

# Finite Temperature Spin Correlations in Quantum Magnets with a Spin Gap

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## Quantum Magnets at $T=0$

From coherent singlet to paramagnet

- Large gap : Coupled spin-1/2 dimers
- Small gap : Haldane spin-1 chain

Conclusions



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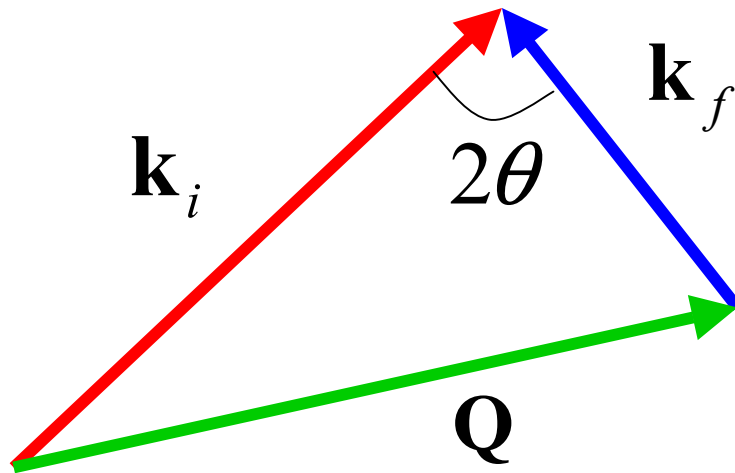
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# Magnetic Neutron Scattering



$$\mathbf{Q} = \mathbf{k}_i - \mathbf{k}_f$$

$$\hbar\omega = E_i - E_f$$

$$S^{\alpha\beta}(\mathbf{Q}, \omega) = \frac{1}{2\pi\hbar} \int dt e^{-i\omega t} \frac{1}{N} \sum_{\mathbf{R}\mathbf{R}'} e^{i\mathbf{Q}\cdot(\mathbf{R}-\mathbf{R}')} \langle S_{\mathbf{R}}^{\alpha}(0) S_{\mathbf{R}'}^{\beta}(t) \rangle$$



**SPINS Cold neutron triple axis spectrometer at NCNR**

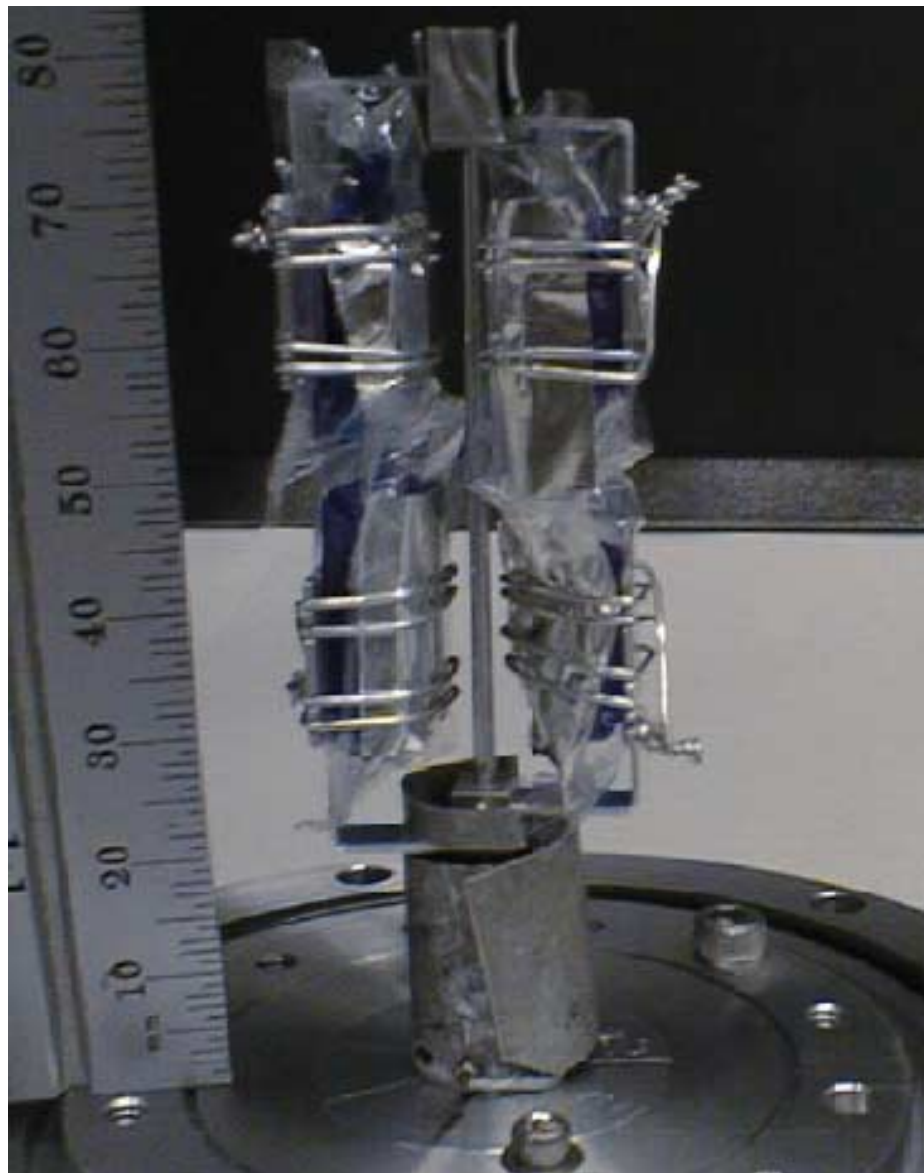
# Focusing analyzer system on SPINS



$\text{Y}_2\text{BaNiO}_5$   
Ito, Oka, and Takagi

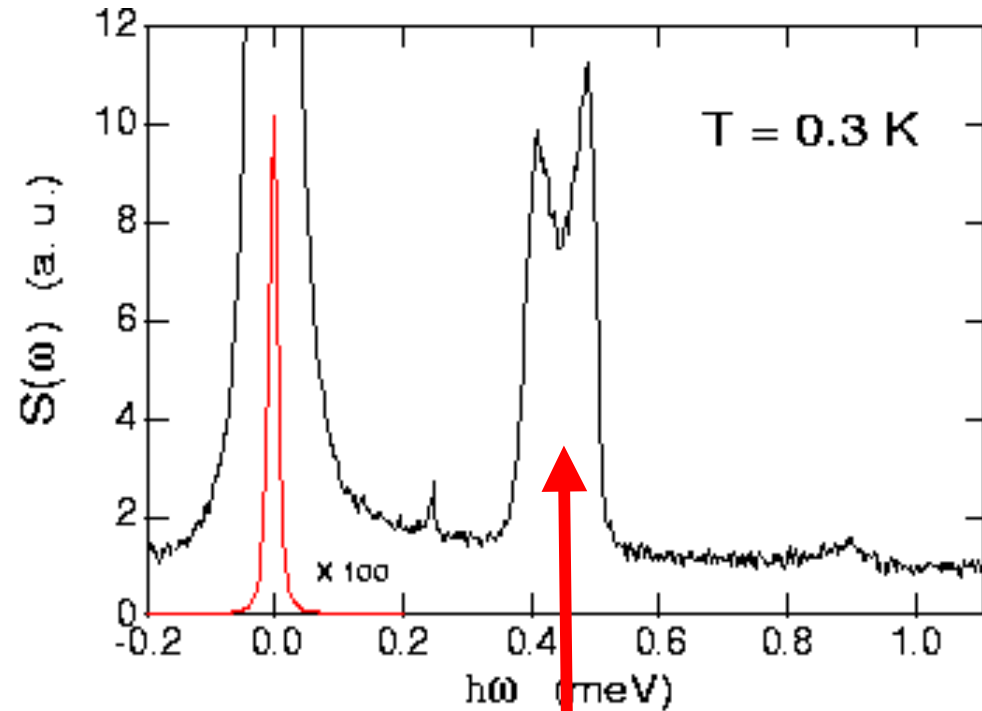
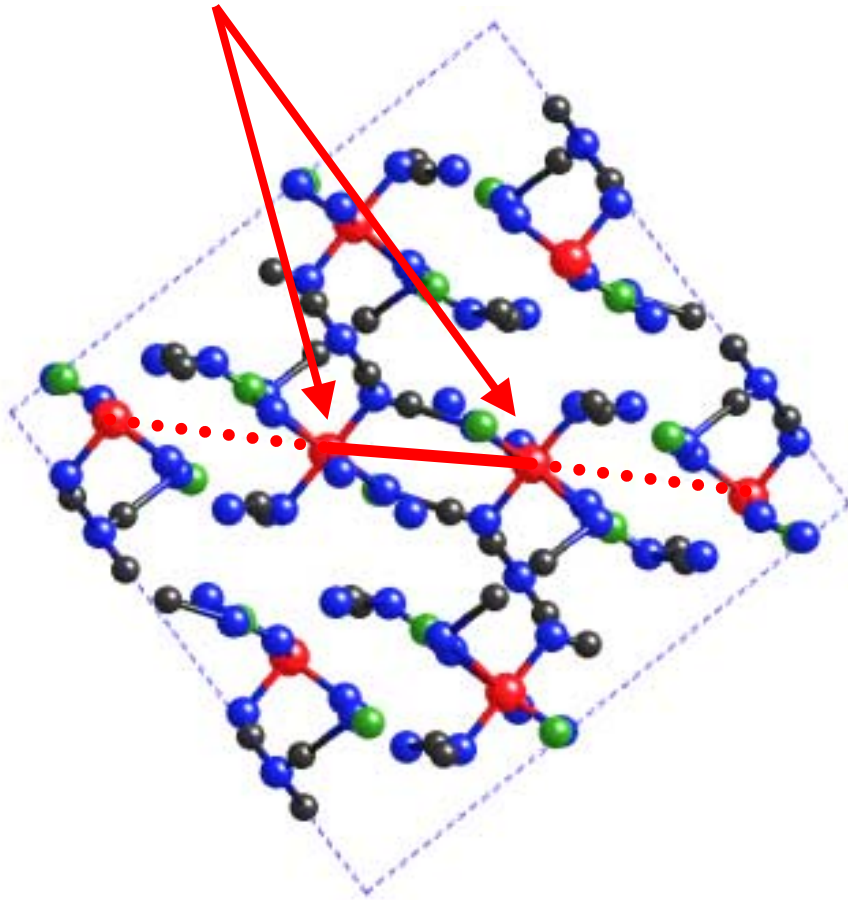


