c = 12 Moonshine

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Based on: arXiv:1406.5502 w. Cheng, Dong, Duncan, Kachru, Wrase;
What is moonshine?

Connection between 2 mathematical objects:

- Finite groups
- Modular forms

What are these things and why do we care about them in physics?
First I’ll introduce the basic objects and their context in physics...
Finite groups

- Describe discrete symmetries of physical objects

Monster, $|M| \approx 10^{53}$
Largest Mathieu group, $|M_{24}| \approx 10^{8}$

Conway, $|Co_1| \approx 10^{18}$

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Finite groups

- Describe discrete symmetries of physical objects
- Classified in the last century (finite simple groups)

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Finite groups

- Describe discrete symmetries of physical objects
- Classified in the last century (finite simple groups)
- 18 infinite families + 26 sporadic groups

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What is moonshine?

Outline
- Monstrous moonshine
- Conway moonshine
- Conclusions

Modular forms

- General form:
  \[ f \left( \frac{a\tau + b}{c\tau + d} \right) = (c\tau + d)^k f(\tau) \]
  for \( \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z}) \)
  and \( \tau \in \mathbb{H} \).

- \( k \) is called the weight

- \( f(q) = \sum c_n q^n, \quad q = e^{2\pi i \tau} \)

Upper half plane, \( \mathbb{H} \).
Appearance in string theory

- In string theory we consider 2d sigma models, maps from worldsheet to target manifold
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- For compact target manifold, discrete string spectrum $\rightarrow$ discrete symmetry groups.
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- In string theory we consider 2d sigma models, maps from worldsheet to target manifold.
- For compact target manifold, discrete string spectrum $\rightarrow$ discrete symmetry groups.
- For one-loop partition function, worldsheet is torus $\rightarrow$ any trace function should be invariant under $SL(2, \mathbb{Z})$. 
Elliptic genus and Jacobi forms

For supersymmetric string theories, we can consider the elliptic genus:

\[ Z_{\mathcal{T}}(\tau, z) = \text{tr}_{H_{\mathcal{T}, RR}} \left( (-1)^{F_R + F_L} y^{J_0} q^{H_L} q^{H_R} \right) \]

Like a partition function, but only counts BPS states weighted by energy and U(1) charge

Topological

Compact manifold $\iff$ Modular, holomorphic function

**Table 1: Coxeter numbers, exponents, and Frame shapes**

<table>
<thead>
<tr>
<th>Frame shape</th>
<th>Coxeter number</th>
<th>Exponents</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>$-2, -1, 0$</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>$-3, -1, 1$</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>$-3, 1, 3$</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>$-4, -2, -1, 1$</td>
</tr>
<tr>
<td>E</td>
<td>5</td>
<td>$-5, -3, -1, 1, 2$</td>
</tr>
</tbody>
</table>

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Sarah M. Harrison  c = 12 Moonshine
Mock modular forms

\[ \hat{h}(\tau) = h(\tau) + (4i)^{w-1} \int_{-\bar{\tau}}^{\infty} (z + \tau)^{-w} f(-\bar{z}) \, dz \]

Modular form (non-holomorphic) \hspace{1cm} Mock modular form (holomorphic)

Shadow

These functions tend to appear in physics considering manifolds which are non-compact, and in characters of superalgebras.
What is moonshine?

A relationship between characters of certain finite groups and coefficients of modular forms

Finite groups \(\xrightarrow{\text{MOONSHINE}}\) Modular forms
Why is this interesting?

- Besides the fact that symmetries are vital to our understanding of physical systems, modular forms appear in all corners of string theory and physics in general (black holes, $AdS_3$ gravity, CFT, string dualities/topological strings, chiral algebras, theories with electric-magnetic duality...)

- This subject unites many fascinating areas of mathematics including group theory, number theory, algebra, and geometry
Monstrous moonshine

Conway moonshine

Conclusions
The $j$-function

- The $j$-function is the unique weight zero modular function with only a pole at $\tau = i\infty$, up to constant and normalization.
- It has the expansion
  \[
  j(\tau) = \frac{1}{q} + 744 + 196884q + 21493760q^2 + 864299970q^3 + \ldots
  \]
- Famous observation of McKay:
  \[
  196884 = 1 + 196883
  \]
The \( j \)-function

\[
j(\tau) = \frac{1}{q} + 744 + 196884q + 21493760q^2 + 864299970q^3 + \ldots
\]

\( 196884 = 1 + 196883 \)

Dimension of smallest irreducible rep of monster group!!

