma_n} = e^{-b} = R_0 \]
- Observed: UA1, Tevatron, LHC

Poisson [Peskin & Schroder; Rainwater, Summers, Zeppenfeld (1997)]

- Ratios are not constant
  \[ \sigma_n^{exclv} \sim e^{-\bar{n}} \bar{n}^n / n! \]
  \[ \Rightarrow \frac{\sigma_{n+1}}{\sigma_n} = \frac{\bar{n}}{n + 1} \]
- Observed: e.g. photons at LEP
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Observed Scaling Patterns

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- Observed: e.g. photons at LEP

\[ pp \rightarrow jj + \text{jets at 7 TeV} \]
\[ \frac{\sigma_{n+1}}{\sigma_n} \quad pp \rightarrow jj + \text{gap jets at 7 TeV} \]
\[ \bar{p}_\perp > 100\text{GeV} \quad \Delta y > 3.2 \]
Jet scaling in the data

Finding patterns in current analysis

- Both patterns observed in QCD processes (depending on cuts)
- For staircase: \( R_{\text{excl}} = R_{\text{incl}} \)
  
  \[ R_{\text{excl}} = \frac{\bar{n}}{n+1} \quad \iff \quad R_{\text{incl}} = \left( \frac{(n+1) e^{-\bar{n}} \bar{n}^{-(n+1)}}{\Gamma(n+1) - n\Gamma(n, \bar{n}) + 1} \right)^{-1} \]

\[ \gamma (Z/\sigma) / N_{\geq 1} \]

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QED and the emergence of Poisson scaling

Basic synopsis of Poisson radiation pattern from QED [Peskin & Schroder; Weinberg]

- Fully factorized form of the matrix element (Eikonal approximation)
- Phase space factor $1/n!$ for identical bosons in the final state

$$\Rightarrow \sigma_n \sim \frac{L^n}{n!} e^{-L} \quad \text{with} \quad L \sim \frac{\alpha}{\pi} \log \left( \frac{E_{\text{hard}}}{E_{\text{soft}}} \right)$$

Crucial theorem: Addition of independent Poisson processes

- Suppose two Poisson processes $N_1$ and $N_2$ with Poisson expectations $\bar{n}_1$ and $\bar{n}_2$ are independent. The counting process $N$ defined by $N(t) = N_1(t) + N_2(t)$ is a Poisson process with rate function $\bar{n}$ given by $\bar{n} = \bar{n}_1 + \bar{n}_2$.

$$\Rightarrow \text{All QED processes give Poisson} \quad [\text{in soft collinear limit}]$$
Important differences between QED and QCD

Deviation from Poisson must be the result of one of the following

1. QED Poisson scaling is derived in the soft-collinear limit

2. QCD has (logarithmically equivalent) subsequent splittings (gluon 3-pt)

3. Different kinematics due to PDFs
Important differences between QED and QCD

Deviation from Poisson must be the result of one of the following

1. QED Poisson scaling is derived in the soft-collinear limit
   \[
   \Rightarrow \text{Corrections due to hard matrix elements}
   \]

2. QCD has (logarithmically equivalent) subsequent splittings (gluon 3-pt)

3. Different kinematics due to PDFs
Corrections due to the hard matrix element

Require photons to be widely separated

- Full amplitude contains finite terms \([\text{only log terms exponentiate}]\)
- De-emphasize log terms by scanning final state with Durham measure; i.e. only keep particles with

\[
2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij}) > y_{\text{cut}}^s.
\]

- Destroys the Poisson scaling, but cannot be responsible for a staircase
- Ratios are pushed in the wrong direction \((\text{not surprising due to phase space})\)
Important differences between QED and QCD

Deviation from Poisson must be the result of one of the following

1. QED Poisson scaling is derived in the soft-collinear limit
   \[ \Rightarrow \text{Corrections due to hard matrix elements} \]

2. QCD has (logarithmically equivalent) subsequent splittings (gluon 3-pt)
   \[ \Rightarrow \text{Secondary emissions} \]

3. Different kinematics due to PDFs
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Statistical model for an iterated Poisson process