$\text{Cu}(\text{NO}_3)_2 \cdot 2.5 \text{D}_2\text{O}$   
Guangyong Xu



# Simple example of "Quantum" magnet

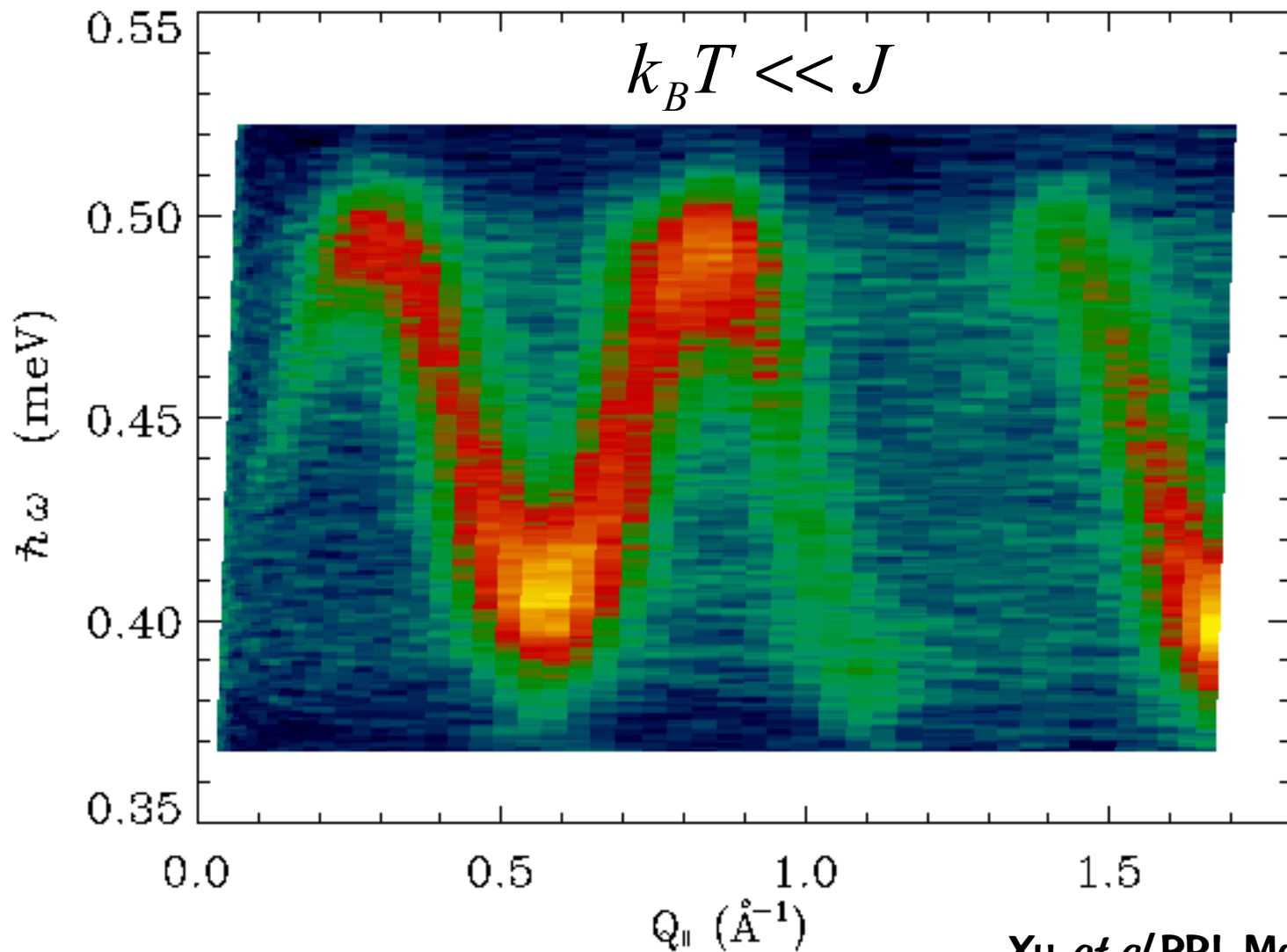
**Cu(NO<sub>3</sub>)<sub>2</sub>·2.5D<sub>2</sub>O** : dimerized spin-1/2 system



**Only Inelastic  
magnetic scattering**

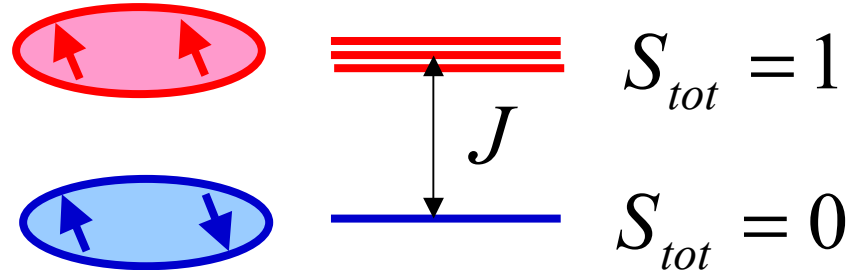
# Dispersion relation for triplet waves

Dimerized spin-1/2 system: copper nitrate

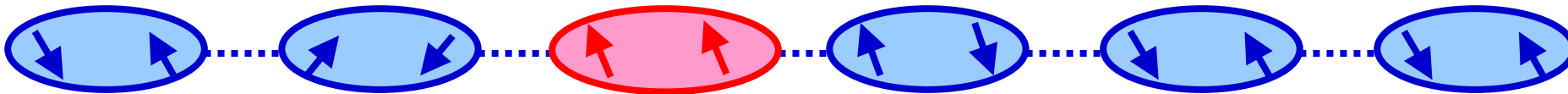


# Qualitative description of excited states

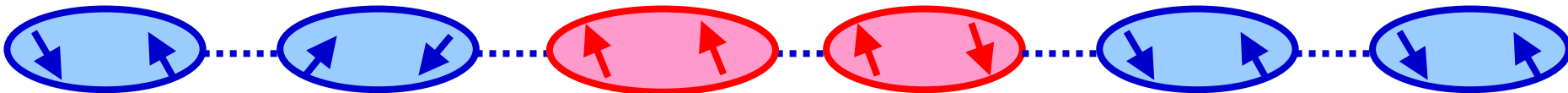
- A spin-1/2 pair with AFM exchange has a singlet - triplet gap:



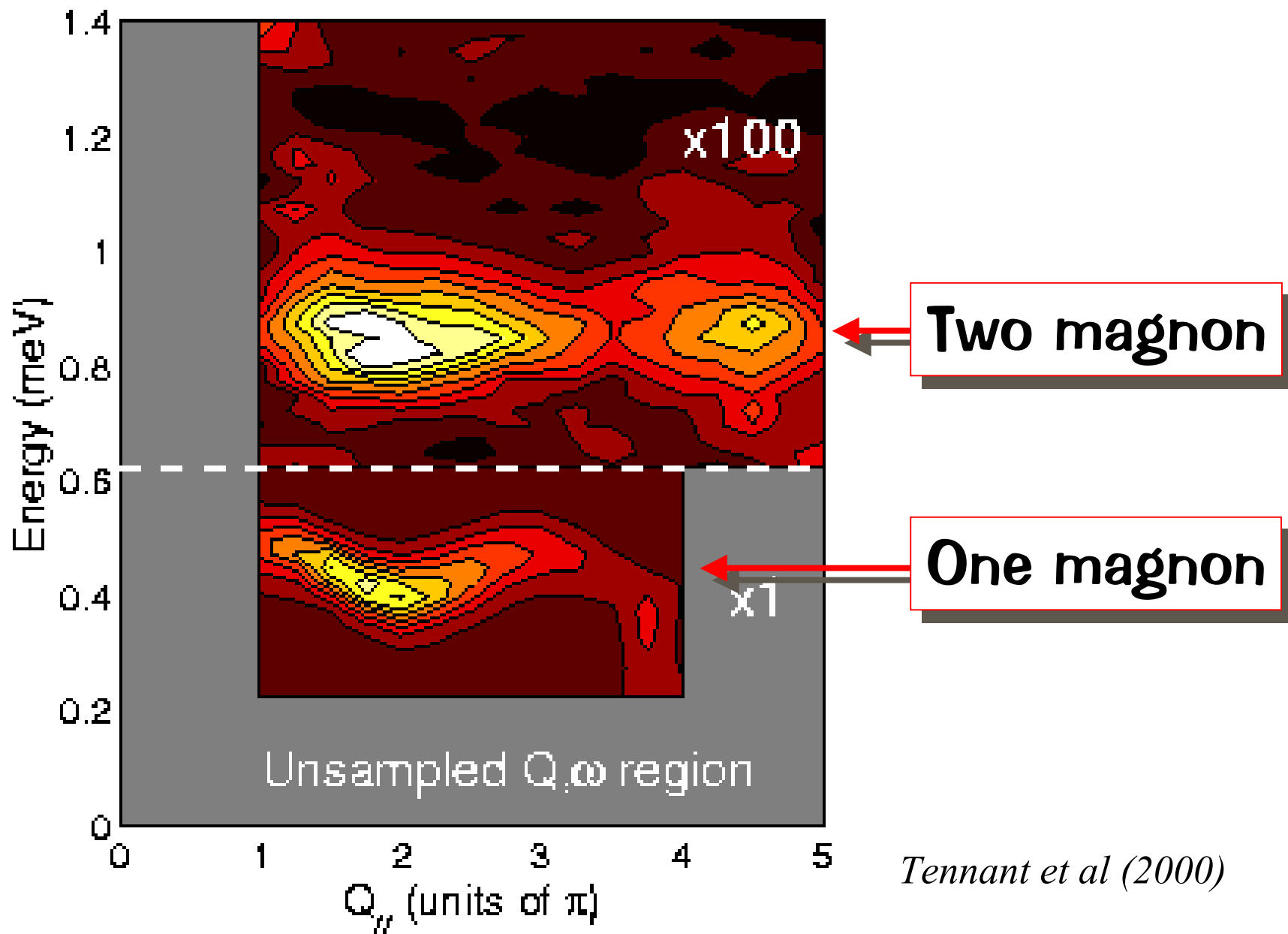
- Inter-dimer coupling allows coherent triplet propagation and produces well defined dispersion relation



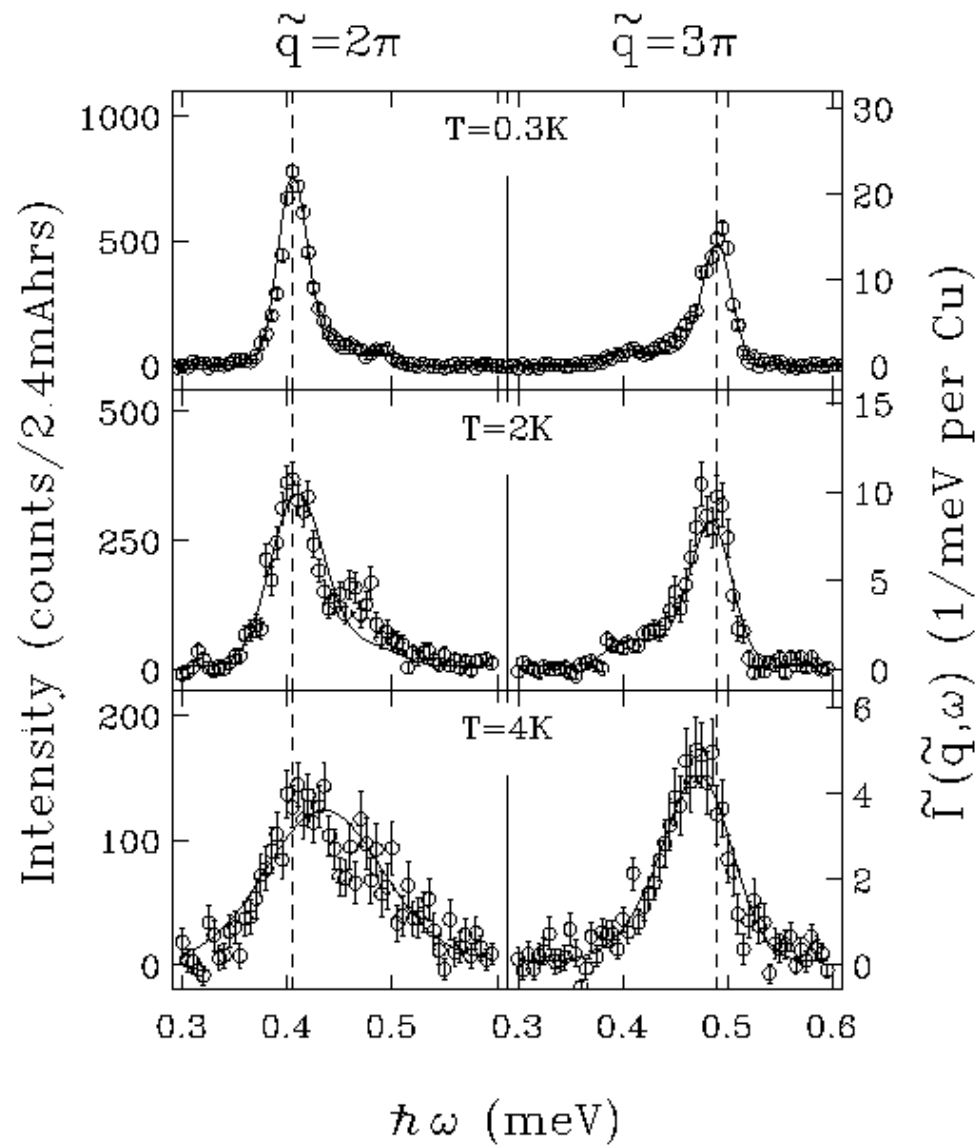
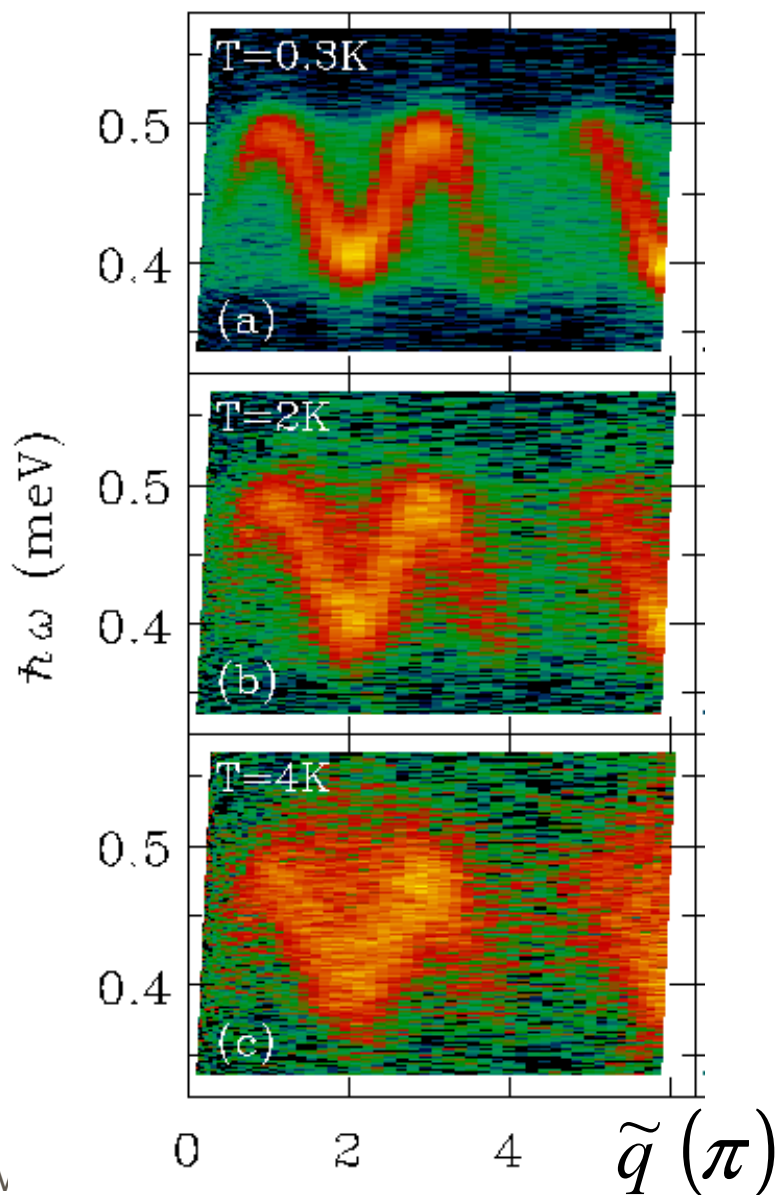
- Triplets can also be produced in pairs with total  $S_{tot} = 1$



