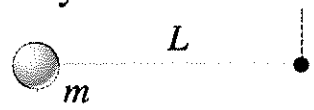
Classical Physics 171.105
Final Exam, December 17, 2002

The exam contains 5 short questions and 7 long questions. Each short question counts 5 points and you should be able to write down the answer with almost no calculation. Each long question counts 15 points. EXPLAIN YOUR REASONING! LOOK AT BOTH SIDES OF PAGES. Equations for translational and rotational dynamics and formulae for moments of inertia are given on the back pages. Calculators are not needed or allowed. Express all answers in terms of the variables defined in the problem and standard constants like g. You may use one 8.5x11" sheet of notes.

SHORT QUESTIONS

1. A ball of mass m is attached to a (massless) rod of length L that is free to pivot about the other end. At time t<0 the rod is horizontal. It is released with no initial velocity at t=0. What is the angular velocity about the pivot point when the mass is at the bottom of its circular trajectory?



2. A cube floats in water of density ρ_0 with $1/10^{th}$ of its volume out of the water. What is the average density of the cube?

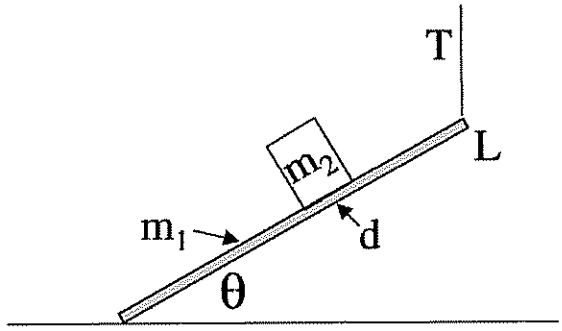
3. A harmonic oscillator has spring constant k and mass m. There is a very small amount of damping. An external oscillating force with variable frequency ω is applied to the mass. Describe – very qualitatively – how the amplitude of the response varies as ω changes from 0 to ∞.

4. A train of mass m moving with velocity v_1 along a straight track collides with another train of equal mass that is initially stationary. If the collision is perfectly inelastic, what is the difference between initial and final kinetic energies? What happened to this energy?

5. If no mechanical energy is lost in the above collision, what are the final velocities of the two trains. Be sure to explain why.

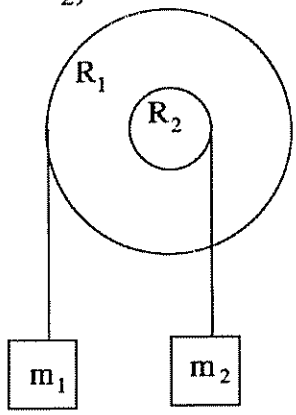
LONG QUESTIONS

6. A board of uniformly distributed mass m_1 and length L has one end resting on the ground and the other suspended from a vertical string. A weight of mass m_2 is placed a distance d away from the end on the ground. Call the tension in the string T and the normal and friction forces exerted by the ground N and f.
   a) Write the equations the assembly must satisfy for it to be in static equilibrium.
   b) What is the tension in the string?
   c) What are the values of N and f?