Thompson:

\[
21493760 = 1 + 196883 + 21296876
\]

\[
864299970 = 2 \times 1 + 2 \times 196883 + 21296876 + 842609326
\]

Also monster irreps!
Monstrous moonshine

What does this mean?
Frenkel-Lepowsky-Meurman:

\[ V = V_{-1} \oplus V_1 \oplus V_2 \oplus V_3 \oplus \ldots \]

\[ V_{-1} = \rho_0, \quad V_1 = \rho_1 \oplus \rho_0, \quad V_2 = \rho_2 \oplus \rho_1 \oplus \rho_0, \ldots \]

\[ j(\tau) - 744 = \dim(V_{-1}) q^{-1} + \sum_{i=1}^{\infty} \dim(V_i) q^i \]

Construction: orbifold (gets rid of 744) of bosonic strings on 24-dimensional Leech lattice

optimal way to pack 24-dimensional spheres
**McKay-Thompson series**

How do we know the symmetry group is there?

- j-function is a partition function of a CFT

\[ Z(\tau) = \text{Tr}(q^{L_0}) = j(\tau) \] counts total number of states at each energy level

- we can “twine” by elements of symmetry group and get many more interesting modular functions

\[ Z[g](\tau) = \text{Tr}(g q^{L_0}) = T[g](\tau) \]

operator acting on states

these functions have nice modular properties under group which preserves g-twisted boundary conditions
Monstrous moonshine

One of the defining characteristics of monstrous moonshine is the Genus Zero property:
If $\mathbb{H}/\Gamma$ for $\Gamma \in SL(2, \mathbb{R})$ is genus zero, then there exists a unique $\Gamma$-invariant holomorphic function satisfying $T_\Gamma(\tau) = q^{-1} + O(q)$ as $\tau \to i\infty$

These are called “principal moduli”, and these functions are precisely the McKay-Thompson series of the FLM module; however, the genus zero property has picked them out without any knowledge of the module.
Monstrous moonshine

Summary:
- Partition function of strings on (orbifolded) Leech lattice
- Modular j-function
- Monster symmetry group
Monstrous moonshine + physics

- Bosonic string compactified on Leech lattice is interesting curiosity...
Monstrous moonshine + physics

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- This is a $c = 24$ 2d Conformal Field Theory
- It is also conjectured to be dual to pure quantum gravity in 3d Anti-de Sitter space (Witten) with action

$$S = \frac{1}{16\pi G} \int d^3x \sqrt{-g} (R - 2\Lambda)$$

where $\Lambda = -1/256G^2$ and AdS radius $\ell^2 = -1/\Lambda$
Monstrous moonshine + physics

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- This is the most negative possible value of the c.c.
Monstrous moonshine + physics

Why?

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Monstrous moonshine + physics

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$$Z_{\text{vac}} = q^{-c/24} \prod_{n=2}^{\infty} \frac{1}{(1-q^n)}$$

to the partition function, corresponding to the identity operator and its Virasoro descendants in the dual CFT
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to the partition function, corresponding to the identity operator and its Virasoro descendants in the dual CFT  
- This is not modular...
Monstrous moonshine + physics

Why?

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Monstrous moonshine + physics

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- CFTs for which this hold with $c = 24k$ are known as “extremal”
- For $c = 24$, this is simply $j(\tau) - 744 = \frac{1}{q} + 196884q + \ldots$
Monstrous moonshine + physics

Assumptions, problems and open questions

- Holomorphic factorization for $c = 24k$, not proven; Chiral gravity?
Monstrous moonshine + physics

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Monstrous moonshine + physics

Assumptions, problems and open questions

- Holomorphic factorization for $c = 24k$, not proven; Chiral gravity?
- Extremal CFTs are not known for $k > 1$, and in many cases their symmetry groups cannot be this large (Gaiotto)
- There are subtleties about which geometries contribute to the CFT partition function—which saddles should one include? (Maloney-Witten)
Mathieu moonshine

Eguchi-Ooguri-Tachikawa, K3 elliptic genus:

\[
Z(\tau, z) = \frac{\theta_2^2(\tau, z)}{\eta^3(\tau)} \left( a \mu(\tau, z) + q^{-1/8} \left( b + \sum_{n=1}^{\infty} t_n q^n \right) \right)
\]

24 massless multiplets
expand this function in N=4 superconformal characters

\[2 \times 45, 2 \times 231, 2 \times 770, 2 \times 2277, 2 \times 5796, \ldots\]

(Sums of) dimensions of irreducible representations of the largest Mathieu group M24!

