New Supergravities from Supersymmetry Breaking

SUSY98

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From string theory we know that the real world has eight supersymmetries,

$N = 8$

↳ $N = 1$

↳ $N = 0$

In this talk, we will restrict ourselves to

$N = 2$

↳ $N = 1$

N=2 Algebra

$$\{Q_\alpha,\, \bar{Q}_{\dot\alpha}\} \;=\; 2\,\sigma^m_{\alpha\dot\alpha}\,P_m$$
$$\{S_\alpha,\, \bar{S}_{\dot\alpha}\} \;=\; 2\,\sigma^m_{\alpha\dot\alpha}\,P_m$$

Folk Theorem:

$$Q\,|0\rangle \;=\; \bar{Q}\,|0\rangle \;=\; 0$$
$$\Rightarrow \qquad H\,|0\rangle \;=\; 0$$
$$\Rightarrow \qquad (\bar{S}S + S\bar{S})\,|0\rangle \;=\; 0$$

" $\Rightarrow$ "

$$S\,|0\rangle \;=\; \bar{S}\,|0\rangle \;=\; 0$$

Partial supersymmetry breaking appears impossible …. ?

Loopholes:

- Spontaneously broken charges don't exist

$$\{\bar{Q}_{\dot\alpha},\ J^1_{\alpha m}\} \ = \ 2\,\sigma^n_{\alpha\dot\alpha}\,T_{mn}$$

$$\{\bar{S}_{\dot\alpha},\ J^2_{\alpha m}\} \ = \ 2\,\sigma^n_{\alpha\dot\alpha}\,(v^4\eta_{mn} + T_{mn})$$

- In covariantly-quantized supergravity theories, the Hilbert space does not have positive norm.

$\psi_{m\alpha}$ is a "gauge field."

Examples:

- Branes

  N=1 supersymmetric 3-brane in D=6
  (Hughes, Liu and Polchinski)

- Nonlinear realizations

  Chiral and vector Goldstone multiplets
  (Bagger and Galperin)

- Linear realizations

  N=2 vector multiplet
  (Antoniadis, Partouche and Taylor)

Questions:

- How does partial breaking work in the presence of gravity?

$$T_{mn}$$

$$v^4 \eta_{mn} + T_{mn}$$

Ferrara, Girardello, Porrati; Zinov'ev

- How does partial breaking work in the presence of a cosmological constant?

$$\Lambda \neq 0$$

Cecotti, Girardello, Porrati

In this talk we'll focus on the theories with nonlinear realizations. We'll couple them to supergravity – to lowest non-trivial order.

We'll find

- Partial breaking in flat space motivates a new representation for the N=1 massive spin-3/2 multiplet.
- This representation gives rise to a new N=2 supergravity and a new N=2 supersymmetry algebra.
- Partial breaking in AdS space can also give rise to a new N=2 algebra….

Technique:

- Construct the Lagrangian and super-symmetry transformations for the massive spin 3/2 multiplet.

- Unhiggs the representation by adding appropriate Goldstone fields and coupling to gravity.

We'll see that the basic technique is the same in flat and AdS space....

The massive N=1 spin-3/2 multiplet contains the following spins:

$$\begin{pmatrix} & \frac{3}{2} & \\ 1 & & 1 \\ & \frac{1}{2} & \end{pmatrix}$$

- The traditional representation contains one spin-3/2 fermion, one spin-1/2 fermion, and two vectors, all with identical mass.
- An alternative representation has with the same fermions, but just one vector and one antisymmetric tensor.

Each multiplet has a role to play in the theory of partial supersymmetry breaking!

Traditional Representation:

Lagrangian

$$
\begin{aligned}
\mathcal{L} \;=\; & \epsilon^{mn\rho\sigma}\overline{\psi}_m\overline{\sigma}_n\partial_\rho\psi_\sigma - \mathrm{i}\overline{\zeta}\overline{\sigma}^m\partial_m\zeta - \frac{1}{4}\mathcal{A}_{mn}\bar{\mathcal{A}}^{mn} \\
& - \frac{1}{2}m^2\mathcal{A}_m\bar{\mathcal{A}}^m \\
& + \frac{1}{2}m\,\zeta\zeta \;+\; h.c. \\
& - m\,\psi_m\sigma^{mn}\psi_n \;+\; h.c.
\end{aligned}
$$

Ferrara,van Nieuwenhuizen

Transformations

$$
\begin{aligned}
\delta_\eta\mathcal{A}_m \;&=\; 2\psi_m\eta - \mathrm{i}\frac{2}{\sqrt{3}}\bar{\zeta}\bar{\sigma}_m\eta - \frac{2}{\sqrt{3}m}\partial_m(\zeta\eta) \\
\delta_\eta\zeta \;&=\; \frac{1}{\sqrt{3}}\bar{\mathcal{A}}_{mn}\sigma^{mn}\eta - \mathrm{i}\frac{m}{\sqrt{3}}\sigma^m\bar{\eta}\mathcal{A}_m \\
\delta_\eta\psi_m \;&=\; \frac{1}{3m}\partial_m(\bar{\mathcal{A}}_{rs}\sigma^{rs}\eta + 2\mathrm{i}m\sigma^n\bar{\eta}\mathcal{A}_n) \\
&\quad - \frac{\mathrm{i}}{2}(H_{+mn}\sigma^n + \frac{1}{3}H_{-mn}\sigma^n)\bar{\eta} \\
&\quad - \frac{2}{3}m(\sigma_m{}^n\bar{\mathcal{A}}_n\eta + \bar{\mathcal{A}}_m\eta)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{A}_m \;&=\; A_m + \mathrm{i}B_m \\
H_{\pm mn} \;&=\; \mathcal{A}_{mn} \pm \frac{\mathrm{i}}{2}\epsilon_{mnrs}\mathcal{A}^{rs}
\end{aligned}
$$