- Extension of the (pure Poisson) exponentiation model [Rainwater, Zeppenfeld]
- Simple analogy; each emission generates a new Poisson process with separate Poisson parameter $\bar{n}'$

$$P(4, \bar{n}, \bar{n}') \sim e^{-\bar{n} - 4\bar{n}'} \frac{\bar{n}^4}{4!}$$
Statistical model for an iterated Poisson process

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- Simple analogy; each emission generates a new Poisson process with separate Poisson parameter $\bar{n}'$

\[
P(4, \bar{n}, \bar{n}') \sim e^{-\bar{n} - 4\bar{n}'} \frac{\bar{n}^4}{4!} + \frac{1}{2} e^{-\bar{n} - 4\bar{n}'} \bar{n}^3 \bar{n}'
\]
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\]
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Statistical model for an iterated Poisson process

- Limit of “gluon” dominated ($\bar{n}' \gg \bar{n}$) evolution gives flat staircase scaling
- Iterated Poisson process can compute exact jet rates in DLLA [double-leading log]

$$\bar{n} \sim \alpha_s^2 C_F \left( \int_{Q_0}^Q \frac{dz}{z} \log \frac{Q}{z} \right)^2 \quad \bar{n}' \sim \alpha_s^2 C_A \int_{Q_0}^Q \frac{dz}{z} \log \frac{Q}{z} \int_{Q_0}^z \frac{dz'}{z'} \log \frac{Q}{z'}$$
Subsequent splittings and the emergence of staircase

More realistic model: \( e^+ e^- \rightarrow q \bar{q} + \text{jets} \)

- Leading log and next-to-leading log jet rates available for the Durham measure (we calculate to \( \mathcal{O}(\alpha^4) \)) [Catani, Dokshitzer, Olsson, Turnock, Webber (1991); Webber (2010)]

\[
L \equiv \log \frac{1}{\gamma_{\text{cut}}} \quad \text{and} \quad a \equiv \frac{\alpha_S}{\pi}
\]

- Purely abelian terms from \( qg \) splitting exponentiate

\[
f_D^2 = 1 - a \left( \frac{C_F}{2} \right) L^2 + a^2 \left( \frac{C_F}{4} + \frac{C_F C_A}{48} \right) L^4 - a^3 \left( \frac{C_F^2}{16} + \frac{C_F^2 C_A}{96} + \frac{C_F^2 C_A^2}{960} \right) L^6 + a^4 \left( \frac{C_F^4}{384} + \frac{C_F^2 C_A}{1920} + \frac{C_F C_A^2}{21504} \right) L^8
\]

\[
f_D^3 = a \left( \frac{C_F}{2} \right) L^2 - a^2 \left( \frac{C_F^2}{4} + \frac{C_F C_A}{48} \right) L^4 + a^3 \left( \frac{C_F^3}{16} + \frac{C_F^3 C_A}{2880} \right) L^6 - a^4 \left( \frac{C_F^4}{96} + \frac{C_F^3 C_A}{512} + \frac{C_F^2 C_A^2}{5120} \right) L^8
\]

\[
f_D^4 = a^2 \left( \frac{C_F^2}{8} + \frac{C_F C_A}{48} \right) L^4 - a^3 \left( \frac{C_F^3}{16} + \frac{C_F^3 C_A}{48} + \frac{7 C_F C_A^2}{2880} \right) L^6 + a^4 \left( \frac{C_F^4}{64} + \frac{C_F^3 C_A}{128} + \frac{C_F^2 C_A^2}{512} + \frac{C_F C_A^3}{5120} \right) L^8
\]

\[
f_D^5 = a^3 \left( \frac{C_F^3}{48} + \frac{C_F^2 C_A}{96} + \frac{C_F^2 C_A^2}{720} \right) L^6 - a^4 \left( \frac{C_F^4}{96} + \frac{3 C_F^2 C_A^2}{1280} + \frac{41 C_F C_A^3}{161280} \right) L^8
\]

\[
f_D^6 = a^4 \left( \frac{C_F^4}{384} + \frac{C_F^3 C_A}{384} + \frac{7 C_F^2 C_A^2}{7680} + \frac{17 C_F C_A^3}{161280} \right) L^8
\]
Subsequent splittings and the emergence of staircase