Infinite tower of massive multiplets
Mathieu moonshine

- Representations are governed by a mock modular form
- No explicit construction of a theory (CFT or otherwise) with $M_{24}$ symmetry which naturally yields this mock modular form
Conway moonshine

- Original proposal by FLM: a $c = 12$ SCFT composed of 8 bosons on the $E_8$ root lattice and their Fermi superpartners
- Another description by Duncan in terms of 24 free chiral fermions with a $\mathbb{Z}_2$ orbifold
- Choosing an $\mathcal{N} = 1$ supercharge breaks $Spin(24)$ to $Co_0$, group of automorphisms of the Leech lattice

Partition function takes the form

$$\frac{1}{\sqrt{q}} + 276\sqrt{q} + 2048q + 11202q^{3/2} + \ldots$$
Conway moonshine

The twined partition functions in this model

\[ Z_g = \text{Tr}_g q^{L_0 - c/24} \]

under element in the Conway group are normalized principal moduli for genus zero groups.
Conway moonshine

Why study this model? Some reasons

- As in the case of the monster CFT, this $c = 12$ CFT is conjectured to be dual to pure sugra in $AdS_3$ (Witten)
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- As in the case of $K3$ sigma models ($c = 6$), interesting and suggestive properties were observed in elliptic genera of complex 4-manifolds which correspond to $c = 12$ worldsheet CFTs
Conway moonshine

Why study this model? Some reasons

▷ As in the case of the monster CFT, this $c = 12$ CFT is conjectured to be dual to pure sugra in $AdS_3$ (Witten)

▷ As in the case of $K3$ sigma models ($c = 6$), interesting and suggestive properties were observed in elliptic genera of complex 4-manifolds which correspond to $c = 12$ worldsheet CFTs

▷ This is a precise toy model for understanding such symmetries which may arise in superstring compactifications
Holonomy and superstring compactification

- In string theory there is a relation between special holonomy groups of compactification manifolds and spacetime SUSY.
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- For superstring models the list is:
  - $SU(n)$, Calabi Yau
  - $Sp(n)$, HyperKaehler
  - G2
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For superstrings these yield enhanced worldsheet SUSY:
- $SU(n)$, Calabi Yau $\iff \mathcal{N} = (2,2)$
- $Sp(n)$, HyperKaehler $\iff \mathcal{N} = (4,4)$
- G2 $\iff \mathcal{N} = (1,1) + \text{tricritical Ising}$
- Spin(7) $\iff \mathcal{N} = (1,1) + \text{Ising}$
Holonomy and superstring compactification

- For $c = 12$, we can have Calabi-Yau, HyperKaehler, and Spin(7) manifolds.
- The elliptic genus of these theories exhibit intriguing properties, similar to that of the $K3$ surface, but a connection with moonshine is difficult to make precise for a general manifold since the elliptic genus depends on some moduli.
- However, the chiral CFT described is an arena in which we can make very similar connections very precise.
Conway moonshine

How to construct enhanced SUSY algebras in our chiral CFT?

- The key is to first construct an $R$-current using some number of the free fermions
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- Example: $\mathcal{N} = 4$ Choose three of the fermions to generate an $SU(2)$:

$$J_i = -i\epsilon_{ijk}\lambda_j\lambda_k, i, j, k \in \{1, 2, 3\}.$$  (1)
Conway moonshine

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- Example: $\mathcal{N} = 4$ Choose three of the fermions to generate an $SU(2)$:
  \[ J_i = -i\epsilon_{ijk}\lambda_j\lambda_k, \quad i, j, k \in \{1, 2, 3\}. \]  \[ (1) \]
- Check they form an affine $SU(2)$ algebra with level 2 through the OPE:
  \[ J_i(z)J_j(0) \sim \frac{1}{z^2}\delta_{ij} + \frac{i}{z}\epsilon_{ijk}J_k(0). \]  \[ (2) \]
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How to construct enhanced SUSY algebras in our chiral CFT?