New Representation:

Lagrangian

$$\begin{aligned}\mathcal{L} &= \epsilon^{pqrs}\bar{\psi}_p\bar{\sigma}_q\partial_r\psi_s - \mathrm{i}\bar{\zeta}\bar{\sigma}^m\partial_m\zeta - \frac{1}{4}A_{mn}A^{mn} + \frac{1}{2}v^m v_m \\ &\quad - \frac{1}{2}m^2 A_m A^m - \frac{1}{4}m^2 B_{mn}B^{mn} \\ &\quad + \frac{1}{2}m\,\zeta\zeta \; + \; h.c. \\ &\quad - m\,\psi_m\sigma^{mn}\psi_n \; + \; h.c.\end{aligned}$$

Transformations

$$\begin{aligned}\delta_\eta A_m &= (\psi_m\eta + \bar{\psi}_m\bar{\eta}) + \frac{\mathrm{i}}{\sqrt{3}}(\bar{\eta}\bar{\sigma}_m\zeta - \bar{\zeta}\bar{\sigma}_m\eta) - \frac{1}{\sqrt{3}m}\partial_m(\zeta\eta + \bar{\zeta}\bar{\eta}) \\ \delta_\eta B_{mn} &= \frac{2}{\sqrt{3}}\left(\eta\sigma_{mn}\zeta + \frac{\mathrm{i}}{2m}\partial_{[m}\bar{\zeta}\bar{\sigma}_{n]}\eta\right) \; + \; h.c. \\ &\quad + \mathrm{i}\eta\sigma_{[m}\bar{\psi}_{n]} + \frac{1}{m}\eta\psi_{mn} \; + \; h.c. \\ \delta_\eta\zeta &= \frac{1}{\sqrt{3}}A_{mn}\sigma^{mn}\eta - \frac{\mathrm{i}m}{\sqrt{3}}\sigma^m\bar{\eta}A_m - \frac{1}{\sqrt{3}}m\sigma_{mn}\eta B^{mn} - \frac{1}{\sqrt{3}}v_m\sigma^m\bar{\eta} \\ \delta_\eta\psi_m &= \frac{1}{3m}\partial_m\left(A_{rs}\sigma^{rs}\eta + 2\mathrm{i}m\sigma^n\bar{\eta}A_n\right) - \frac{\mathrm{i}}{2}(H^A_{+mn}\sigma^n + \frac{1}{3}H^A_{-mn}\sigma^n)\bar{\eta} \\ &\quad - \frac{2}{3}m(\sigma_m{}^n A_n\eta + A_m\eta) \\ &\quad + \frac{1}{3m}\partial_m\left(2v_n\sigma^n\bar{\eta} - m\sigma^{rs}\eta B_{rs}\right) - \frac{2\mathrm{i}}{3}(v_m + \sigma_{mn}v^n)\eta \\ &\quad - \frac{\mathrm{i}m}{3}(B_{mn}\sigma^n\bar{\eta} + \mathrm{i}\epsilon_{mnrs}B^{nr}\sigma^s\bar{\eta})\end{aligned}$$

where

$$v_m = \frac{1}{2}\epsilon_{mnrs}\partial^n B^{rs}$$

To unhiggs the representations we need to add Goldstone and gauge fields.

Which ones?

Chiral multiplet:

$$\left[\delta_{\eta^2}, \delta_{\eta^1}\right] \phi \ = 2v^2\, \eta^2 \eta^1$$

Two Goldstone scalars suggest that we need two vectors....

$$\underbrace{\begin{pmatrix} 2 \\ \frac{3}{2} \end{pmatrix} \quad \begin{pmatrix} \frac{3}{2} \\ 1 \end{pmatrix}}_{\text{N=2 supergravity}} \qquad \underbrace{\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \quad \begin{pmatrix} & \frac{1}{2} & \\ 0 & & 0 \end{pmatrix}}_{\text{N=1 matter}}$$

N=2 supergravity N=1 matter

One vector is the N=2 graviphoton and the other is N=1 matter.

Lagrangian

$$
\begin{aligned}
e^{-1}\mathcal{L} \;=&\\
&-\frac{1}{2\kappa^2}\mathcal{R}+\epsilon^{mnrs}\overline{\psi}_{mi}\overline{\sigma}_n D_r\psi^i_s-\mathrm{i}\overline{\chi}\,\overline{\sigma}^m D_m\chi-\mathrm{i}\overline{\lambda}\overline{\sigma}^m D_m\lambda-\mathcal{D}^m\phi\overline{\mathcal{D}_m\phi}\\
&-\frac{1}{4}\mathcal{A}_{mn}\overline{\mathcal{A}}^{mn}-\frac{1}{\sqrt{2}}m(\psi^2_m\sigma^m\overline{\lambda}\;+\;h.c.)-m(\mathrm{i}\psi^2_m\sigma^m\overline{\chi}\;+\;h.c.)\\
&-\sqrt{2}m(\mathrm{i}\lambda\chi\;+\;h.c.)-\frac{1}{2}m(\chi\chi\;+\;h.c.)-m\,(\psi^2_m\sigma^{mn}\psi^2_n\;+\;h.c.)\\
&-\frac{\kappa}{4}(\epsilon_{ij}\psi^i_m\psi^j_n\overline{H}^{mn}_+\;+\;h.c.)-\frac{\kappa}{\sqrt{2}}(\chi\sigma^m\overline{\sigma}^n\psi^1_m\overline{\mathcal{D}_n\phi}\;+\;h.c.)\\
&-\frac{\kappa}{2\sqrt{2}}(\overline{\lambda}\overline{\sigma}_m\psi^1_n\overline{H}^{mn}_-\;+\;h.c.)-\frac{\kappa}{\sqrt{2}}\epsilon^{mnrs}(\overline{\psi}_{m2}\overline{\sigma}_n\psi^1_r\overline{\mathcal{D}_s\phi}\;+\;h.c.)
\end{aligned}
$$

