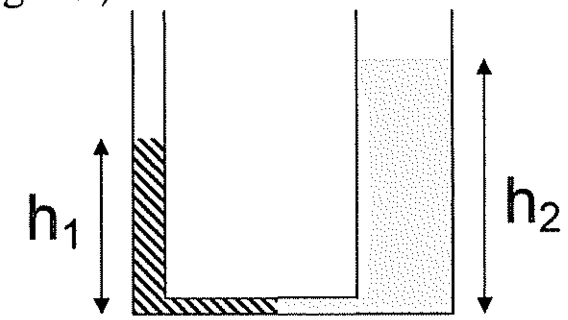
Classical Physics 171.105: Final Exam, December 14, 2004

EXPLAIN YOUR REASONING! LOOK AT BOTH SIDES OF PAGES. The number of points for each of the TWELVE questions is indicated. Equations for translational and rotational dynamics and formulae for moments of inertia are given on the back pages. Calculators are not needed or allowed. Express all answers in terms of the variables defined in the problem and standard constants like g. You may use one 8.5x11" sheet of notes.

1) (5 pts) The two arms of the tube below contain different fluids. What is the ratio of their densities? (The height of the bottom arm of the tube is negligible.)



2) (5 pts) An observer hears a siren on an ambulance as it comes directly at the observer and then moves away with the same speed v_s. Express the ratio of the frequency as it approaches, f_1, to the frequency, f_2, as it departs in terms of v_s and the sound velocity v_0.

3) (8 pts) The pressure of water in a fire hydrant is larger than atmospheric pressure by *p*. A fire hose connected to the hydrant emits a vertical spray from a point at the same height as the hydrant. Neglect viscosity in your answers.

a) What is the speed v_0 of the water just after it leaves the hose? Assume that the nozzle is much narrower than the hose.

b) How large must *p* be for the jet to reach the top of a building of height *H* above the hydrant?

4) (8 pts) A car of mass *M* needs to be lifted by a man who can exert a maximum force $F = Mg/3$. Describe three entirely different ways that this might be achieved using mechanical advantage. Be explicit about how a factor of 3 would be achieved.

5) (8 pts) A train of mass M moving at velocity v hits a stationary train of the same mass. After the collision the first train moves at v/10.

a) What is the velocity v_f of the train that had been stationary?

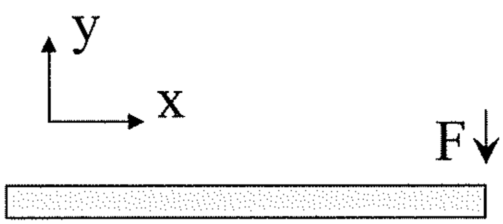
b) What is the change in kinetic energy in the collision?

c) Where did this energy go?

6) (8 pts) A stick of length L and mass M lies on frictionless ice. The diagram shows a view looking down on the ice from above. A force F is applied to the right end for a very short interval Δt. Ignore the rotation and translation during this time.

(a) What is the velocity of the center of mass of the stick after the force has ended?

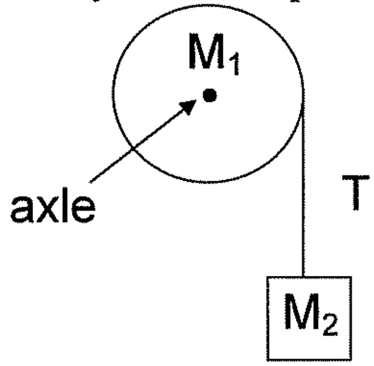
(b) What is the angular speed around the center of mass?



7) (10 pts) A solid sphere of uniform density, radius R and total mass M_1 rotates about a frictionless horizontal axle through its center. A string is wrapped around it at the widest point and attached to a mass M_2. (The string does not slip on the sphere).

a) When the mass is released from rest, what is the tension in the string?

b) After the mass has fallen a distance H below its initial height, what is the angular velocity ω of the sphere?