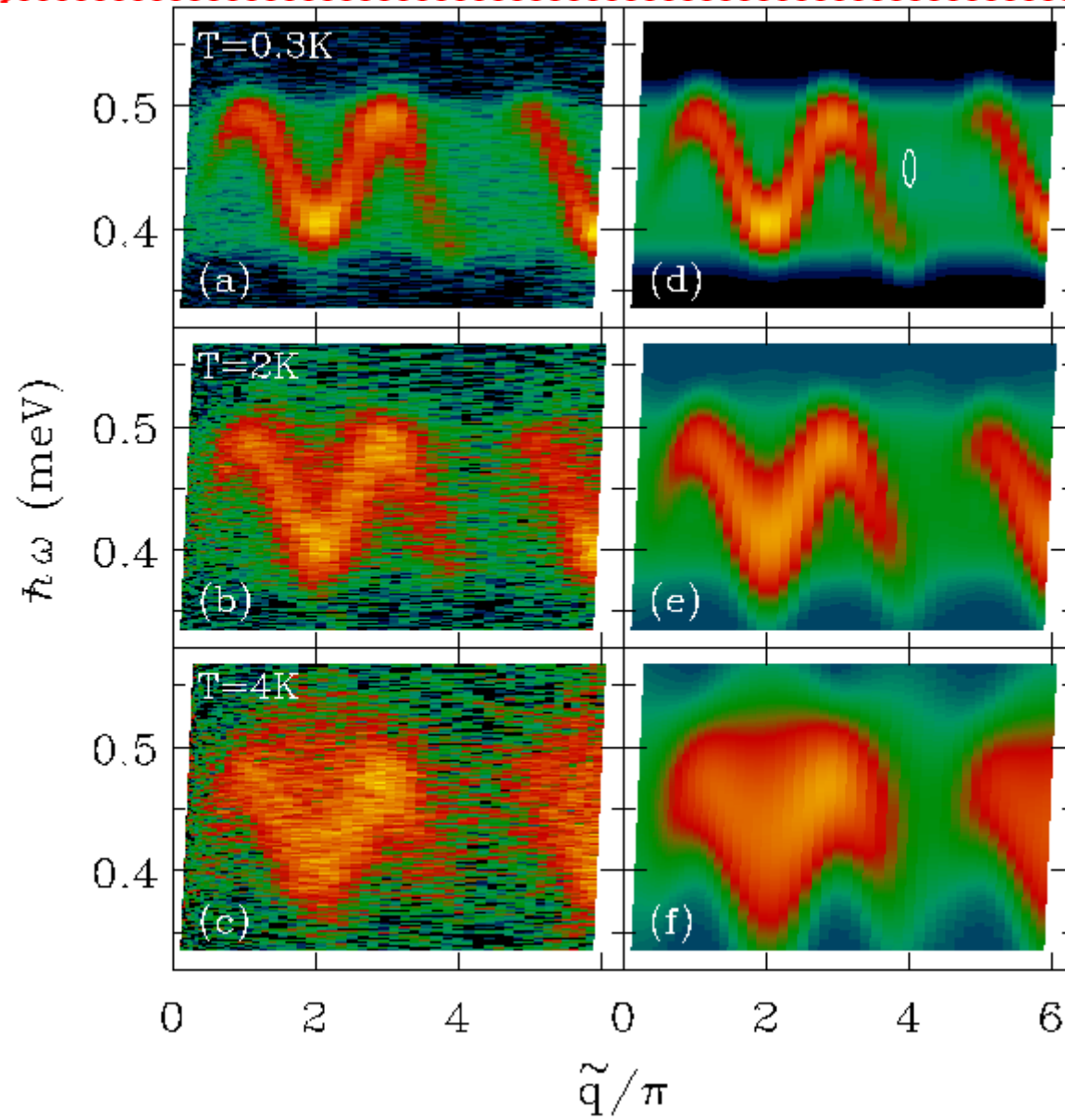
# Creating **two** triplets with **one** neutron



# Heating coupled dimers



# SMA fit to scattering data



**T-Parameters  
extracted from fit**

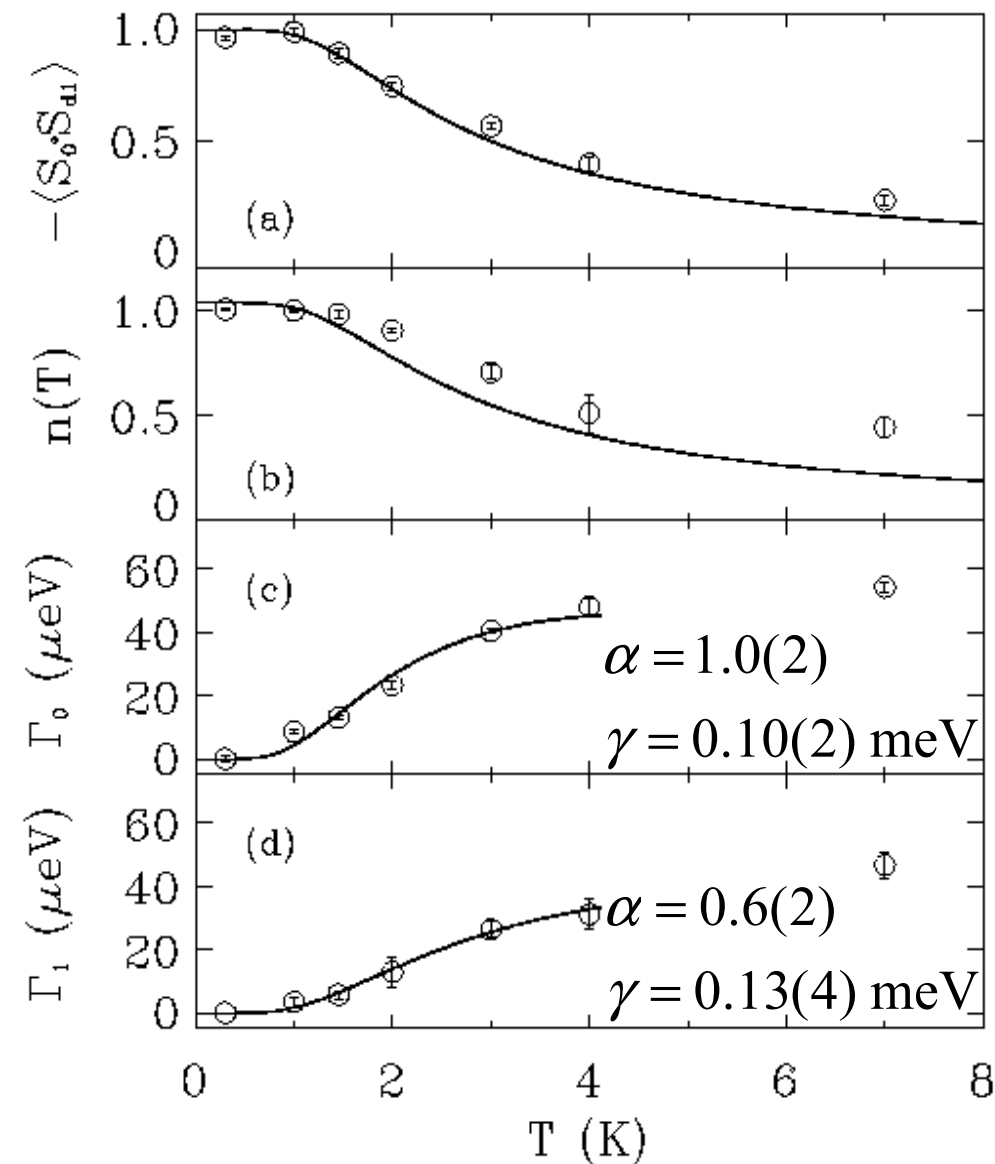
$$\langle S_0 S_d \rangle_T$$

$$\varepsilon(\tilde{q}) = J_1 + n(T) \frac{J_2}{2} \cos \tilde{q}$$

$$\Gamma(\tilde{q}) = \Gamma_0 + \frac{\Gamma_1}{2} \cos \tilde{q}$$

**More than 1000 data  
points per parameter!**

# T-dependence of singlet-triplet mode



$$-\langle \mathbf{S}_0 \cdot \mathbf{S}_{\bar{d}_0} \rangle_T = S(S+1)(\rho_{S=0} - \rho_{S=1})$$