Transformations

$$m \;=\; \kappa v^2$$

$$
\begin{aligned}
\delta e^a_m \;&=\; \mathrm{i}\kappa(\eta^i\sigma^a\overline{\psi}_{mi}+\bar{\eta}_i\bar{\sigma}^a\psi^i_m)\\
\delta\psi^i_m \;&=\; \frac{2}{\kappa}D_m\eta^i\\
&\quad+\;\left(-\frac{\mathrm{i}}{2}\hat{H}_{+mn}\sigma^n\overline{\eta}_1+\sqrt{2}\overline{\mathcal{D}_m\phi}\eta^1-\kappa\psi^1_m(\bar{\chi}\bar{\eta}_1)+\mathrm{i}v^2\sigma_m\overline{\eta}_2\right)\delta_2{}^i\\
\delta\mathcal{A}_m \;&=\; 2\epsilon_{ij}\psi^i_m\eta^j+\sqrt{2}\overline{\lambda}\overline{\sigma}_m\eta^1\\
\delta\lambda \;&=\; \frac{\mathrm{i}}{\sqrt{2}}\overline{\hat{\mathcal{A}}}_{mn}\sigma^{mn}\eta^1-\mathrm{i}\sqrt{2}v^2\eta^2\\
\delta\chi \;&=\; \mathrm{i}\sqrt{2}\sigma^m\hat{\mathcal{D}}_m\phi\overline{\eta}_1+2v^2\eta^2\\
\delta\phi \;&=\; \sqrt{2}\chi\eta^1
\end{aligned}
$$

Irreducible representation!

where

$$\begin{aligned} \mathcal{A}_m &= A_m + \mathrm{i}B_m \\ \mathcal{A}_{mn} &= \partial_m \mathcal{A}_m - \partial_n \mathcal{A}_m \\ H_{\pm mn} &= \mathcal{A}_{mn} \pm \frac{\mathrm{i}}{2} \epsilon_{mnrs} \mathcal{A}^{rs} \end{aligned}$$

and

$$\begin{aligned} \hat{\mathcal{D}}_m \phi &= \partial_m \phi - \frac{\kappa}{\sqrt{2}} \psi^1_m \chi - \frac{1}{\sqrt{2}} \kappa v^2 \mathcal{A}_m \\ \hat{\mathcal{A}}_{mn} &= \mathcal{A}_{mn} + \kappa \psi^2_{[m} \psi^1_{n]} - \frac{\kappa}{\sqrt{2}} \bar{\lambda} \bar{\sigma}_{[n} \psi^1_{m]} \end{aligned}$$

Supercovariant!

Vector multiplet:

$$\left[\delta_{\eta^2}, \delta_{\eta^1}\right] B_m = 2\mathrm{i}v^2 \left(\eta^2\sigma_m\bar{\eta}^1 - \eta^1\sigma_m\bar{\eta}^2\right)$$

One Goldstone vector suggests that we need one antisymmetric tensor....

$$\underbrace{\begin{pmatrix} 2 \\ \frac{3}{2} \end{pmatrix} \quad \begin{pmatrix} \frac{3}{2} \\ 1 \end{pmatrix}}_{\text{N=2 supergravity}} \qquad \underbrace{\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \quad \begin{pmatrix} 0 \\ \frac{1}{2} \\ 0 \end{pmatrix}}_{\text{N=1 matter}}$$

The real scalar in the linear multiplet suggests that we also need one real vector....

Lagrangian

$$
\begin{aligned}
e^{-1}\mathcal{L} \; =& \\
& - \frac{1}{2\kappa^2}\mathcal{R} + \epsilon^{pqrs}\bar{\psi}_{pi}\bar{\sigma}_q D_r \psi^i_s - \mathrm{i}\bar{\chi}\bar{\sigma}^m D_m \chi - \mathrm{i}\bar{\lambda}\bar{\sigma}^m D_m \lambda \\
& - \frac{1}{2}\mathcal{D}^m\phi\mathcal{D}_m\phi - \frac{1}{4}\mathcal{F}^A_{mn}\mathcal{F}^{Amn} - \frac{1}{4}\mathcal{F}^B_{mn}\mathcal{F}^{Bmn} + \frac{1}{2}v^m v_m \\
& - \frac{1}{\sqrt{2}}m\,(\psi^2_m\sigma^m\bar{\lambda} \; + \; h.c.) - m(\mathrm{i}\psi^2_m\sigma^m\bar{\chi} \; + \; h.c.) \\
& - \sqrt{2}m(\mathrm{i}\lambda\chi \; + \; h.c.) - \frac{1}{2}m(\chi\chi \; + \; h.c.) - {\color{red}m}\,(\psi^2_m\sigma^{mn}\psi^2_n \; + \; h.c.) \\
& - \frac{\kappa}{2\sqrt{2}}(\epsilon_{ij}\psi^i_m\psi^j_n\mathcal{F}^{Amn}_- \; + \; h.c.) - \frac{\kappa}{2}(\chi\sigma^m\bar{\sigma}^n\psi^1_m\mathcal{D}_n\phi \; + \; h.c.) \\
& - \frac{\kappa}{2}(\bar{\lambda}\bar{\sigma}_m\psi^1_n\mathcal{F}^{Bmn}_+ \; + \; h.c.) - \frac{\kappa}{2}(\epsilon^{pqrs}\bar{\psi}^2_p\bar{\sigma}_q\psi^1_r\mathcal{D}_s\phi \; + \; h.c.) \\
& + \mathrm{i}\,\frac{\kappa}{2}(\chi\sigma^m\bar{\sigma}^n\psi^1_m v_n \; + \; h.c.) + \mathrm{i}\,\frac{\kappa}{2}(\epsilon^{pqrs}\bar{\psi}^2_p\bar{\sigma}_q\psi^1_r v_s s \; + \; h.c.)
\end{aligned}
$$