More realistic model: \( e^+ e^- \rightarrow q\bar{q} + \text{jets} \)

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\[
L \equiv \log \frac{1}{y_{\text{cut}}} \quad \text{and} \quad a \equiv \frac{\alpha_S}{\pi}
\]

- Purely Abelian terms from \( qg \) splitting exponentiate

\[
f^D_2 = \exp \left[ -\frac{a C_F L^2}{2} \right]
\]

\[
f^D_3 = \left( \frac{a C_F L^2}{2} \right) \exp \left[ -\frac{a C_F L^2}{2} \right] - a^2 \left( \frac{C_F C_A}{48} \right) L^4 + a^3 \left( \frac{C_F^2 C_A}{96} + \frac{C_F C_A^2}{960} \right) L^6 - a^4 \left( \frac{C_F^3 C_A}{384} + \frac{C_F^2 C_A^2}{1920} + \frac{C_F C_A^3}{21504} \right) L^8
\]

\[
f^D_4 = \frac{1}{2!} \left( \frac{a C_F L^2}{2} \right)^2 \exp \left[ -\frac{a C_F L^2}{2} \right] + a^2 \left( \frac{C_F C_A}{48} \right) L^4 - a^3 \left( \frac{C_F^2 C_A}{48} + \frac{7C_F C_A^2}{2880} \right) L^6 + a^4 \left( \frac{C_F^3 C_A}{128} + \frac{C_F^2 C_A^2}{512} + \frac{C_F C_A^3}{5120} \right) L^8
\]

\[
f^D_5 = \frac{1}{3!} \left( \frac{a C_F L^2}{2} \right)^3 \exp \left[ -\frac{a C_F L^2}{2} \right] + a^3 \left( \frac{C_F^2 C_A}{96} + \frac{C_F C_A^2}{720} \right) L^6 - a^4 \left( \frac{C_F^3 C_A}{128} + \frac{3C_F^2 C_A^2}{1280} + \frac{41C_F C_A^3}{161280} \right) L^8
\]

\[
f^D_6 = \frac{1}{4!} \left( \frac{a C_F L^2}{2} \right)^4 \exp \left[ -\frac{a C_F L^2}{2} \right] + a^4 \left( \frac{C_F^3 C_A}{384} + \frac{7C_F^2 C_A^2}{7680} + \frac{17C_F C_A^3}{161280} \right) L^8
\]
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Subsequent splittings and the emergence of staircase

More realistic model: $e^+e^- \rightarrow q\bar{q} + \text{jets}$

- Leading log and next-to-leading log jet rates available for the Durham measure (we calculate to $\mathcal{O}(\alpha^4)$) [Catani, Dokshitzer, Olsson, Turnock, Webber (1991); Webber (2010)]

$$L \equiv \log \frac{1}{y_{\text{cut}}} \quad \text{and} \quad a \equiv \frac{\alpha_S}{\pi}$$

- Non-abelian terms do not simply exponentiate!

$$f_{2D}^D = \exp \left[-\frac{a_{cf} L^2}{2}\right]$$

$$f_{3D}^D = \left(\frac{a_{cf} L^2}{2}\right) \exp \left[-\frac{a_{cf} L^2}{2}\right] - a^2 \left(\frac{C_F C_A}{48}\right) L^4 + a^3 \left(\frac{C_F^2 C_A}{96} + \frac{C_F C_A^2}{960}\right) L^6 - a^4 \left(\frac{C_F^3 C_A}{384} + \frac{C_F C_A^2}{1920} + \frac{C_F C_A^3}{21504}\right) L^8$$

$$f_{4D}^D = \frac{1}{2!} \left(\frac{a_{cf} L^2}{2}\right)^2 \exp \left[-\frac{a_{cf} L^2}{2}\right] - a^2 \left(\frac{C_F C_A}{48}\right) L^4 + a^3 \left(\frac{C_F^2 C_A}{96} + \frac{7C_F C_A^2}{2880}\right) L^6 + a^4 \left(\frac{C_F^3 C_A}{128} + \frac{C_F C_A^2}{512} + \frac{C_F C_A^3}{5120}\right) L^8$$