8) (12 pts) A cylindrical tube of height L is closed at the bottom. Air is blown over the top to make music. The sound velocity of the air is v. The pressure must vanish at the top of the tube and its derivative vanishes at the bottom. Sketch the variation of pressure with height or write an equation for pressure vs. height in answering the following questions. (Neglect the small change in static pressure due to gravity and end effects from the radius.)

a) What are the wavelength λ and frequency f of the lowest resonance of the tube?

b) What are λ and f for the next lowest resonance?

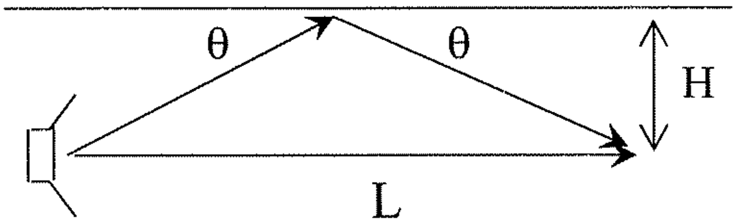
c) What are λ and f for the 3rd lowest resonance?

9) (12 pts) Sound of frequency f is generated by a loud speaker and travels with sound velocity v. The sound travels either directly to a listener a distance L away or bounces off the ceiling and then goes to the listener. There is no phase shift at the ceiling and the incoming and outgoing sound travel at the same angle θ relative to the ceiling. The ceiling is a height H above the listener.

a) Write an equation for the values of H and L where the wave bouncing off the ceiling arrives in phase with the wave that travels in a straight line. (Eliminate θ from your equation.)

b) Write an equation for the values of H and L where the wave bouncing off the ceiling is exactly out of phase from the wave that travels in a straight line.

c) Describe qualitatively whether complete destructive interference will occur and why or why not.

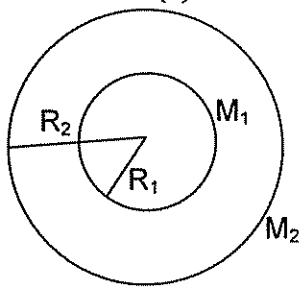


10)(15 pts) Two spherical shells have the same center. They have masses M_1 and M_2, and radii R_1 and R_2, respectively. Set the gravitational potential U to zero at radius $r = \infty$, and remember that U is related to the force by integration.

a) What are the magnitude of the force $|\vec{F}(r)|$ and the gravitational potential U(r) for a small satellite of mass m at a distance r from the center when $r > R_2 > R_1$?

b) What are $|\vec{F}(r)|$ and U(r) for $R_1 < r < R_2$?

c) What are $|\vec{F}(r)|$ and U(r) for $r < R_1$?



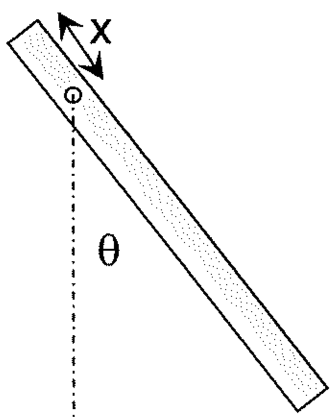
11) (16 pts) A stick of length L and mass M rotates around a hole drilled at a distance $x < L/2$ from the top end as shown in the figure below and to the left. Ignore the thickness of the stick and assume it has a uniform mass per unit length.