Transformations

$$m \; = \; \kappa v^2$$

$$
\begin{aligned}
\delta_\eta e^a_m \; &= \; \mathrm{i}\,\kappa(\eta^i\sigma^a\overline{\psi}_{mi} + \bar{\eta}_i\bar{\sigma}^a\psi^i_m) \\
\delta_\eta \psi^1_m \; &= \; \frac{2}{\kappa}D_m\eta^1 \\
\delta_\eta A_m \; &= \; \sqrt{2}\epsilon_{ij}(\psi^i_m\eta^j + \bar{\psi}^i_m\bar{\eta}^j) \\
\delta_\eta B_m \; &= \; \bar{\eta}^1\bar{\sigma}_m\lambda + \bar{\lambda}\bar{\sigma}_m\eta^1 \\
\delta_\eta B_{mn} \; &= \; 2\eta^1\sigma_{mn}\chi \; + \; h.c. \\
& \quad + \mathrm{i}\,\eta^1\sigma_{[m}\bar{\psi}^2_{n]} + \mathrm{i}\,\eta^2\sigma_{[m}\bar{\psi}^1_{n]} \; + \; h.c.
\end{aligned}
$$

Irreducible representation!

$$\delta_\eta \lambda = \mathrm{i}\,\mathcal{F}^B_{mn}\sigma^{mn}\eta^1 - \mathrm{i}\sqrt{2}v^2\eta^2$$

$$\delta_\eta \chi = \mathrm{i}\,\sigma^m\bar{\eta}^1\hat{\mathcal{D}}_m\phi - \hat{v}_m\sigma^m\bar{\eta}^1 + 2v^2\eta^2$$

$$\delta_\eta \psi^2_m = \frac{2}{\kappa}D_m\eta^2 + \mathrm{i}v^2\sigma_m\bar{\eta}^2 - \frac{\mathrm{i}}{\sqrt{2}}\hat{\mathcal{F}}^A_{+mn}\sigma^n\bar{\eta}^1 + \hat{\mathcal{D}}_m\phi\eta^1 + \kappa\big((\bar{\psi}^1_m\bar{\chi})\eta - (\bar{\chi}\bar{\eta})\psi^1_m\big) - \mathrm{i}\,\hat{v}_m\eta^1$$

$$\delta_\eta \phi = \chi\eta^1 + \bar{\chi}\bar{\eta}^1$$

where

$$\mathcal{D}_m\phi = \partial_m\phi - \frac{m}{\sqrt{2}}(A_m + B_m)$$

$$\mathcal{F}^A_{mn} = \partial_{[m}A_{n]} + \frac{m}{\sqrt{2}}B_{mn}$$

$$\mathcal{F}^B_{mn} = \partial_{[m}B_{n]} - \frac{m}{\sqrt{2}}B_{mn}$$

and

$$\hat{\mathcal{D}}_m\phi = \mathcal{D}_m\phi - \frac{\kappa}{2}(\psi^1_m\chi + \bar{\psi}^1_m\bar{\chi})$$

Supercovariant!

$$\hat{\mathcal{F}}^A_{mn} = \mathcal{F}^A_{mn} + \frac{\kappa}{\sqrt{2}}(\psi^2_{[m}\psi^1_{n]} + \bar{\psi}^2_{[m}\bar{\psi}^1_{n]})$$

$$\hat{\mathcal{F}}^B_{mn} = \mathcal{F}^B_{mn} - \frac{\kappa}{2}(\bar{\lambda}\bar{\sigma}_{[n}\psi^1_{m]} + \bar{\psi}^1_{[m}\bar{\sigma}_{n]}\lambda)$$

$$\hat{v}_m = v_m + (\mathrm{i}\kappa\psi^1_n\sigma_m{}^n\chi \;+\; h.c.) + (-\frac{\mathrm{i}\kappa}{2}\epsilon_m{}^{nrs}\psi^1_n\sigma_r\bar{\psi}^2_s \;+\; h.c.)$$

These Lagrangians have N=2 supersymmetry

- N=1 realized linearly – not broken
- N=2 realized nonlinearly – broken!

From the transformations we see

- $\zeta = \frac{1}{\sqrt{3}}\chi + i\sqrt{2}\lambda$ does not shift
- $\nu = \frac{1}{\sqrt{3}}\lambda - i\sqrt{2}\chi$ does!

It is the Goldstone fermion for N=2 supersymmetry, spontaneously broken to N=1.

In the chiral case,

$$\left[\delta_{\eta^2}, \delta_{\eta^1}\right] \phi \;=\; 2v^2\, \eta^2 \eta^1$$

$$\left[\delta_{\eta^2}, \delta_{\eta^1}\right] \mathcal{A}_m \;=\; \frac{2}{\kappa}\, \partial_m \eta^2 \eta^1$$

The complex scalar ϕ is the Goldstone boson for a gauged central charge.

Moreover, in the unitary gauge,

$$\phi \;=\; \nu \;=\; 0$$

The Lagrangian reduces to the usual representation for the massive N=1 spin-3/2 multiplet.

In the vector case,

$$\left[\delta_{\eta^2}, \delta_{\eta^1}\right] B_m \; = 2\mathrm{i}v^2 \left(\eta^2 \sigma_m \bar{\eta}^1 - \eta^1 \sigma_m \bar{\eta}^2\right)$$

$$\left[\delta_{\eta^2}, \delta_{\eta^1}\right] B_{mn} \; = \frac{2\mathrm{i}}{\kappa} \partial_{[m}(\eta^2 \sigma_{n]} \bar{\eta}^1 - \eta^1 \sigma_{n]} \bar{\eta}^2)$$

The real vector B_m is the Goldstone boson for a gauged vectorial central extension of the N=2 algebra.

