Classical Physics 171,105
Final Exam Fall 2001

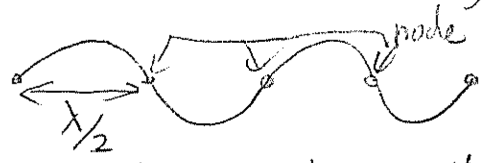
1) One could use an inclined plane, pulleys a lever or a hydraulic system. The force could be decreased but the work would remain constant.

2) W could be converted into potential energy rather than dissipated, This corresponds to an extra height h given by $Mgh = W$ or $h = W/Mg$.

3) Since the spring represents a central force the problem reduces to that of a single reduced mass $\mu = \frac{m \; 3m}{m+3m} = \frac{3}{4}m$.
Thus $\omega = \sqrt{4k/3m}$.

4) As the truck is approaching the frequency is Doppler shifted to a higher pitch. As the truck approaches it moves perpendicular to the listener and there is no Doppler shift. Once past the pitch begins to drop. The frequency shifts up when the truck approaches because the motion of the truck shortens the wavelength.

5) The wave length $\lambda = v/f = 388 m/s / 776 Hz = 0.5 m$
If there are 3 nodes, the length is $4 \cdot \lambda/2 = 1m$.



6) a) Your weight would double. b) 1/4 since $F = G\frac{Mm}{r^2}$

7) a) The change in potential energy goes into the kinetic energy so $\frac{1}{2}m_1 v_1^2 = m_1 g d \Rightarrow v_1 = \sqrt{2gd}$

b) Have conservation of energy and momentum,

$$m_1 v_1 = m_1 v_1' + m_2 v_2' \quad , \quad m_1 v_1^2 = m_1 v_1'^2 + m_2 v_2'^2$$

$$v_2' = m_1 (v_1 - v_1')/m_2 \qquad m_1 v_1^2 = m_1 v_1'^2 + \frac{m_1^2}{m_2}(v_1 - v_1')^2$$

$$m_2 (v_1^2 - v_1'^2) = m_1 (v_1 - v_1')^2$$

$$m_2 (v_1 + v_1') = m_1 (v_1 - v_1')$$

$$(m_2 - m_1) v_1 = -(m_1 + m_2) v_1'$$

$$v_1' = -\frac{m_2 - m_1}{m_1 + m_2} v_1 = -\frac{1}{3} v_1 \quad \text{for } m_2 = 2m_1$$

$$v_2' = \frac{1}{2}(v_1 - -\tfrac{1}{3}v_1) = \frac{2}{3} v_1$$

Check $\quad m_1 v_1 = m_1(-\frac{1}{3}v_1) + 2m_1 \cdot \frac{2}{3} v_1 = m_1 v_1$ ✓

$$m_1 v_1^2 = m_1 \cdot \frac{1}{9} v_1^2 + 2 \cdot \frac{4}{9} m_1 v_1^2 = m_1 v_1^2 \ ✓$$

c) If the collision were perfectly inelastic still have momentum conservation $\quad m_1 v_1 = (m_1 + m_2) v_1'$

$$v_1' = v_2' = \frac{1}{3} v_1$$

d) Difference between initial and final energy

$$\frac{1}{2} m_1 v_1^2 - \frac{1}{2} 3m_1 \cdot \left(\frac{1}{3} v_1\right)^2 = \frac{1}{2} m_1 v_1^2 \left(1 - \frac{1}{3}\right) = \frac{1}{3} m_1 v_1^2$$

This is $\frac{2}{3}$ of initial energy.

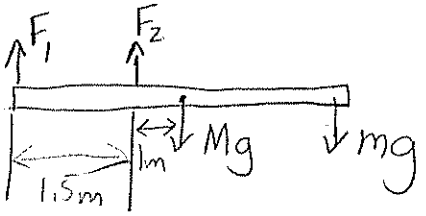
8) The distance traveled is given by the initial velocity v_0 times the time to fall to the ground. This time is independent of v_0. The initial v_0 is obtained by equating the kinetic energy to the potential energy in the spring

$$\frac{1}{2} m v_0^2 = \frac{1}{2} k x^2$$

or
$$v_0 = \sqrt{\frac{k}{m}}\, x$$

Thus the distance moved is proportional to v_0 which is proportional to x. If $x = 1.10\,cm$ gives $2.20\,m - 0.27\,m = 1.93\,m$, Then need x to be $1.10\,cm \cdot \frac{2.20\,m}{1.93\,m} = 1.25\,cm$.

