Classical Physics 171.105

Final Exam, Dec. 14, 2004

1. The pressures must be equal at the bottom and at the top. Thus

$$\rho_1 g h_1 = \rho_2 g h_2 \qquad \rho_1/\rho_2 = h_2/h_1$$

2. On approach $\lambda_1 = (v_0 - v_s)/f$

$$f_1 = v_0/\lambda_1 = \frac{v_0}{v_0 - v_s} \cdot f$$

$$f_2 = \frac{v_0}{v_0 + v_s} f \qquad \frac{f_1}{f_2} = \frac{v_0 + v_s}{v_0 - v_s}$$

3. a) From Bernoulli's equation

$$\tfrac{1}{2}\rho v^2 + p + U = \text{const}$$

If the hose is wide can neglect its velocity

Then $\quad \tfrac{1}{2}\rho v_0^2 = p \qquad v_0 = \sqrt{2P/\rho}$

b) Time for water to reach peak height is v_0/g. Distance moved is same as distance moved falling from a stop in free fall

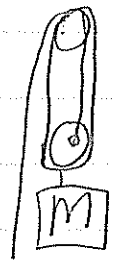
$$H = \tfrac{1}{2} g t^2 = \tfrac{1}{2} v_0^2/g = \frac{1}{2g} \cdot \frac{2p}{\rho}$$

$$p = \rho g H$$

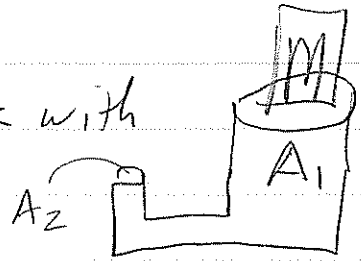
Can also use energy conservation $\tfrac{1}{2}\rho v_0^2 = \rho g H$

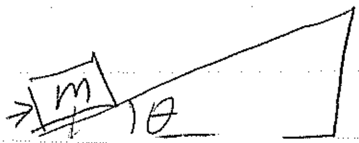
4). Lever with leverarm 3 times as long as side under car.

Pulley with three turns



Hydraulic jack with $A_1 = 3A_2$





Ramp with $\sin\theta = 1/3$

5. a) $p_i = mv \qquad p_f = m\frac{v}{10} + m v_f$

$$v_f = \frac{9}{10} v$$

b) $K_i = \frac{1}{2} m v^2$

$$K_f = \frac{1}{2} m \cdot \left(\frac{v}{10}\right)^2 + \frac{1}{2} m \left(\frac{9}{10} v\right)^2 = \frac{1}{2} m v^2 \left(\frac{82}{100}\right)$$

$$K_f - K_i = -\frac{9}{100} m v^2$$

c) This energy went into internal energy of the system.

6 a) $J = F\Delta t = \Delta p_{cm}$

$v_{cm} = F\frac{\Delta t}{m}$ directed down, along $-\hat{y}$ direction

b)

$$I\alpha = \tau = -F \cdot \frac{L}{2}$$

$$w = \int_0^{\Delta t} dt\, \alpha = -\frac{FL}{2I} \cdot \Delta t$$

rotates clockwise so $w < 0$

7. a) Torque $= -TR$ $\quad \alpha = -\frac{TR}{I}$ $\quad I = \frac{2}{5} m_1 R^2$

$$\alpha = -\frac{5T}{2m_1 R}$$

Now use $a = \alpha R$ $\quad (\alpha < 0 \Rightarrow a < 0)$

and $m_2 a = T - m_2 g$ $\quad -\frac{5m_2}{2m_1} T = T - m_2 g$

$$T = \frac{m_2 g}{1 + 5m_2/2m_1} = \frac{2m_1}{2m_1 + 5m_2} \cdot m_2 g$$

b) Use energy conservation

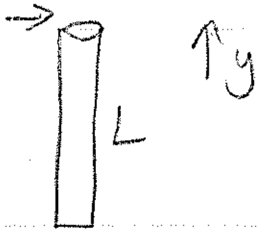
$$\Delta U = m_2 g H = \frac{1}{2} I \omega^2 + \frac{1}{2} m_2 v^2$$

$$= \frac{1}{2} \cdot \frac{2}{5} m_1 R^2 \omega^2 + \frac{1}{2} m_2 \omega^2 R^2$$

$$= \omega^2 R^2 \left[\frac{1}{5} m_1 + \frac{1}{2} m_2 \right]$$

$$\omega^2 = \frac{g H \cdot m_2}{R^2 \left(\frac{1}{5} m_1 + \frac{1}{2} m_2 \right)}$$

8.a



$p = p_0 \sin(ky + \phi)$

Choose $y = 0$ at top, then
$\phi = 0$ gives $p = 0$ at top.