In addition, the real scalar ϕ is the Goldstone boson for a gauged central charge.

In the unitary gauge, with

$$\phi \; = \; B_m \; = \; \nu \; = \; 0$$

The Lagrangian reduces to the new representation for the massive N=1 spin-3/2 multiplet.

In each case, the <u>second supercurrent</u> is

$$J^2_{m\alpha} \;=\; v^2\,(\sqrt{6}i\,\sigma_{\alpha\dot\alpha m}\bar\nu^{\dot\alpha} + 4\,\sigma_{\alpha\beta mn}\psi^{2n\beta})$$

Computing, one finds

$$\{\,\bar Q_{\dot\alpha},\,J^1_{m\alpha}\,\} \;=\; 2\,\sigma^n_{\alpha\dot\alpha}\,T_{mn}$$

$$\{\,\bar S_{\dot\alpha},\,J^2_{m\alpha}\,\} \;=\; 2\,\sigma^n_{\alpha\dot\alpha}\,T_{mn}$$

Because of the <u>second gravitino</u>,

$$\langle 0|\,(\bar S S + S\bar S)\,|0\rangle \;=\; 0$$

even though

$$S\,|0\rangle \;\neq\; 0 \qquad\qquad \bar S\,|0\rangle \;\neq\; 0$$

The <u>supergravity couplings</u> exploit the <u>second loophole</u> to the <u>no-go theorem</u>!

Note:

The chiral case is a truncation of the supergravity coupling found by Ferrara, Girardello and Porrati and by Zinov'ev.

(Their results were based on linear N=2 supersymmetry, They involved N=2 vector and hyper multiplets.)

The vector case is completely new. It contains a new realization of N=2 supergravity.

The couplings presented here are minimal and model-independent.

They describe the superHiggs effect in the low-energy effective theories that result from partial supersymmetry breaking....

Partial Breaking in AdS Space:

OSP(2,4) algebra

$$\{Q^i_\alpha, \bar{Q}_{j\dot{\beta}}\} = 2\sigma^a_{\alpha\dot{\beta}} R_a \delta^i_j$$

$$\{Q^i_\alpha, Q^{\beta j}\} = 2\mathrm{i}\Lambda \sigma^{ab}{}_\alpha{}^\beta M_{ab}\delta^{ij} + 2\mathrm{i}\delta_\alpha{}^\beta \epsilon^{ij} T$$

$$\left[M_{ab}, Q^i\right] = -\mathrm{i}\sigma_{ab} Q^i$$

$$\left[R_a, Q^i\right] = \frac{1}{2}\Lambda \sigma_a \bar{Q}_i$$

$$\left[T^{ij}, Q^k\right] = \mathrm{i}\Lambda(\delta^{jk} Q^i - \delta^{ik} Q^j)$$

As $\Lambda \to 0$

$$R_m \to P_m$$

$T \to$ one real central charge....

How does partial breaking work in AdS space?

To construct the coupling, follow the same procedure….

Start with the massive N=1 spin-3/2 multiplet in AdS space.

$$D(E+\tfrac{1}{2},\tfrac{3}{2}) \oplus D(E,1) \oplus D(E+1,1) \oplus D(E+\tfrac{1}{2},\tfrac{1}{2})$$

where $E > 2$ and

$D(E,s)$ denotes the eigenvalues under $U(1) \times SU(2) \subset SO(3,2)$

Lagrangian

$$
\begin{aligned}
e^{-1}\mathcal{L} &= e^{-1}\epsilon^{mnrs}\overline{\psi}_m\overline{\sigma}_n\nabla_r\psi_s - \mathrm{i}\overline{\zeta}\overline{\sigma}^m\nabla_m\zeta - \frac{1}{4}A_{mn}A^{mn} - \frac{1}{4}B_{mn}B^{mn} \\
&\quad - \frac{1}{2}(m^2 - m\Lambda)A_mA^m - \frac{1}{2}(m^2 + m\Lambda)B_mB^m \\
&\quad + \frac{1}{2}m\,\zeta\zeta \ + \ h.c. \\
&\quad - m\,\psi_m\sigma^{mn}\psi_n \ + \ h.c.
\end{aligned}
$$