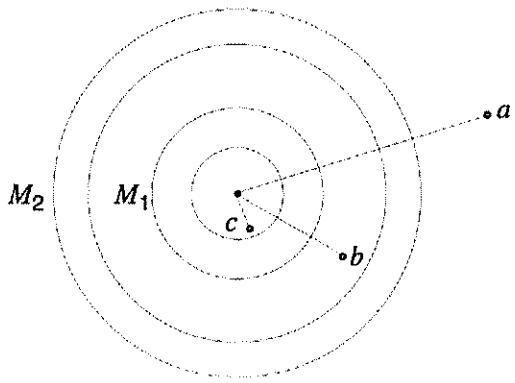


massless

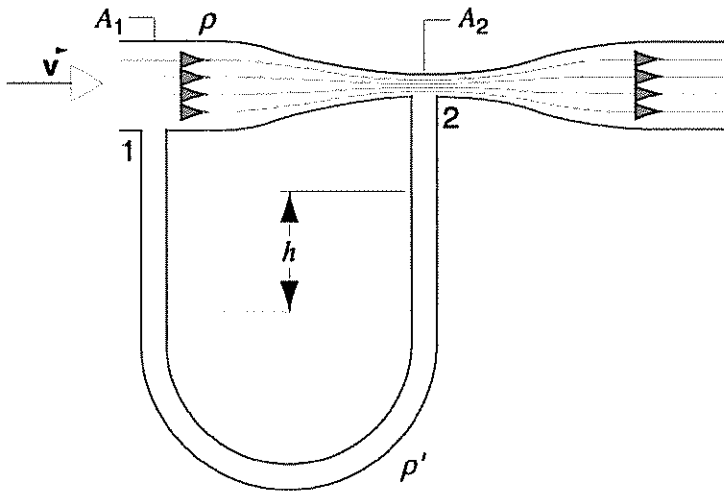
7. Two disks of radius R_1 and R_2 are attached so that they rotate together. Two masses are suspended from ropes wound around the rims of the wheels as shown.
   a) If $R_1=3R_2$, what is the ratio m_1/m_2 when the system is in static equilibrium?
   b) If the two masses are equal, find the angular acceleration α of the wheels and indicate the direction of rotation.
   c) What are the tensions, T_1 and T_2, in the two ropes? For part b?



8. Two concentric shells of uniform density having masses M_1 and M_2 are situated as shown below. Find the force on a particle of mass m when the particle is located either at
   a) r=a
   b) r=b
   c) r=c
   d) What is the escape velocity (minimum initial speed to escape from the masses) from point a?



9. A gas of constant density ρ flows through the horizontal tube in the diagram below. The cross-sectional area decreases from A_1 at the inlet to A_2 at point 2. In steady-state there is no flow of the gas or fluid in the U-shaped tube below. Ignore viscosity.
   a) What is the ratio of the velocity v_1 at the inlet to the velocity v_2 at point 2?
   b) What is the difference between the pressures p_1 and p_2 at points 1 and 2?
   c) What is the height difference h in the U-shaped tube if the fluid it contains has density ρ'.



10. A bicycle wheel of mass M is attached by its hub to a spring of force constant k. The other end is attached to a wall. Assume that all of the mass of the wheel is at its rim of radius R and the spring is massless. The wheel rolls without slipping on the floor and friction can be ignored. Motion is confined to the x-axis and the spring exerts no force when x=0. The hub is displaced to x=d and released at time t=0.

a) Show that the equation of motion for the center of mass of the wheel corresponds to simple harmonic motion and give an expression for the oscillation frequency ω?

b) Write the equation for the hub's position x as a function of time t.

c) When does the wheel first reach its maximum translational kinetic energy and what is the maximum value?

d) When does the wheel first reach its maximum rotational kinetic energy and what is the maximum value.

11. An organ pipe has length L and the air inside has sound velocity v. Use the fact that the pressure is zero at an open end of a tube and maximum at a closed end to answer the following two questions.

a) What are the lowest two resonant frequencies if both ends of the tube are open?

b) What are the lowest two resonant frequencies if a cap is placed to close off one end?

c) The wavelength of the lowest frequency sound that most people hear is 17m. What is the shortest length pipe you could use with this harmonic frequency? Would it be open or closed?

12. (22 pts) Two load speakers emit sinusoidal pressure waves of frequency ω and traveling at speed v in the $+x$ direction. One is located at the origin and one on the x-axis at $x=-L$. The two speakers are connected to the same source so the waves are emitted in phase.

a) Write the combined pressure wave along the x-axis for $x>0$. Assume that the pressure waves from each speaker have constant amplitude p_m for all x.

b) What is the problem with the assumption above if sound is emitted in all directions?

c) For what values of ω will the two waves completely cancel at $x>0$?

d) For what values of ω will there be perfect constructive interference?