$$f_{5D}^D = \frac{1}{3!} \left(\frac{a_{cf} L^2}{2}\right)^3 \exp \left[-\frac{a_{cf} L^2}{2}\right] + a^3 \left(\frac{C_F^2 C_A}{96} + \frac{C_F C_A^2}{720}\right) L^6 - a^4 \left(\frac{C_F^3 C_A}{128} + \frac{3C_F^2 C_A^2}{1280} + \frac{41C_F C_A^3}{161280}\right) L^8$$

$$f_{6D}^D = \frac{1}{4!} \left(\frac{a_{cf} L^2}{2}\right)^4 \exp \left[-\frac{a_{cf} L^2}{2}\right] + a^4 \left(\frac{C_F^3 C_A}{384} + \frac{7C_F^2 C_A^2}{7680} + \frac{17C_F C_A^3}{161280}\right) L^8$$
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Final state gluon cascades

- Analytic results from "jet calculus" for pure YM [Veniziano, Konishi]

\[ \sigma^{q \rightarrow q + ng} = e^{-\left(\frac{C_F}{C_A}\right)Y_A_0} \frac{\Gamma\left(\frac{C_F}{C_A} + n\right)}{\Gamma\left(\frac{C_F}{C_A}\right)} \frac{\Gamma\left(\frac{C_F}{C_A} + n\right)}{n!} \]

- Staircase or geometric scaling returns for large \( n \) [small violation due to \( C_F \neq C_A \) effects]
- Jet rates from the parton shower confirm this [cross-over behavior \( n \approx 8 \)]
Parton shower in a nutshell

1. Generate core process with set of outgoing particle momenta
Parton shower in a nutshell

1. Generate core process with set of outgoing particle momenta
2. Attach to each external line Sudakov form factor [no-splitting probability]

$\Delta^i(q) = \exp \left\{ - \int_{Q_0}^{q} dt \, \Gamma^i(q, t) \right\}.$
Parton shower in a nutshell

1. Generate core process with set of outgoing particle momenta
2. Attach to each external line Sudakov form factor [no-splitting probability]
   \[ \Delta^i(q) = \exp \left\{ - \int_{Q_0}^{q} dt \Gamma^i(q, t) \right\} . \]
3. Integrate virtuality \( q \) from hard scale to hadronization scale
Parton shower in a nutshell

1. Generate core process with set of outgoing particle momenta
2. Attach to each external line Sudakov form factor $\Delta_i(q) = \exp\left\{- \int_{Q_0}^{q} dt \Gamma_i(q, t) \right\}$.
3. Integrate virtuality $q$ from hard scale to hadronization scale
4. With each splitting, attach a new Sudakov and repeat
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Final state parton cascade

1. All radiative emissions start off as a Poisson process
\[ \Delta_q = 1 - \Gamma_q + \frac{1}{2} \Gamma^2_q + \cdots = \left\langle \frac{1}{\Delta} \right\rangle + \left\langle \frac{1}{\Delta^2} \right\rangle + \cdots \]

2. Secondary emissions break Poisson scaling \([e.g. \; e^+e^- \rightarrow \text{jets}]\)
\[ \left\langle \frac{1}{\Delta} \right\rangle \sim \Gamma_q \otimes \Gamma_g \]

3. Relative size of primary and secondary splitting processes give us \(\tilde{n}'\)
\[ \frac{\tilde{n}'}{\tilde{n}} \sim \left\langle \frac{1}{\Delta} \right\rangle / \left\langle \frac{1}{\Delta^2} \right\rangle \]
Statistical model for an iterated Poisson process

- Leading log jet rate ratios in $e^+ e^- \rightarrow \text{jets}$ flatter than Poisson \cite{higher multiplicities}
- Still not enough to completely explain the data in Drell-Yan
Important differences between QED and QCD

Deviation from Poisson must be the result of one of the following

1. QED Poisson scaling is derived in the soft-collinear limit.
   Corrections due to hard matrix elements

2. QCD has (logarithmically equivalent) subsequent splittings (gluon 3-pt)
   ⇒ Secondary emissions ✓

3. Different kinematics due to PDFs
   ⇒ Relative cost of an additional jet depends on $n$
Estimating the effect due to PDFs