$$n(T) = \rho_{S=0} - \rho_{S=1}$$

$$\Gamma(T) = \gamma \left( \frac{J_1}{k_B T} \right)^\alpha \exp(-J_1/k_B T)$$

# Types of Quantum magnets

- **Definition:** small or vanishing frozen moment at low T:

$$|\langle \mathbf{S} \rangle| \ll S \quad \text{for } k_B T \ll J$$

- **Conditions that yield quantum magnetism**

- Low effective dimensionality

- Low spin quantum number

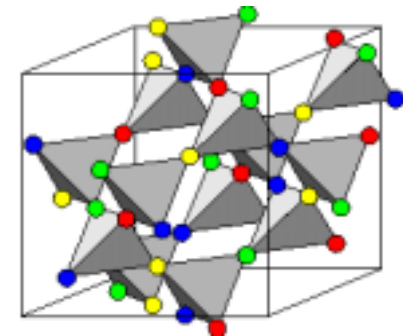
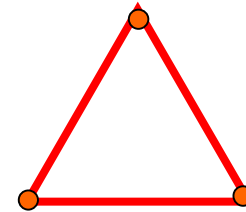
- geometrical frustration

- dimerization

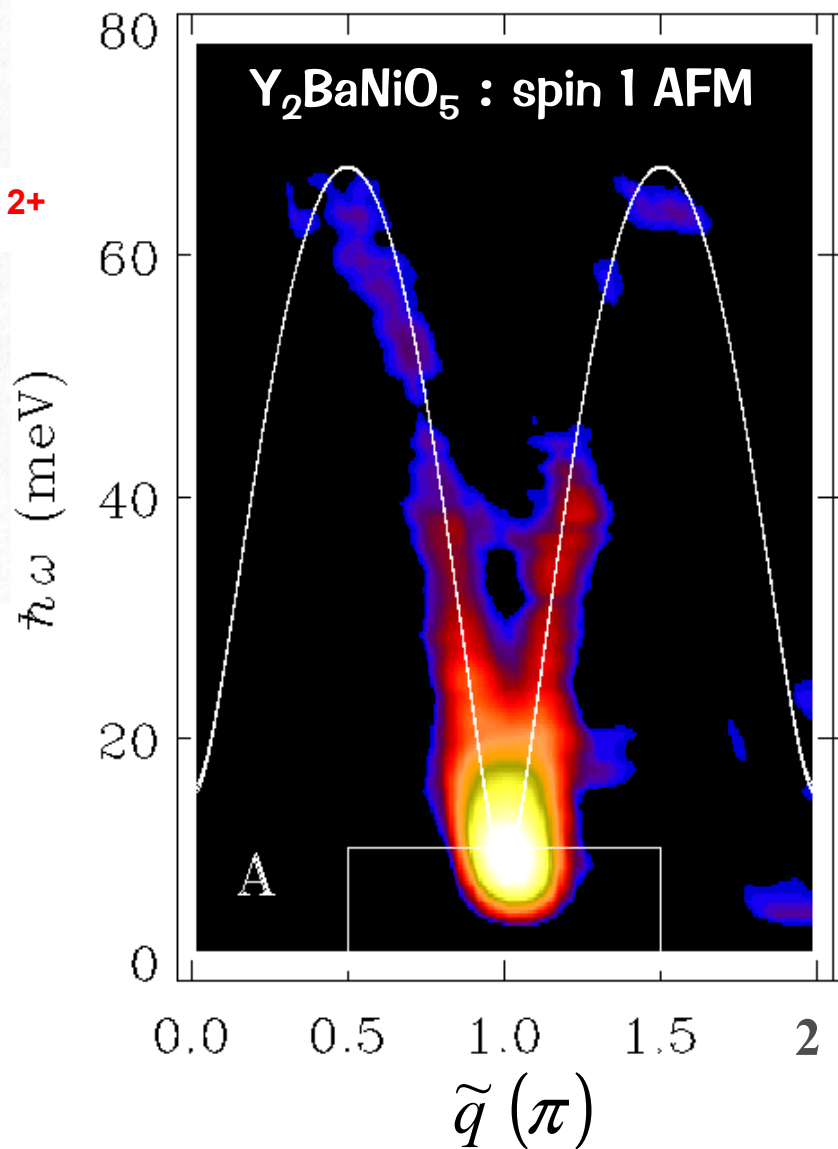
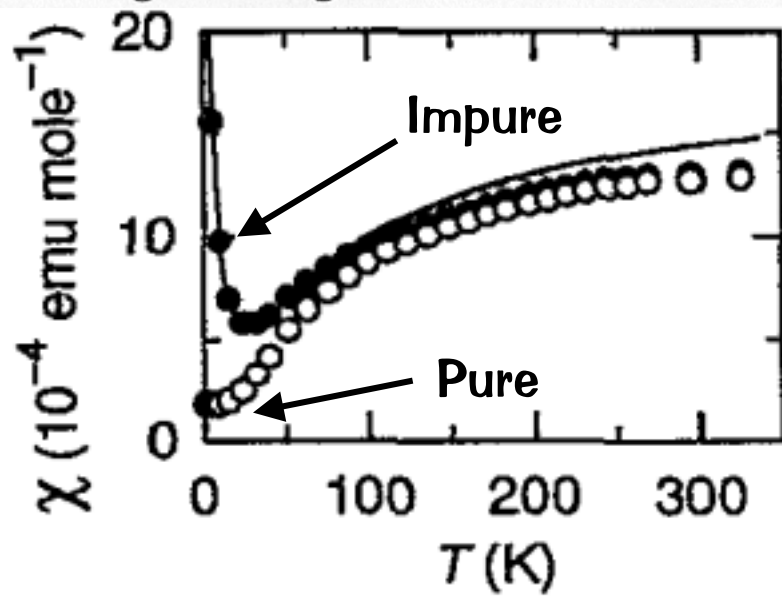
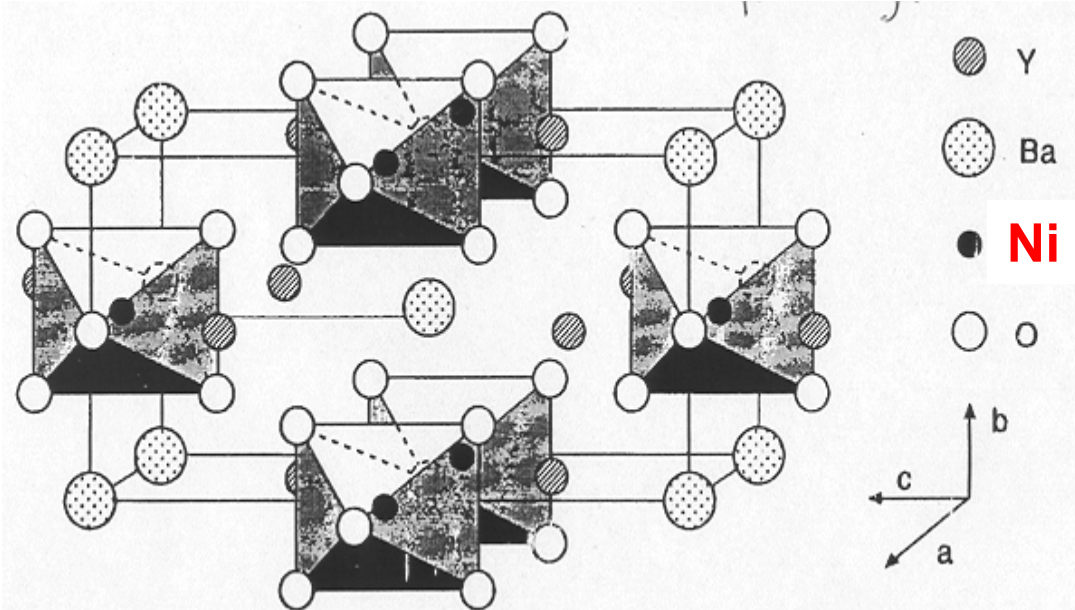
- weak connectivity