- Fill out the rest of the $\mathcal{N} = 4$ algebra by computing OPE with $\mathcal{N} = 1$ generators
- This yields a full copy of the $\mathcal{N} = 4$ SCA
- Can similarly construct a $\mathcal{N} = 2$ SCA by choosing 2 fermions to generate the $U(1)$ $R$-current
- Can similarly construct a $SW(3/2, 2)$ SCA ($\text{Spin}(7)$) by choosing 1 fermion to generate the Ising $R$-current
Global symmetries

Once we construct the extended superalgebras, the model no longer has a $Co_0$ global symmetry group. What symmetry group is preserved by each algebra?

- A choice of $n$ fermion(s) breaks $Co_0$ to a subgroup which preserves an $n$-plane in the Leech lattice:

<table>
<thead>
<tr>
<th>Superalgebra</th>
<th>Geometrical Representation</th>
<th>Global symmetry group</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{N} = 0$</td>
<td>$\mathbb{R}^{24}$</td>
<td>$Spin(24)$</td>
</tr>
<tr>
<td>$\mathcal{N} = 1$</td>
<td>$\Lambda_{Leech}$</td>
<td>$Co_0$</td>
</tr>
<tr>
<td>Spin(7)</td>
<td>$\Lambda_{Leech}$, fixed 1-plane</td>
<td>$M_{24}$</td>
</tr>
<tr>
<td>$\mathcal{N} = 2$</td>
<td>$\Lambda_{Leech}$, fixed 2-plane</td>
<td>$M_{23}$</td>
</tr>
<tr>
<td>$\mathcal{N} = 4$</td>
<td>$\Lambda_{Leech}$, fixed 3-plane</td>
<td>$M_{22}$</td>
</tr>
</tbody>
</table>
Global symmetries

We can see this explicitly by looking at the decomposition of the graded partition function in terms of characters of these SCAs: For example, computing

$$Z(\tau, z) = \text{Tr}(-1)^F q^{L_0 - c/24} y^{J_0}$$

and decomposing into $\mathcal{N} = 4$ characters, we get:

$$Z(\tau, z) = 21 \text{ch}_{3; \frac{1}{2}, 1}^{\mathcal{N}=4} + \text{ch}_{3; \frac{1}{2}, 0}^{\mathcal{N}=4}$$

$$+ (560 \text{ch}_{3; \frac{3}{2}, \frac{1}{2}}^{\mathcal{N}=4} + 8470 \text{ch}_{3; \frac{5}{2}, \frac{1}{2}}^{\mathcal{N}=4} + 70576 \text{ch}_{3; \frac{7}{2}, \frac{1}{2}}^{\mathcal{N}=4} + \ldots)$$

$$+ (210 \text{ch}_{3; \frac{3}{2}, 1}^{\mathcal{N}=4} + 4444 \text{ch}_{3; \frac{5}{2}, 1}^{\mathcal{N}=4} + 42560 \text{ch}_{3; \frac{7}{2}, 1}^{\mathcal{N}=4} + \ldots)$$
Global symmetries

These are $M_{22}$ representations: $21, 210, 560 = 280 + \overline{280}$, ....

$$Z(\tau, z) = 21 \text{ch}_{3;\frac{1}{2},0}^{N=4} + \text{ch}_{3;\frac{1}{2},1}^{N=4}$$

$$+ (560 \text{ch}_{3;\frac{3}{2},\frac{1}{2}}^{N=4} + 8470 \text{ch}_{3;\frac{5}{2},\frac{1}{2}}^{N=4} + 70576 \text{ch}_{3;\frac{7}{2},\frac{1}{2}}^{N=4} + \ldots)$$

$$+ (210 \text{ch}_{3;\frac{3}{2},1}^{N=4} + 4444 \text{ch}_{3;\frac{5}{2},1}^{N=4} + 42560 \text{ch}_{3;\frac{7}{2},1}^{N=4} + \ldots)$$

$$= (\psi_{1,1}(\tau, z))^{-1} \left( 24 \mu_{3;0}(\tau, z) + \sum_{r \in \mathbb{Z}/6\mathbb{Z}} h_r(\tau) \theta_3,r(\tau, z) \right)$$

and they are encoded in a mock modular form.
Global symmetries

The same holds for $\mathcal{N} = 2$ and $M_{23}$ representations: 23, 231, 770, ...