- The ratio of exclusive cross-sections contains a ratio of PDF's evaluated at different typical scales

\[ R_{(n+1)/n} \sim \frac{f(x_{n+1}, Q_{n+1})}{f(x_n, Q_n)} \]

- Measure of the suppression on exclusive multiplicities given by the (discrete) 2nd-derivative with respect to \( x \).

\[ B(n, Q) = \frac{|f(x_{n+1}, Q_{hard})|^2}{f(x_n, Q_{hard}) f(x_{n+2}, Q_{hard})} \]

- Two representative kinematic extremes
  1. \( Z \) recoils against jets
  2. Each jet costs moves \( x \) by \( \delta x = p_\perp / \sqrt{s} \)

- Choose factorization scale below the jet scale [exclusive jet rates]
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Estimating the effect due to PDFs

Lowest bin most affected and effect decreases rapidly with multiplicity
Important differences between QED and QCD

Deviation from Poisson must be the result of one of the following

1. QED Poisson scaling is derived in the soft-collinear limit
   ⇒ Corrections due to hard matrix elements

2. QCD has subsequent splittings via gluon 3-point vertex
   ⇒ Secondary emissions ✓

3. Different kinematics due to PDFs
   ⇒ Relative cost of an additional jet depends on $n$ ✓
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The return to Poisson scaling in QCD

Observation: Poisson distribution returns when we require a hard final state jet

- PDF effect disappears as we move to higher $x$ [example Drell-Yan]

- Large logarithm drives evolution along a single line
- Primary (abelian) emission off the hard color line are dynamically enhanced [see generating functional formalism]
- Note that inducing a large initial state logarithm (e.g. cut on the lepton invariant mass) is not sufficient to induce the Poisson behaviour [again fixed energy jet rates imply this]
Summary slide on the origins of scaling patterns

1. Jet ratios show two scaling patterns:
   - Poisson or staircase

2. Poisson scaling is a fundamental result which can most easily be seen by expanding the Sudakov form factor off a given hard line.

3. Staircase scaling is a fundamental consequence of QCD for large $n$, and a fortuitous semi-coincidence of two effects in the general case:
   - secondary emissions and PDF suppression.

4. We can undo both effects by imposing a large final state logarithm, and return to Poisson scaling.
Applications
Deserted island physics
Deserted island physics: calculate the (normalized) Drell Yan $N_{\text{jet}}$ ratios

Computational method

1. Find favorite parton shower MC
2. Wait a while (couple of weeks?)
3. Compute each $N_{\text{jet}}$ cross-section
4. Divide rates to obtain ratios

Scaling arguments

1. Everything starts as a Poisson
2. Add 1st order inhomogeneity [from $g \to gg$ splitting functions]
   \[ \tilde{n} \sim 1 \quad \tilde{n}' \sim \frac{C_A}{12C_F} \]
3. Evaluate PDF function B
4. Fold together!
The origin of jet scaling patterns (Illustration)

1. All radiative emissions start off as a Poisson process
The emergence of scaling in QCD jets

The origin of jet scaling patterns (Illustration)

1. All radiative emissions start off as a Poisson process
2. Secondary emissions break Poisson scaling [model non-homogenous Poisson process]
The emergence of scaling in QCD jets

The origin of jet scaling patterns (Illustration)

1. All radiative emissions start off as a Poisson process
2. Secondary emissions break Poisson scaling [model non-homogenous Poisson process]
3. PDF kinematics mean that cost of producing additional jets is $n$ dependent
The emergence of scaling in QCD jets

The origin of jet scaling patterns (Illustration)
Jet Vetos in Higgs searches

Higgs searches via Weak Boson Fusion

- Early sensitivity in $H \rightarrow WW^*/\tau\tau/\gamma\gamma$ [both Atlas and CMS dedicated WBF searches]
- Large background from $V(V) + \text{jets at } O(\alpha_s^2)$

![Higgs WBF signal and Z + jets background diagrams]

Central Jet Veto [or how QCD can actually help us for once!]