Note that including rotation of the ball just changes m to $m + \frac{I}{R^2}$ and has no effect on answer

9a) 

$m = 60\,kg$

Net force vanishes $\quad F_1 + F_2 - Mg - mg = 0$

Net torque also vanishes. Choose to evaluate about right post because of later parts of problem

$$\tau_{ext} = -Mg \cdot 1m - mg \cdot 3.5m - F_1 \cdot 1.5m = 0$$

$$F_1 = -\left(1\tfrac{2}{3} Mg + \tfrac{7}{3} mg\right) = -\left(\tfrac{20}{3} + 140\right) 9.8\,N = -1400\,N$$

$$F_2 = Mg + mg - F_1 = \tfrac{5}{3} Mg + \tfrac{10}{3} mg = 2100\,N$$

b) The contribution to I from the diver is $m \cdot 3.5m^2$. The contribution from the board is

$$I_b = \frac{ML^2}{12} + M \cdot (1m)^2 \quad \text{using parallel axis theorem}$$

$$I = 60\,kg \cdot 3.5m^2 + 10\,kg\left(\frac{25m^2}{12} + 1m^2\right)$$

$$= (735 + 30.8)\,kg\,m^2$$
$$= 770\,kg\,m^2$$

c) The torque is just the opposite of what the left pedestal used to provide $\quad \tau = F_1 \cdot 1.5m = -2100\,Nm$

d) $\quad \alpha = \tau / I = -2100\,Nm / 770\,kg\,m^2 = -2.7\,s^{-2}$

As a vector torque and α are in to page $\Rightarrow$ negative

10. Bernoulli's equation gives

$$p + \rho g y + \frac{1}{2}\rho v^2 = \text{const.}$$

a) Net flux must be constant since water incompressible and in steady state

$$A \cdot 5\,m/s = \frac{A}{2} \cdot v_{lower} \Rightarrow v_{lower} = 10\,m/s$$

b)

$$p_{upper} + \rho g y_{upper} + \frac{1}{2}\rho \cdot v_{upper}^2 = p_{lower} + \rho g y_{lower} + \frac{1}{2}\rho\,(2v_{upper})^2$$

$$p_{lower} - p_{upper} = \rho g \cdot 10m + \frac{1}{2}\rho v_{upper}^2\,(1-4)$$

$$= \rho g \cdot 10m - \frac{3}{2}\rho\, v_{upper}^2$$

Inserting ρ, g and v_{upper}

$$p_{lower} - p_{upper} = 10^3\,kg\,m^{-3} \cdot 9.8\,\frac{m}{s^2} \cdot 10m - \frac{3}{2} \cdot 10^3\,kg\,m^{-3} \cdot (5\,m/s)^2$$

$$= (9.8 \cdot 10^4 - 3.8 \cdot 10^4)\,Pa$$

$$= 6.0 \cdot 10^4\,Pa$$

c) $\Delta p = 0$

$$10m \cdot g = \frac{3}{2} v_{upper}^2$$

$$v_{upper}^2 = \frac{20}{3}m \cdot 9.8\,\frac{m}{s^2} = 65.3\,\frac{m^2}{s^2}$$

$$v_{upper} = 8\,m/s$$

11 a) The bougancy force must balance the weight Mg of the log. It is just ρ times the displaced volume so

$$Mg = \rho g A L \Rightarrow M = \rho A L$$

b) The bougancy force is decreased by the amount $\rho A x g$.

This means there is a net downward restoring force

$$F = -\rho A g x$$

c) The force is just that of a spring with $k = \rho A g$ and the mass is $\rho A L$. The equations are those of a harmonic oscillator with $w = \sqrt{k/m} = \sqrt{\rho A g / \rho A L} = \sqrt{g/L}$ (note right units)

The period is $\frac{2\pi}{w} = 2\pi\sqrt{L/g}$

12a) The extra distance from the upper speaker to the listener is 1m. The condition for constructive interference is that this length is an integral number of wave lengths

$$n\lambda = 1m$$

or $\lambda = \frac{1m}{n}$ with n = any integer

b) For destructive interference have an extra half wave length $(n+\frac{1}{2})\lambda = 1m$

$$\lambda = \frac{1m}{n+1/2}$$

c) If $v = 340 m/s$ then $f = \frac{v}{\lambda} = \frac{340 m/s}{1m} \cdot (n+1/2)$

$$= 340\ Hz \cdot (n+1/2)$$

The lowest values are 170Hz and $\frac{3}{2} \cdot 340 = 510\ Hz$

d) The line parallel to the x-axis and midway between speakers