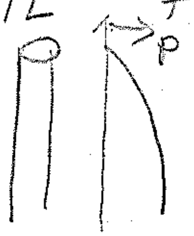
$p = p_0 \sin ky$

$\frac{dp}{dy} = k p_0 \cos ky \quad = 0 \text{ for } y = -L$

$$kL = \frac{\pi}{2} + n\pi$$

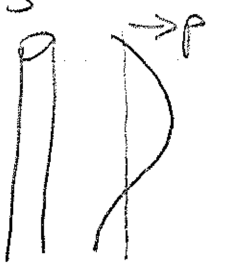
$$\lambda = \frac{2\pi}{k} = \frac{2\pi L}{\frac{\pi}{2} + n\pi} = \frac{4L}{1+2n}$$

a) $\lambda = 4L \qquad f = v/\lambda = \frac{v}{4L}$

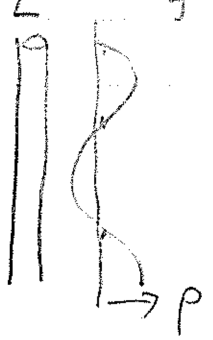


1/4 wave length in tube

b) $\lambda = \frac{4}{3}L \qquad f = \frac{v}{4/3 L} = \frac{3v}{4L}$



c) $\lambda = \frac{4}{5}L \qquad f = \frac{5v}{4L}$



19) a)

Constructive interference requires path length differs by $\lambda = v/f$

Path hitting ceiling has length $2 \cdot \sqrt{\left(\frac{L}{2}\right)^2 + H^2}$

$2\sqrt{\left(\frac{L}{2}\right)^2 + H^2} - L = n\lambda$ for constructive interference

b) $2\sqrt{\left(\frac{L}{2}\right)^2 + H^2} - L = \left(n + \frac{1}{2}\right)\lambda$ for destructive

c) Complete destructive interference will not be observed because the reflected wave will have a smaller amplitude. One reason is that it travels farther. Another is energy lost in reflection

10

a) Since both shells inside r

$$|\vec{F}| = \frac{G(m_1+m_2)\,m}{r^2} \quad \text{and } U = -\frac{Gm(m_1+m_2)}{r}$$

b) For $r < R_2$ no force from outer shell

$$|\vec{F}| = \frac{G m_1 m}{r^2}.$$

No change in potential from outer shell but change in term for inner shell is just like point charge

$$U = -\frac{G m m_1}{r} - \frac{G m m_2}{R_2}$$

c) $|\vec{F}| = 0$ $\quad U = \text{const} = -\frac{Gmm_1}{R_1} - \frac{Gmm_2}{R_2}$

11 a) $I = \frac{1}{3} m \ell^2$ for rod about end

$$I = \frac{1}{3} \cdot \left(\frac{x}{L}\right) m \cdot x^2 + \frac{1}{3}\left(\frac{L-x}{L}\right) m \cdot (L-x)^2$$

$$= \frac{1}{3} m L^2 \left(\left(\frac{x}{L}\right)^3 + \left(\frac{L-x}{L}\right)^3\right)$$

Can also use parallel axis theorm

$$I = I_{cm} + m h^2 = \frac{1}{12} m L^2 + m \left(\frac{L}{2} - x\right)^2$$

$$= \frac{1}{3} m L^2 - m L x + m x^2$$

b) $I \alpha = \tau$ $\tau = -\underbrace{mg}_{\text{force}} \cdot \underbrace{\left(\frac{L}{2} - x\right)}_{\text{distance from Center of mass}} \sin\theta$

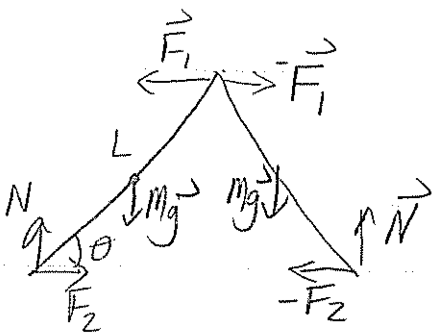
$$I \frac{d^2\theta}{dt^2} = -mg\left(\frac{L}{2} - x\right)\sin\theta \approx -mg\left(\frac{L}{2} - x\right) \cdot \theta$$

c) $$\omega^2 = + \frac{mg}{I}\left(\frac{L}{2} - x\right)$$

$$f = \frac{1}{2\pi}\sqrt{\frac{mg}{I}\left(\frac{L}{2} - x\right)}$$

d) $$\theta = \theta_0 \cos(\omega t)$$

12



a) Consider board on left. In equilibrium the total force vanishes

$$N - Mg = 0 \quad \text{and} \quad F_2 - F_1 = 0$$

Total torque about any axis also vanishes
Choose bottom so N, F_2 don't contribute

$$\tau = + F_1 \underbrace{L \sin\theta} - Mg \cdot \underbrace{\frac{L}{2} \cos\theta} = 0$$

$\perp$ component of distance

$$F_1 = Mg \frac{1}{2} \cot\theta = F_2 \qquad N = Mg$$

b) Must have $\mu_s \geq \frac{F_2}{N} = \frac{1}{2} \cot\theta$

$$\mu_{min} = \frac{1}{2} \cot\theta$$