- Widely separated (in $\eta$) tagging jets and veto “in-between” QCD activity
- Signal gives less soft in-between QCD radiation [Rainwater, Zeppenfeld]
- $N_{\text{jet}}$ distribution give us valuable information on central jets

![Central jet veto diagram]
Predicting Central Jet veto efficiencies

Weak Boson Fusion cuts

1. Identify tagging jets with $|y_1 - y_2| > 4.4$ and $m_{jj} > 600$ GeV
2. Veto events with an additional central jet satisfying $p_\perp > 20$ GeV
3. Define the 2-jet exclusive cross section for signal and background

Theoretical difficulties in the WBF jet veto computation

- The second step induces a large logarithm $\sim \log \left( \frac{p_{\text{veto}}}{p_{\text{tag}}} \right)$
- Cannot trust fixed order calculations (single emission probability $> 1$)
- Analytic resummation available in some cases [Delgado, Forshaw, Marzani, Seymour]
- The only general method involves Parton-Shower [preferably with matrix element matching]
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Validation of theoretical tools in $H \rightarrow \tau \tau$ [EG, Plehn, Schumann]

Before Weak Boson Fusion cuts

- WBF and $Z+$ jets background follow a flat staircase distribution

Veto Probabilities

- Can use Poisson fit to calculate veto probability taking into account all multi-jet contributions.

$$P_{veto} = e^{-\bar{n}}$$

- Many (data rich) processes available for comparison [QCD gap fraction, $Z/W/\gamma +$ jets]
Validation of theoretical tools in $H \rightarrow \tau\tau$ [EG, Plehn, Schumann]

After Weak Boson Fusion cuts

- $Z+\text{jets}$ background quickly becomes Poisson, while WBF does not

Veto Probabilities

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Validation of tools and scaling hypothesis

Atlas study on jets between gaps as a function of $p_{\perp}$ and $\Delta y$

based on RIVET public-analysis from ATLAS JHEP 1109 (2011) 053

Same for $Z +$ jets analysis/figures
Searches for new physics: Autofocus [EG; Englert, Plehn, Schumann, Schichtel]

1. Take interesting processes (jets + $E_T/\text{leptons} ...$) and plot exclusive ratio distributions
2. New physics signals expected in bins which break scaling pattern [either Poisson or staircase]
3. Complementary discriminating power with simple cut and count.
Conclusions

- Ratio of $N_{\text{jet}}$ distributions interesting from a theory perspective and crucial for precise predictions for jet vetos.
- Many opportunities to study the Poisson distribution in other $n_{\text{jet}}$ distributions, for example
  - $Z/\gamma +$ jets with a large leading jet $p_\perp$.
  - $Z+$ jets in WBF type of configurations (large $\eta$ separations), with large average jet $p_\perp$ of leading two jets.
  - Pure QCD di-jets with a large rapidity gap.
  - $t\bar{t} \rightarrow b\bar{b}WW^* \rightarrow l\nu l\nu +$ jets?
- The fact that QCD cross-section do not at all follow a Poisson distribution (unless a large final state logarithm is present) is the result of a combination of secondary splittings and PDF effects.
- A simple search technique, already present in the data [does not depend on MonteCarlo]
- We suspect more Pheno applications...and we’re working on this!
Central jet vetos in Higgs searches [EG, Plehn, Schumann]

Weak Boson Fusion

- Signal has high $|\eta|$ tagging jets, $Z + \text{jets}$ background more likely to radiate additional jets into this region. Impose WBF cuts

$$y_1 \cdot y_2 < 0 \quad |y_1 - y_2| > 4.4 \quad m_{jj} > 600 \text{ GeV}$$

- Central jet veto makes this channel relevant. Veto all events with an additional jet satisfying

$$p_T^{\text{veto}} > 20 \text{ GeV} \quad \min y_{1,2} < y^{\text{veto}} < \max y_{1,2}$$

- For background, Dipole initiated shower contains a large Logarithm.

$$\bar{n} \sim \log \frac{m_{jj}}{Q_{\text{veto}}}$$

- For signal, large log not induced.
The emergence of scaling in QCD jets

Validation of theoretical tools in $H \rightarrow \tau \tau$ [EG, Plehn, Schumann]

The emergence of Jet scaling patterns

- Through simulation we see that WBF cuts induce Poisson scaling [in background]

Before WBF cuts

![Graph showing data before WBF cuts]

After WBF cuts

![Graph showing data after WBF cuts]

- Our standard $Z/W/\gamma +$ jets $N_{\text{jet}}$ ratios
- For both distribution we can easily compute the survival probability

$$\sigma_{2}^{\text{exclusive}} = \sigma_{2}^{\text{inclusive}} e^{-\bar{n}}$$
The origin of Jet scaling patterns

How can we get a handle on non-exponentiable contributions?