$$\Lambda \neq 0$$

Transformations

$$
\begin{aligned}
\delta_\eta A_m &= \sqrt{1+\epsilon}(\psi_m\eta + \bar{\psi}_m\bar{\eta}) \\
&\quad + \frac{1}{\sqrt{1-\epsilon}}\left(\mathrm{i}\frac{1}{\sqrt{3}}(1-\epsilon)(\bar{\eta}\bar{\sigma}_m\zeta - \bar{\zeta}\bar{\sigma}_m\eta) - \frac{1}{\sqrt{3}m}\partial_m(\zeta\eta + \bar{\zeta}\bar{\eta})\right) \\
\delta_\eta B_m &= \sqrt{1-\epsilon}(-\mathrm{i}\psi_m\eta + \mathrm{i}\bar{\psi}_m\bar{\eta}) \\
&\quad + \frac{1}{\sqrt{1+\epsilon}}\left(-\frac{1}{\sqrt{3}}(1+\epsilon)(\bar{\eta}\bar{\sigma}_m\zeta + \bar{\zeta}\bar{\sigma}_m\eta) + \frac{\mathrm{i}}{\sqrt{3}m}\partial_m(\zeta\eta - \bar{\zeta}\bar{\eta})\right) \\
\delta_\eta\zeta &= \sqrt{1-\epsilon}\left(\frac{1}{\sqrt{3}}A_{mn}\sigma^{mn}\eta - \mathrm{i}\frac{m}{\sqrt{3}}\sigma^m\bar{\eta}A_m\right) \\
&\quad + \sqrt{1+\epsilon}\left(-\frac{\mathrm{i}}{\sqrt{3}}B_{mn}\sigma^{mn}\eta + \frac{m}{\sqrt{3}}\sigma^m\bar{\eta}B_m\right) \\
\delta_\eta\psi_m &= \frac{1}{\sqrt{1+\epsilon}}\left(\frac{1}{3m}\nabla_m(A_{rs}\sigma^{rs}\eta + 2\mathrm{i}m\sigma^n\bar{\eta}A_n) - \frac{\mathrm{i}}{2}(H^A_{+mn}\sigma^n + \frac{1}{3}H^A_{-mn}\sigma^n)\bar{\eta}\right. \\
&\quad \left. - \frac{2}{3}m(\sigma_m{}^n A_n\eta + A_m\eta) - \frac{\mathrm{i}}{2}\epsilon H^A_{+mn}\sigma^n\bar{\eta} - \epsilon m A_m\eta\right) \\
&\quad + \frac{1}{\sqrt{1-\epsilon}}\left(\frac{-\mathrm{i}}{3m}\nabla_m(B_{rs}\sigma^{rs}\eta - 2\mathrm{i}m\sigma^n\bar{\eta}B_n) + \frac{1}{2}(H^B_{+mn}\sigma^n + \frac{1}{3}H^B_{-mn}\sigma^n)\bar{\eta}\right. \\
&\quad \left. + \frac{2}{3}\mathrm{i}m(\sigma_m{}^n B_n\eta + B_m\eta) - \frac{1}{2}\epsilon H^B_{+mn}\sigma^n\bar{\eta} - \mathrm{i}\epsilon m B_m\eta\right)
\end{aligned}
$$

$$\epsilon \ := \ \Lambda/m \qquad\qquad m \ := \ (E-1)\Lambda$$

$$E > 2 \quad \Rightarrow \quad 0 \leq \epsilon \leq 1$$

To unhiggs, note that

$$D(E+\tfrac{1}{2},\tfrac{3}{2}) \oplus D(E,1) \oplus D(E+1,1) \oplus D(E+\tfrac{1}{2},\tfrac{1}{2})$$

$$\xrightarrow{E \to 2}$$

$$\underbrace{D(\tfrac{5}{2},\tfrac{3}{2}) \oplus D(2,1)}_{\text{massless spin-3/2}} \oplus \underbrace{D(3,1) \oplus D(\tfrac{5}{2},\tfrac{1}{2}) \oplus D(\tfrac{3}{2},\tfrac{1}{2}) \oplus D(3,0)}_{\text{massive spin-1}}$$

- The symmetry is $OSP(1,4) \times U(1)$
- The $U(1)$ must be broken because $E > 2$

Lagrangian

$$
\begin{aligned}
e^{-1}\mathcal{L} \; = & \\
& - \frac{1}{2\kappa^2}\mathcal{R} + \epsilon^{mnrs}\overline{\psi}_{im}\overline{\sigma}_n D_r \psi^i_s - \mathrm{i}\overline{\lambda}\overline{\sigma}^m D_m\lambda - \mathrm{i}\overline{\chi}\overline{\sigma}^m D_m \chi \\
& - \frac{1}{4}A_{mn}A^{mn} - \frac{1}{4}B_{mn}B^{mn} - \frac{1}{2}\mathcal{D}_m\phi_A\mathcal{D}^m\phi_A - \frac{1}{2}\mathcal{D}_m\phi_B\mathcal{D}^m\phi_B \\
& - \frac{1}{\sqrt{2}}m\sqrt{1-\epsilon^2}(\psi^2_m\sigma^m\overline{\lambda} \; + \; h.c.) - m\sqrt{1-\epsilon^2}(\mathrm{i}\psi^2_m\sigma^m\overline{\chi} \; + \; h.c.) \\
& - \sqrt{2}m(\mathrm{i}\lambda\chi \; + \; h.c.) - \frac{1}{2}m(\chi\chi \; + \; h.c.) \\
& - (m\psi^2_m\sigma^{mn}\psi^2_n \; + \; h.c.) + (\epsilon m\psi^1_m\sigma^{mn}\psi^1_n \; + \; h.c.) \\
& - \frac{\kappa}{4}\epsilon_{ij}\psi^i_m\psi^j_n(\sqrt{1+\epsilon}H^{mn}_{A-} - \mathrm{i}\sqrt{1-\epsilon}H^{mn}_{B-}) \; + \; h.c. \\
& - \frac{\kappa}{2}\chi\sigma^m\overline{\sigma}^n\psi^1_m(\mathcal{D}_n\phi_A - \mathrm{i}\mathcal{D}_n\phi_B) \; + \; h.c. \\
& - \frac{\kappa}{2\sqrt{2}}\overline{\lambda}\overline{\sigma}_m\psi^1_n(\sqrt{1-\epsilon}H^{mn}_{A+} - \mathrm{i}\sqrt{1+\epsilon}H^{mn}_{B+}) \; + \; h.c. \\
& - \frac{\kappa}{2}\epsilon^{mnrs}\sqrt{\frac{1-\epsilon}{1+\epsilon}}\overline{\psi}_{m2}\overline{\sigma}_n\psi^1_r(\partial_s\phi_A - \mathrm{i}\partial_s\phi_B) \; + \; h.c. \\
& + \frac{\kappa}{2}m\epsilon^{mnrs}\overline{\psi}_{m2}\overline{\sigma}_n\psi^1_r(\sqrt{1+\epsilon}A_s - \mathrm{i}\sqrt{1-\epsilon}B_s) \; + \; h.c. \\
& + 2\kappa\epsilon m\sqrt{\frac{1-\epsilon}{1+\epsilon}}\bar{\psi}_{m2}\bar{\sigma}^{mn}\bar{\psi}_{n1}\phi_A \; + \; h.c. \\
& - \frac{\kappa\epsilon m}{\sqrt{2}}\bar{\lambda}\bar{\sigma}^m\psi^1_m\phi_A \; + \; h.c. \\
& - \mathrm{i}\kappa\epsilon m\bar{\chi}\bar{\sigma}^m\psi^1_m\phi_A \; + \; h.c. \\
& + 3\frac{\epsilon^2 m^2}{\kappa^2}
\end{aligned}
$$