$$Z(\tau, z) = 23 \, \text{ch}_{\frac{3}{2}, \frac{1}{2}, 0}^{\mathcal{N}=2} + \text{ch}_{\frac{3}{2}, \frac{1}{2}, 2}^{\mathcal{N}=2} + (770 \, (\text{ch}_{\frac{3}{2}, \frac{3}{2}, 1}^{\mathcal{N}=2} + \text{ch}_{\frac{3}{2}, \frac{3}{2}, -1}^{\mathcal{N}=2})$$

$$+ 13915 \, (\text{ch}_{\frac{3}{2}, \frac{5}{2}, 1}^{\mathcal{N}=2} + \text{ch}_{\frac{3}{2}, \frac{5}{2}, -1}^{\mathcal{N}=2}) + \ldots$$

$$+ (231 \, \text{ch}_{\frac{3}{2}, \frac{3}{2}, 2}^{\mathcal{N}=2} + 5796 \, \text{ch}_{\frac{3}{2}, \frac{5}{2}, 2}^{\mathcal{N}=2} + \ldots )$$

$$= \psi_{1, -\frac{1}{2}}^{-1} \left( 24 \, \tilde{\mu}_{\frac{3}{2}, 0} \right)$$

$$+ (\! - \! q^{-\frac{1}{24}} + 770 \, q^{23} + 13915 \, q^{47} \! + \ldots \! ) (\theta_{\frac{3}{2}, \frac{1}{2}} + \theta_{\frac{3}{2}, -\frac{1}{2}})$$

$$+ (q^{-\frac{3}{8}} + 231 \, q^{\frac{5}{8}} + 57962q^{\frac{13}{8}} + \ldots ) \theta_{\frac{3}{2}, \frac{3}{2}} \right)$$

(3)
Global symmetries

...And for Spin(7) and $M_{24}$ representations: 23, 253, 1771, . . .

$$Z_{NS} = \tilde{\chi}_0^{NS} + 23\tilde{\chi}_{1/2}^{NS} + (253\chi_{0,1}^{NS} + 7359\chi_{0,2}^{NS} + \ldots) + (1771\chi_{1/16,3}^{NS} + \ldots)$$

$$= P(24\mu^{NS} + q^{-1/120}(-1 + 1771q + \ldots)\Theta_{1/16}^{NS})$$

$$+ q^{-49/120}(1 + 253q + \ldots)\Theta_0^{NS}$$
The groups mentioned are not the only subgroups of $Co_0$ which preserve $n$-planes in the Leech lattice.

However the Mathieu groups are singled out due to the following moonshine property, analogous to the genus zero property:

1. $Z_g(\tau, z) = c_g + y^2 + y^{-2}$ as $\tau \to i\infty$, and
2. $Z_g|\gamma(\tau, z) = c_g,\gamma$ as $\tau \to i\infty$ whenever $\gamma \in SL_2(\mathbb{Z})$ and $\gamma\infty \notin \Gamma_g\infty$. 
Conclusions

- In this free fermion chiral CFT, we have constructed the first explicit moonshine modules for mock modular forms.
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- The geometry of $n$-planes in the Leech lattice governs the global symmetry groups which are preserved by various superalgebras.
- It would be interesting to understand precisely to what extent this module governs symmetries which appear in string compactifications with extended worldsheet supersymmetry.
- ...As well as the relation to gravity and the structure of extremal SCFTs with higher central charge.
Conclusions

What physics can we hope to learn about?

- Conformal field theories on K3 surfaces and more general Calabi-Yau manifolds
- Supersymmetric black hole entropy, wall-crossing, and topological strings
- $AdS_3$ quantum gravity
- BPS states and chiral algebras in supersymmetric field theories
- D-branes and their bound states
- Invariants in geometry and topology coming from SCFT/SQFT (Donaldson, Gromov-Witten, Gopakumar-Vafa, Donaldson-Thomas...)