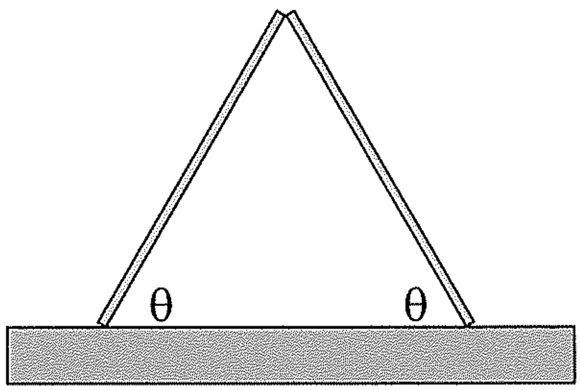
a) What is the moment of inertia I for rotation around the hole? Use the symbol I rather than the expression you find in answering the following parts.

b) Derive the equation for the angular acceleration α as a function of the angle θ. Simplify this equation by making the small angle assumption that $\theta \approx \sin\theta \approx \tan\theta$.

c) What is the characteristic frequency f of oscillation?

d) What is the angle as a function of time if the stick starts from angle θ_0 at time t=0 and its initial angular speed is zero.





12) (15 pts) Two identical boards of mass M and length L lean against each other as shown in the figure above right. Use symmetry in answering the following questions. (Hint: Symmetry requires that the force between the boards at their top is horizontal.)

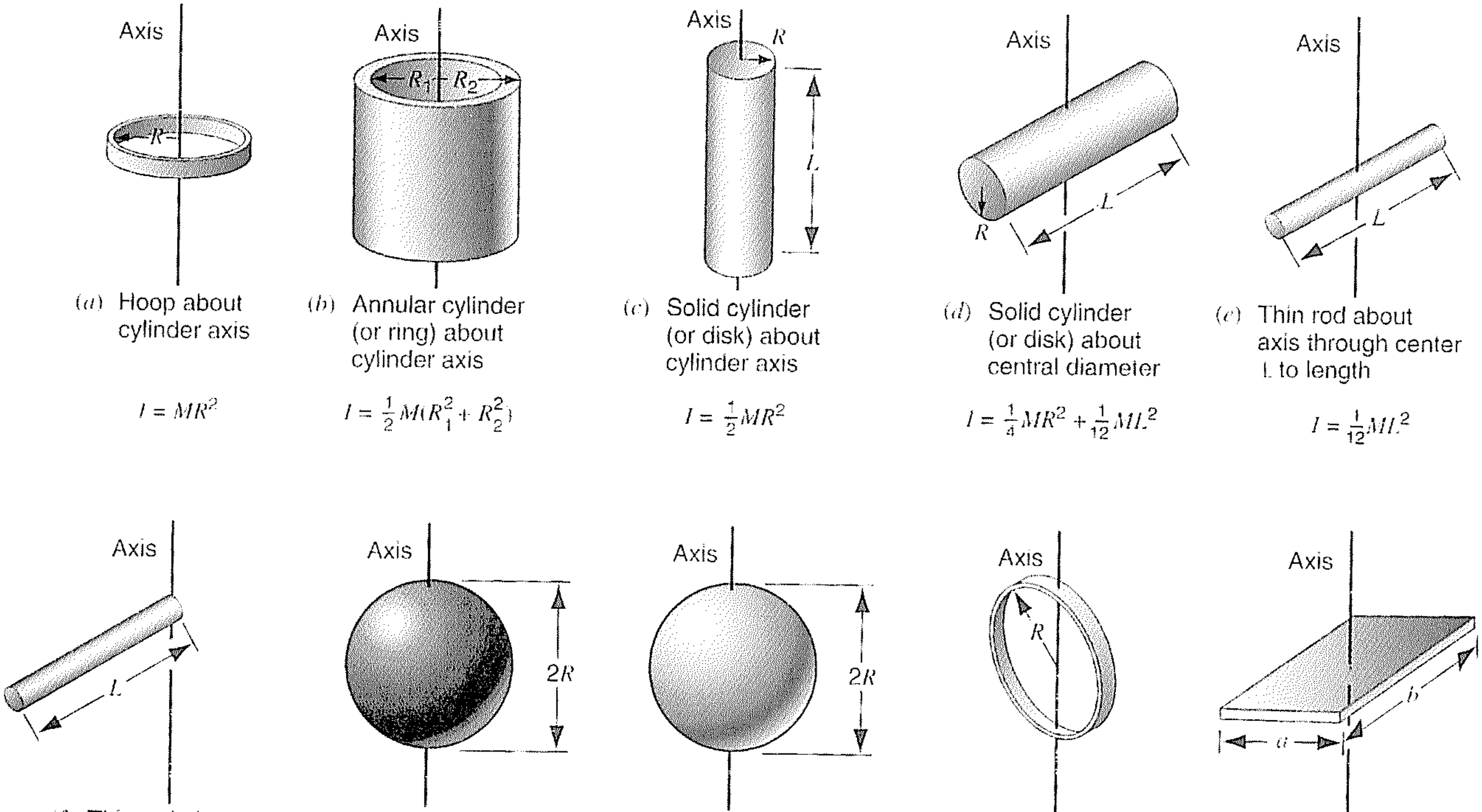
a) In static equilibrium, what are the normal and friction forces exerted by the floor on the left hand board?