Transformations

$$
\begin{aligned}
\delta_\eta e_m^a &= \mathrm{i}\kappa\eta^i\sigma^a\overline{\psi}_{mi} + \mathrm{i}\kappa\bar{\eta}_i\bar{\sigma}^a\psi_m^i \\
\delta_\eta\psi_m^1 &= \frac{2}{\kappa}D_m\eta^1 + \mathrm{i}\frac{\epsilon m}{\kappa}\sigma_m\bar{\eta}^1 \\
\delta_\eta A_m &= \sqrt{1+\epsilon}\,\epsilon_{ij}(\psi_m^i\eta^j + \bar{\psi}_m^i\bar{\eta}^j) + \sqrt{1-\epsilon}\frac{1}{\sqrt{2}}(\bar{\eta}^1\bar{\sigma}_m\lambda + \bar{\lambda}\bar{\sigma}_m\eta^1) \\
\delta_\eta B_m &= \sqrt{1-\epsilon}\,\epsilon_{ij}(-\mathrm{i}\psi_m^i\eta^j + \mathrm{i}\bar{\psi}_m^i\bar{\eta}^j) + \sqrt{1+\epsilon}\frac{\mathrm{i}}{\sqrt{2}}(\bar{\eta}^1\bar{\sigma}_m\lambda - \bar{\lambda}\bar{\sigma}_m\eta^1) \\
\delta_\eta\lambda &= \mathrm{i}\sqrt{1-\epsilon}\frac{1}{\sqrt{2}}\hat{A}_{mn}\sigma^{mn}\eta^1 + \sqrt{1+\epsilon}\frac{1}{\sqrt{2}}\hat{B}_{mn}\sigma^{mn}\eta^1 \\
&\quad + \sqrt{2}\mathrm{i}\epsilon m\phi_A\eta^1 - \mathrm{i}\sqrt{2}\frac{m}{\kappa}\sqrt{1-\epsilon^2}\eta^2 \\
\delta_\eta\chi &= \mathrm{i}\sigma^n\bar{\eta}^1\hat{\mathcal{D}}_n\phi_A - \sigma^n\bar{\eta}^1\hat{\mathcal{D}}_n\phi_B - 2\epsilon m\phi_A\eta^1 \\
&\quad + 2\frac{m}{\kappa}\sqrt{1-\epsilon^2}\eta^2 \\
\delta_\eta\psi_m^2 &= \frac{2}{\kappa}D_m\eta^2 + \mathrm{i}\frac{m}{\kappa}\sigma_m\bar{\eta}^2 \\
&\quad - \frac{\mathrm{i}}{2}\sqrt{1+\epsilon}\hat{H}^A_{+mn}\sigma^n\bar{\eta}^1 - m\sqrt{1+\epsilon}A_m\eta^1 \\
&\quad + \frac{1}{2}\sqrt{1-\epsilon}\hat{H}^B_{+mn}\sigma^n\bar{\eta}^1 + \sqrt{\frac{1-\epsilon}{1+\epsilon}}(\partial_m\phi_A - \mathrm{i}\mathcal{D}_m\phi_B)\eta^1 \\
&\quad - \frac{\kappa}{2}\sqrt{\frac{1-\epsilon}{1+\epsilon}}\psi_m^1(\delta_{\eta^1}\phi_A - \mathrm{i}\delta_{\eta^1}\phi_B) \\
&\quad - \mathrm{i}\epsilon m\sqrt{\frac{1-\epsilon}{1+\epsilon}}\phi_A\sigma_m\bar{\eta}^1 \\
\delta_\eta\phi_A &= \chi\eta^1 + \bar{\chi}\bar{\eta}^1
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{D}_m\phi_A &= \partial_m\phi_A - m\sqrt{1-\epsilon}A_m \\
\mathcal{D}_m\phi_B &= \partial_m\phi_B - m\sqrt{1+\epsilon}B_m
\end{aligned}
$$

and

$$
\begin{aligned}
\hat{\mathcal{D}}_m\phi_A &= \partial_m\phi_A - m\sqrt{1-\epsilon}A_m - \frac{\kappa}{2}(\psi_m^1\chi + \bar{\psi}_m^1\bar{\chi}) \\
\hat{\mathcal{D}}_m\phi_B &= \partial_m\phi_B - m\sqrt{1+\epsilon}B_m + \mathrm{i}\frac{\kappa}{2}(\psi_m^1\chi - \bar{\psi}_m^1\bar{\chi}) \\
\hat{A}_{mn} &= A_{mn} + \frac{\kappa}{2}\sqrt{1+\epsilon}(\psi_{[m}^2\psi_{n]}^1 + \bar{\psi}_{[m}^2\bar{\psi}_{n]}^1) \\
&\quad -\sqrt{1-\epsilon}\frac{\kappa}{2\sqrt{2}}(\bar{\lambda}\bar{\sigma}_{[n}\psi_{m]}^1 + \bar{\psi}_{[m}^1\bar{\sigma}_{n]}\lambda) \\
\hat{B}_{mn} &= B_{mn} - \mathrm{i}\frac{\kappa}{2}\sqrt{1-\epsilon}(\psi_{[m}^2\psi_{n]}^1 - \bar{\psi}_{[m}^2\bar{\psi}_{n]}^1) \\
&\quad +\mathrm{i}\sqrt{1+\epsilon}\frac{\kappa}{2\sqrt{2}}(\bar{\lambda}\bar{\sigma}_{[n}\psi_{m]}^1 - \bar{\psi}_{[m}^1\bar{\sigma}_{n]}\lambda)
\end{aligned}
$$