- interactions with fermions

- **Novel coherent states**

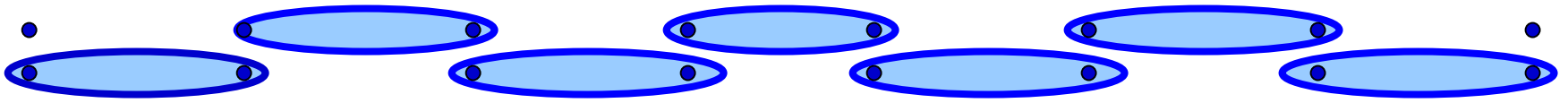


# One dimensional spin-1 antiferromagnet $\text{Y}_2\text{BaNiO}_5$



# Macroscopic singlet ground state of $S=1$ chain

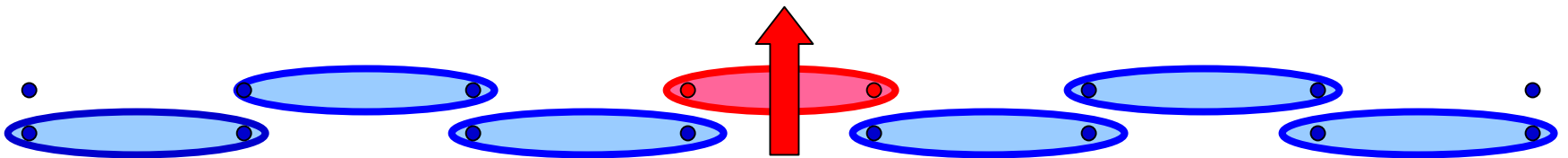
- Magnets with  $2S=nz$  have a nearest neighbor **singlet covering** with full lattice symmetry.



- This is exact ground state for spin projection Hamiltonian

$$\mathcal{H} = \sum_i P_i(S_{tot} = 2) = \sum_i \left( \mathbf{S}_i \cdot \mathbf{S}_{i+1} - \frac{1}{3} (\mathbf{S}_i \cdot \mathbf{S}_{i+1})^2 \right) \approx \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1}$$

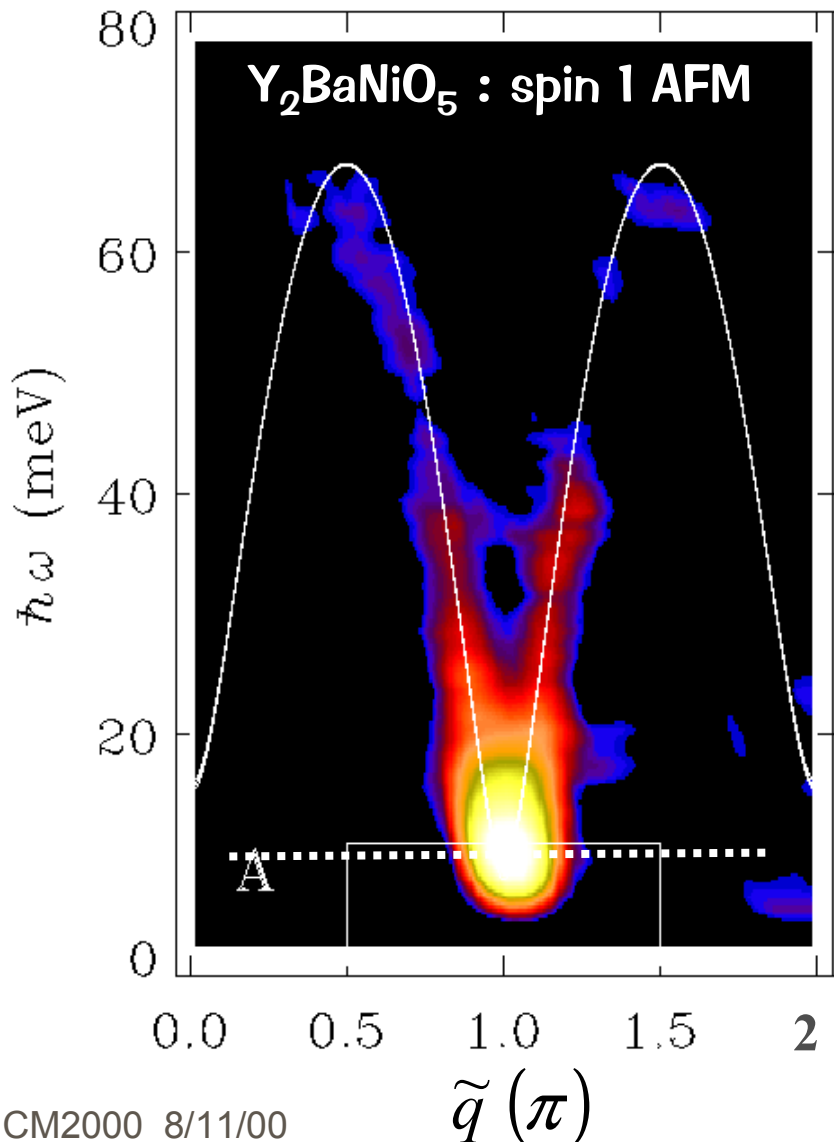
- Excited states are **propagating bond triplets** separated from the ground state by an energy gap  $\Delta \approx J$ .



Haldane PRL 1983

Affleck, Kennedy, Lieb, and Tasaki PRL 1987

# Two length scales in a quantum magnet



## Equal time correlation length

$$S(\tilde{q}) = \hbar \int S(\tilde{q}, \omega) d\omega$$

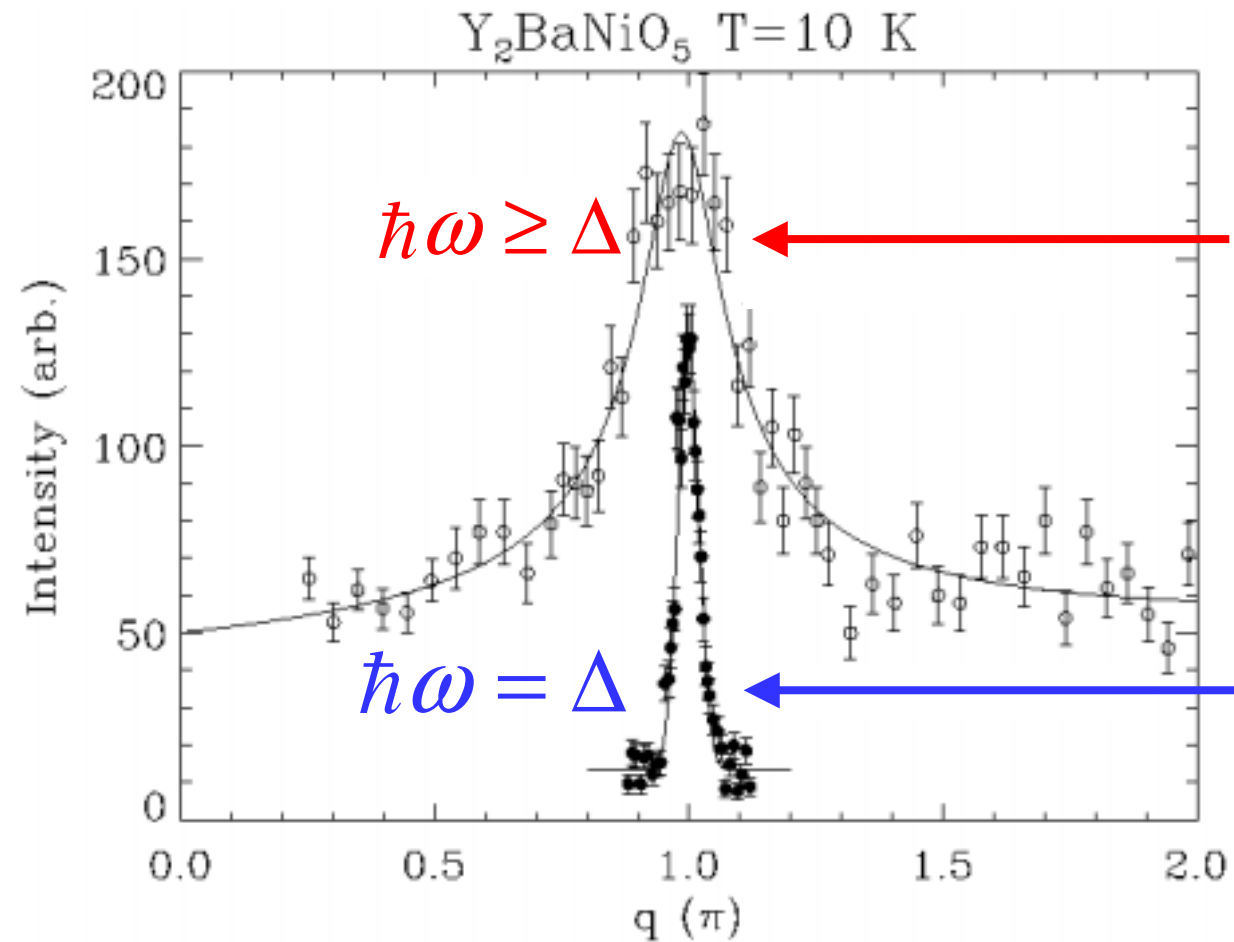
$$S(\tilde{q}) = \frac{1}{N} \sum_{ll'} \langle S_l S_{l'} \rangle \exp(i\tilde{q}(l - l'))$$