- Inhomogenous Poisson processes [each emission emits]

\[ P(n, \bar{n}, \bar{n}') = e^{-\bar{n} - n\bar{n}'} \sum_{i=0}^{n} \left( \frac{(n-1)!}{i!(n-i-1)!(n-i)!} \right) \bar{n}'^i \bar{n}^{n-i} \]

Deviation from Poisson to 1st order \iff compute \( \bar{n}' \)

How can we reasonably estimate PDF effect?

1. Construct “threshold” kinematics for process and cuts (e.g. Drell-Yan)
2. Compute discretized second derivative

\[ B = \frac{|F(x_{n+1}, Q)|^2}{F(x_n, Q)F(x_{n+2}, Q)} \]

3. Multiply ratios by \( B \)
The emergence of scaling in QCD jets

The return to Poisson scaling

Observation: Poisson distribution returns when we require hard final state jets

- PDF effect disappears as we move to higher $x$ [example Drell-Yan]

- Hard final state jet drives evolution along single line [explains why we do not see Poisson scaling in color singlet exchange]

- Exponentiable wrt core process emission off the hard color line are dynamically enhanced [see generating functional formalism]

- Return to fully factorized form of the matrix element
The emergence of scaling in QCD jets

Missing energy searches and laboratories for studying jet scaling

Photons + jets and missing $E_T$ searches [Englert, Plehn, Schumann, Schichtel]

- Ratio of $N_{\text{jet}}$ distributions interesting from a theory perspective and crucial for precise predictions for jet vetos.
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The emergence of scaling in QCD jets

More on Poisson

Experimental Observation

\[ R_{\text{excl}} = \frac{\bar{n}}{n + 1} \]
\[ R_{\text{incl}} = \left( \frac{(n + 1) e^{-\bar{n}} \bar{n}^{-(n+1)}}{\Gamma(n + 1) - n\Gamma(n, \bar{n}) + 1} \right)^{-1} \]
Summary slide on the origins of scaling patterns

**Poisson scaling**

- Poisson scaling is a fundamental result for the soft collinear limit of massless gauge theories [most easily seen by expanding the Sudakov form factor evolving a given hard line]
- Emissions contained in this expansion wrt the core process generate a Poisson distribution while secondary emission break the scaling pattern.
- We expect a Poisson distribution to answer question such as, ”what is the multiplicity distribution of 10GeV photons at LEP” or jets accompanying the process in a $Z + 1$ jet, where the jet has $p_{\perp} > 30$GeV.

**Staircase (or Berends scaling)**

- Not a fundamental consequence of QCD radiation, but rather a fortuitous semi-coincidence of two effects which both act to flatten the ratio distribution
- We expect this type of behavior for processes and selections not driven by a single large final state logarithm, so that instead primary and secondary emission are roughly equally likely.
Leading-log QCD as a statistical process

- For the double-leading-logs in QCD; only physics is in choosing $\bar{n}$ and $\bar{n}'$
- Compared with the generating functional formalism, this is a dramatic simplification

$$P(4, \bar{n}, \bar{n}') \sim e^{-\bar{n}-4\bar{n}'} \frac{\bar{n}^4}{4!} + \frac{1}{2} e^{-\bar{n}-4\bar{n}'} \bar{n}^3 \bar{n}' + \frac{3}{2} e^{-\bar{n}-4\bar{n}'} \bar{n}^2 \bar{n}'^2 + e^{-\bar{n}-4\bar{n}'} \bar{n}\bar{n}'^3$$
Other correlations affecting Poisson scaling

- Color correlation at sub-leading $N$

- Next-to-leading-log contributions and interference effects
- Not strictly Sudakov type logarithms [BFKL, HEJ, ...]
- Finite corrections to the hard matrix element