The couplings depend on

$$\kappa, \quad v, \quad \Lambda$$

with

$$m = \sqrt{\Lambda^2 + \kappa^2 v^4}$$

$$\epsilon = \Lambda/m \qquad\qquad 0 \leq \epsilon \leq 1$$

This Lagrangian describes the spontaneous breaking of N=2 to N=1 in AdS space.

- It has supersymmetry and gauge symmetry
- In unitary gauge, it reduces to the massive case.

Limits:

- $\Lambda \longrightarrow 0,$ fixed $\kappa,\ v$ $\qquad \epsilon \longrightarrow 0$

  ➜ previous case, flat space

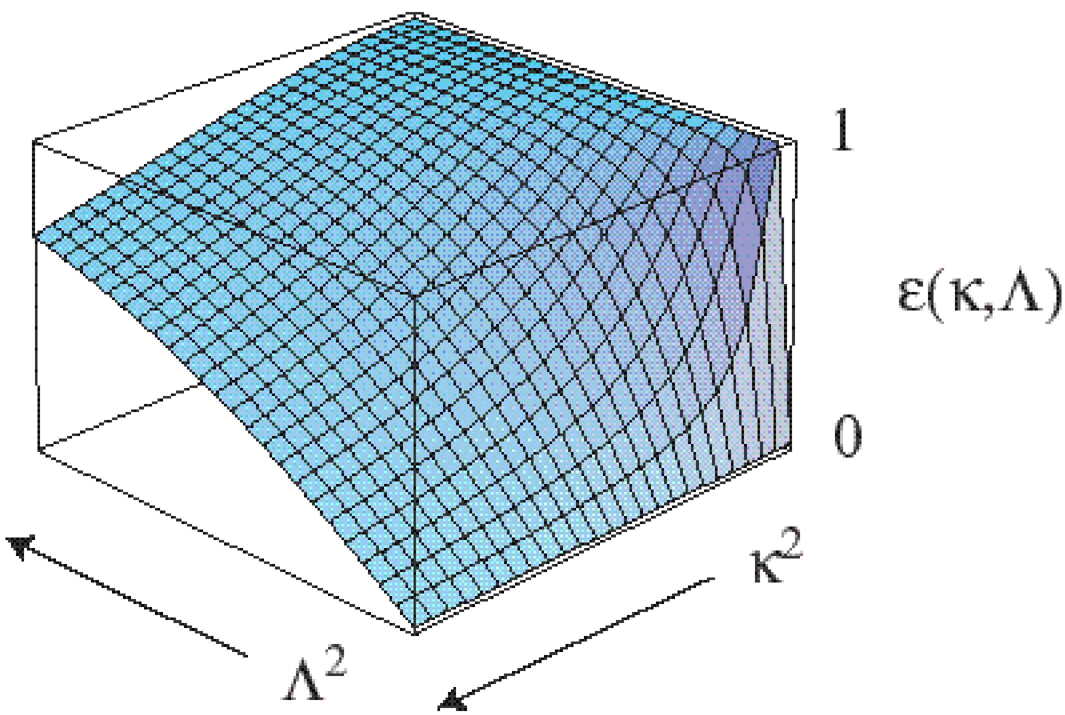
- $\kappa \longrightarrow 0,$ fixed $v,\ \Lambda$ $\qquad \epsilon \longrightarrow 1$

  ➜ partially broken N=2 supersymmetry in a fixed AdS background

Manifold of Supergravities

N=2 supergravity
in AdS space

N=2 supersymmetry
in AdS space



in flat space

Algebra?

$$\kappa \longrightarrow 0$$

AdS

$$\left[\delta_{\eta^2}, \delta_{\eta^1}\right] \phi_A \;=\; 2v^2(\eta^1\eta^2 + \bar{\eta}_1\bar{\eta}_2)$$

$$\left[\delta_{\eta^2}, \delta_{\eta^1}\right] A_m \;=\; 0$$

U(1)

$$\left[\delta_{\eta^2}, \delta_{\eta^1}\right] \phi_B \;=\; -2\mathrm{i}v^2(\eta^1\eta^2 - \bar{\eta}_1\bar{\eta}_2)$$

$$\left[\delta_{\eta^2}, \delta_{\eta^1}\right] B_m \;=\; -\sqrt{2}\mathrm{i}v^2\frac{\partial_m}{m}(\eta^1\eta^2 - \bar{\eta}_1\bar{\eta}_2)$$

When $v, \;\Lambda \neq 0$ the symmetry algebra is

$$\boxed{OSP(2,4) \rtimes U(1)}$$

semi-direct product!

NONLINEARLY REALIZED!

This construction evades

Coleman-Mandula
Haag, Sohnius, Lupusanski

because the broken changes don't exist!

The

$$OSP(2,4) \times U(1)$$

symmetry only exists at the level of the current algebra!

$U(1)$ always broken!

Summary: In this talk we have

- Studied partial breaking in flat space and found a new representation for the N=1 massive spin-3/2 multiplet.

- Unhigged this representation and found a new N=2 supergravity and a new N=2 supersymmetry algebra.

- Discovered the partial breaking in AdS space can also give rise to a new N=2 algebra....

Brane interpretation?