$$\langle S_0 S_l \rangle \propto \frac{1}{\sqrt{l}} \exp\left(-l/\xi\right)$$

## Triplet Coherence length :

length of coherent triplet wave packet

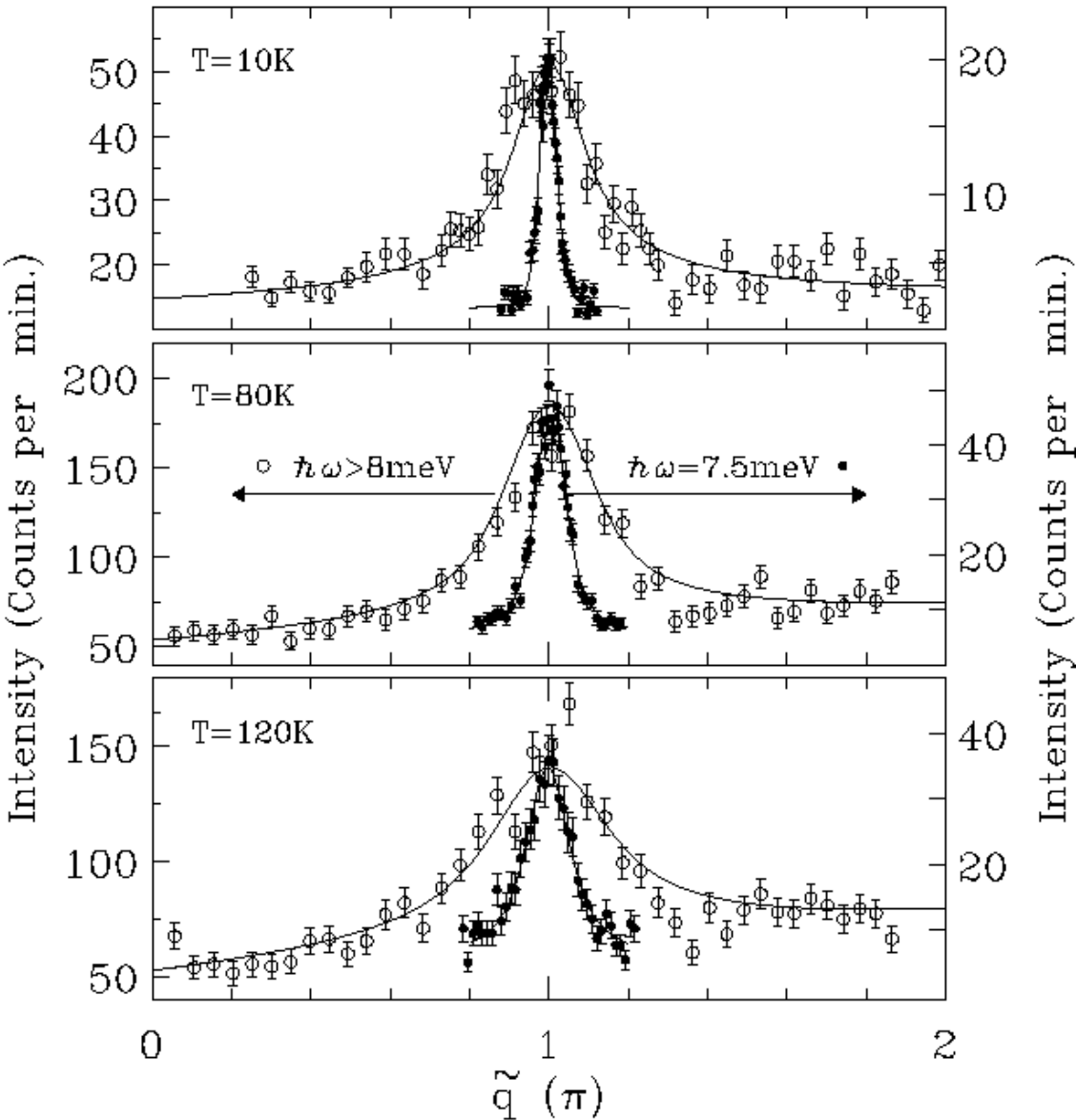
# Coherence in a fluctuating system



Short range G.S. spin correlations

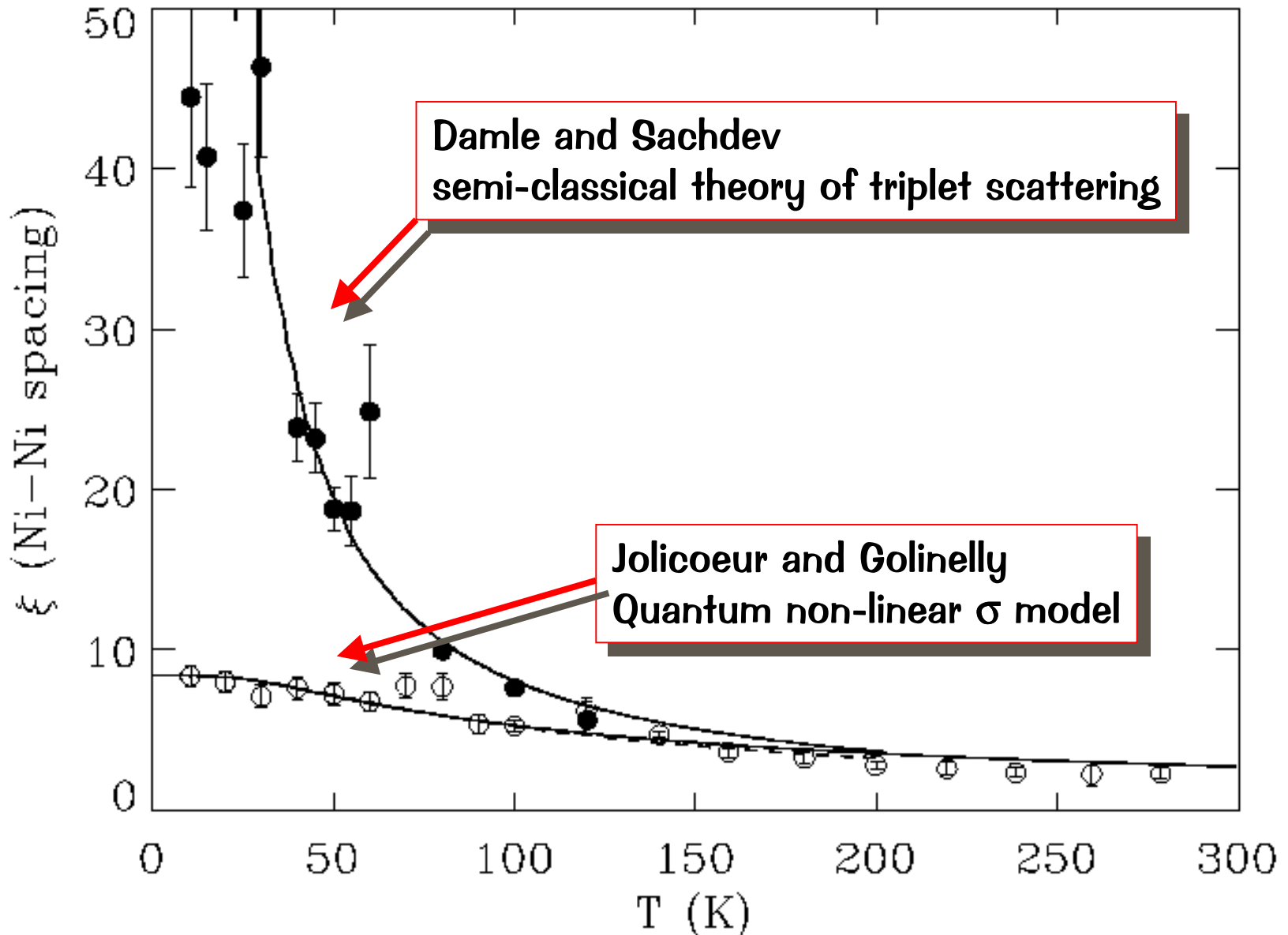
Coherent triplet propagation

# Mix in thermally excited triplets



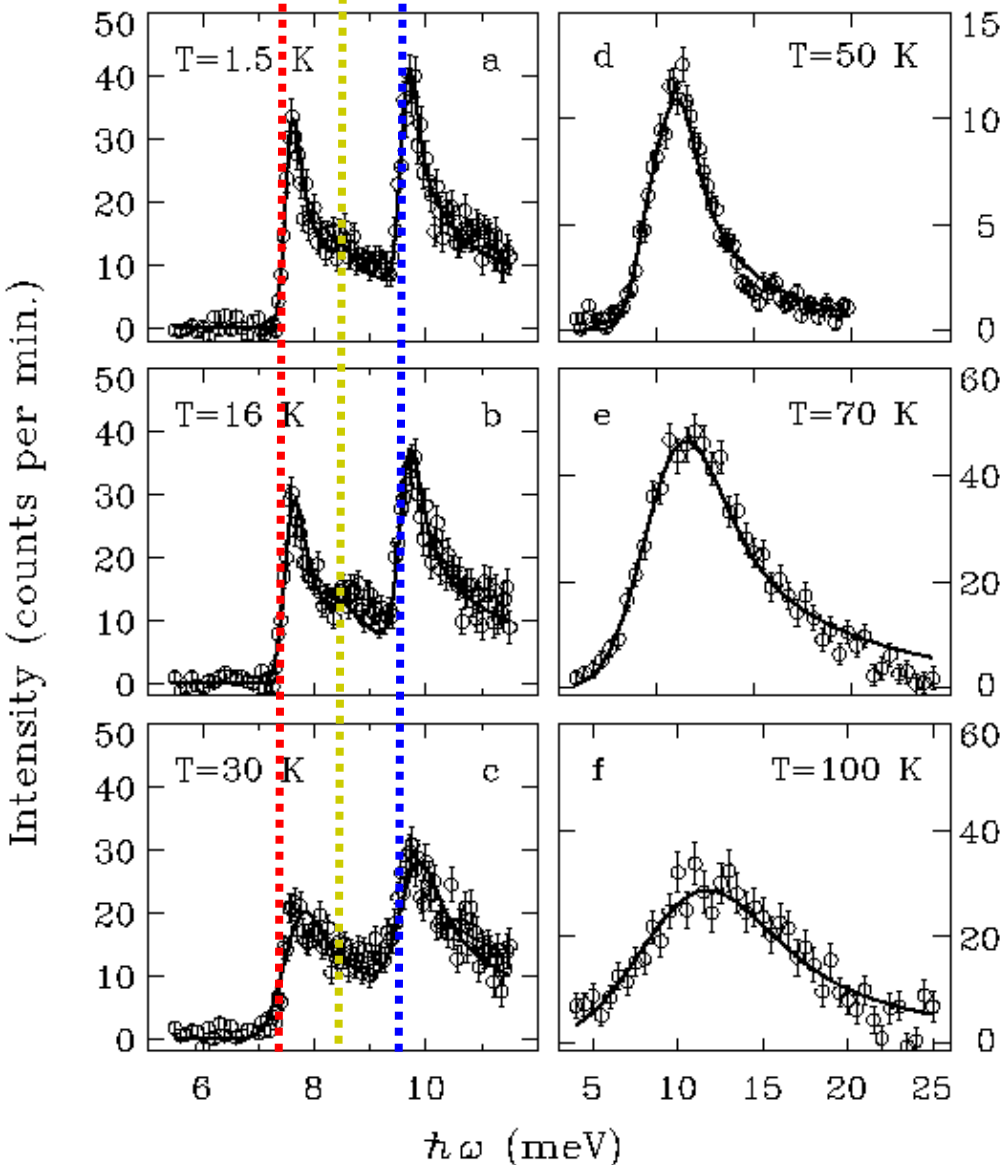
Coherence length  
approaches  
Correlation length  
for  $T \approx \frac{\Delta}{k_B}$

# Coherence and correlation lengths versus T



# $q=\pi$ Triplet creation spectrum versus T

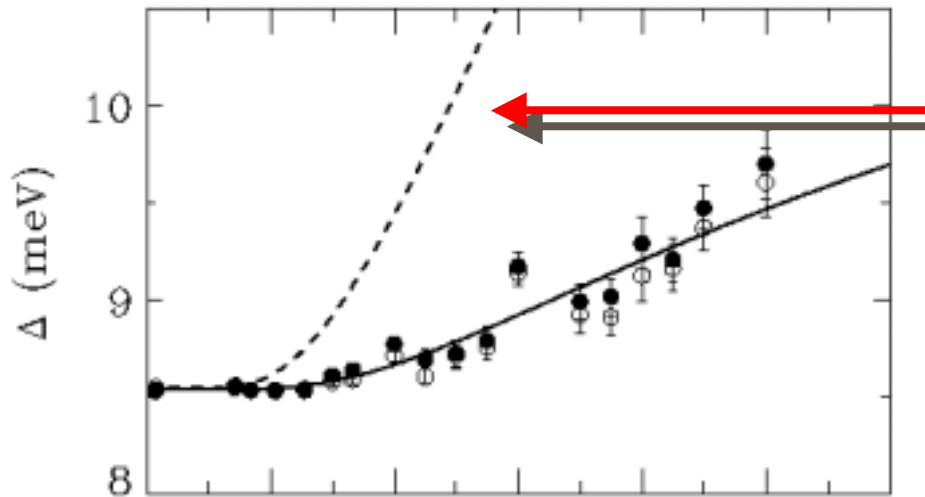
Anisotropy fine structure



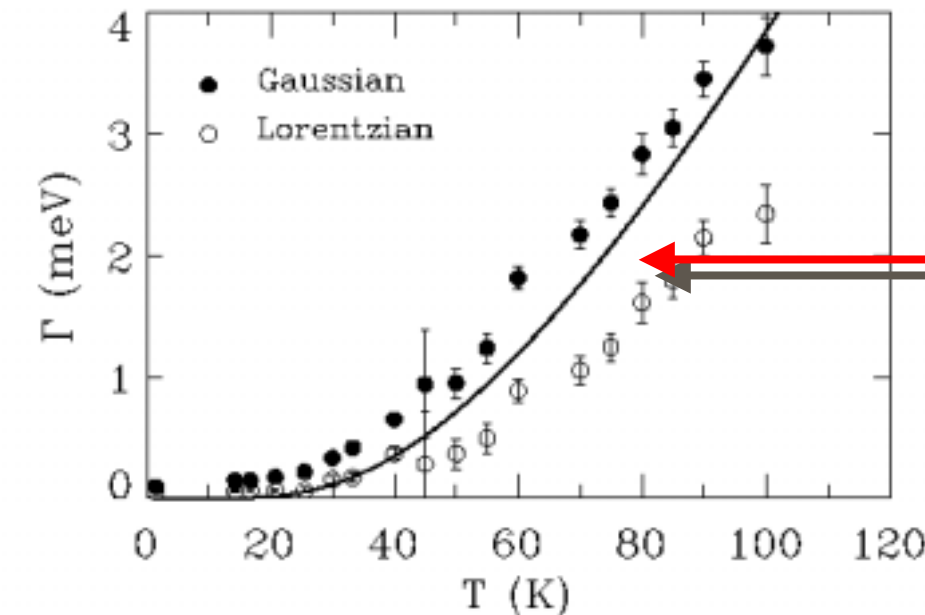
Triplet relaxes due to interaction with thermal triplet ensemble

There is slight "blue shift" with increasing T

# Resonance energy and relaxation rate versus T



**Jolicoeur and Golinelli**  
Quantum non-linear  $\sigma$  model



**Damle and Sachdev**

$$\Gamma = \alpha \rho_{S=1}(T) \sqrt{\langle v^2 \rangle_T}$$
$$= \alpha \frac{3k_B T}{\sqrt{\pi}} \exp\left(-\frac{\Delta}{k_B T}\right)$$

# Conclusions

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## ■ Strong coupling : Alternating spin chain

- Thermally activated triplet relaxation
- Wave-vector dependent relaxation
- Thermally activated band narrowing

## ■ Weak coupling : Haldane spin-1 chain

- Coherence length decreases with mean triplet spacing
- $\sigma$  model accounts for T-dependent equal- $t$  correlation length
- Triplet relaxation due to semi classical triplet scattering
- $\sigma$ -model over estimates thermally activated blue shift

## ■ Notable strong/weak coupling differences

- Different power-law pre-factor to T-dependent relaxation rate
- Theory not yet in place for strong coupling case