b) What is the minimum value of the static coefficient of friction between the floor and the board that maintains static equilibrium?

TABLE 10-1 Review and Comparison of Translational and Rotational Dynamics*

Translational Quantity		*Equation Number*	*Rotational Quantity*	
Velocity	$\vec{v} = d\vec{r}/dt$	2-9	Angular velocity	$\vec{\omega} = d\vec{\phi}/dt$
Acceleration	$\vec{a} = d\vec{v}/dt$	2-16	Angular acceleration	$\vec{\alpha} = d\vec{\omega}/dt$
Mass	m		Rotational inertia	$I = \sum mr^2$
Force	$\vec{F}$		Torque	$\vec{\tau} = \vec{r} \times \vec{F}$
Newton's second law	$\sum \vec{F}_{ext} = m\vec{a}$	4-3	Newton's second law for rotations about a fixed axis	$\sum \tau_{ext\,z} = I\alpha_z$
Equilibrium condition	$\sum \vec{F}_{ext} = 0$	9-22	Equilibrium condition	$\sum \vec{\tau}_{ext} = 0$
Momentum of a particle	$\vec{p} = m\vec{v}$	6-1	Angular momentum of a particle	$\vec{l} = \vec{r} \times \vec{p}$
Momentum of a system of particles	$\vec{P} = M\vec{v}_{cm}$	7-21	Angular momentum of a system of particles	$\vec{L} = I\vec{\omega}$
General form of Newton's second law	$\sum \vec{F}_{ext} = d\vec{P}/dt$	7-23	General form of Newton's second law of rotations	$\sum \vec{\tau}_{ext} = d\vec{L}/dt$
Conservation of momentum in a system of particles for which $\sum \vec{F}_{ext} = 0$	$\vec{P} = \sum \vec{p}_n$ = constant	6-12	Conservation of angular momentum in a system of particles for which $\sum \vec{\tau}_{ext} = 0$	$\vec{L} = \sum \vec{l}_n$ = constant

* Some of these equations apply only under certain special conditions. Be sure you understand the conditions before using these equations. Eq



(a) Hoop about cylinder axis

$I = MR^2$

(b) Annular cylinder (or ring) about cylinder axis

$I = \frac{1}{2}M(R_1^2 + R_2^2)$

(c) Solid cylinder (or disk) about cylinder axis

$I = \frac{1}{2}MR^2$

(d) Solid cylinder (or disk) about central diameter

$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$

(e) Thin rod about axis through center ⊥ to length

$I = \frac{1}{12}ML^2$

(f) Thin rod about axis through one end ⊥ to length

$I = \frac{1}{3}ML^2$

(g) Solid sphere about any diameter

$I = \frac{2}{5}MR^2$

(h) Thin spherical shell about any diameter

$I = \frac{2}{3}MR^2$

(i) Hoop about any diameter

$I = \frac{1}{2}MR^2$

(j) Rectangular plate about ⊥ axis through center

$I = \frac{1}{12}M(a^2 + b^2)$