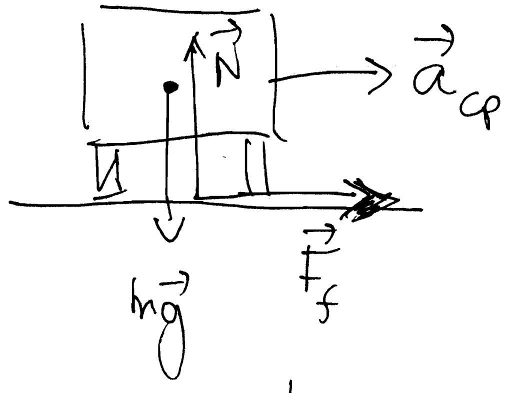
First midterm Fall 2002.
Solutions.

(1.)



$\vec{N} + m\vec{g} + \vec{F}_f = m\vec{a}_{cp}$, whence $N = mg$,

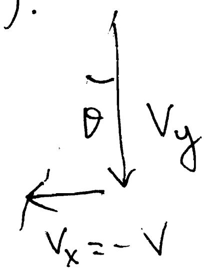
$F_f = m a_{cp} = \frac{mv^2}{R}$

Force of friction cannot exceed μN:

$\frac{mv^2}{R} \le \mu m g \Rightarrow v^2 \le \mu g R$, or

$$\boxed{v \le v_{max} = \sqrt{\mu g R}.}$$

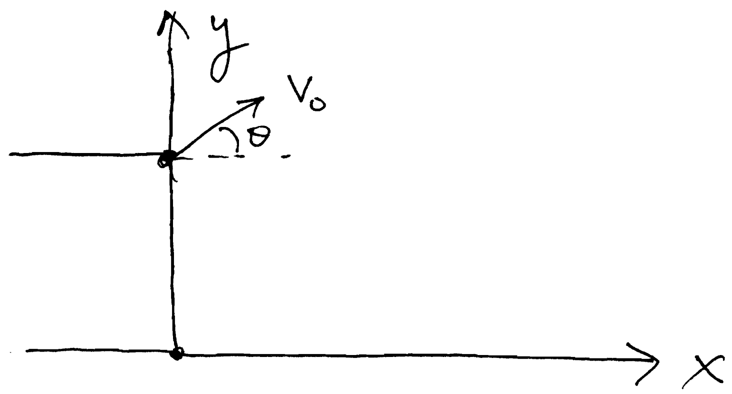
(2.) In driver's frame drops have horizontal velocity $v_x = -v$ and vertical v_y (to be determined).



$\tan\theta = \frac{v}{v_y}$,

hence $\boxed{v_y = v \cot\theta.}$

3.



$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{\vec{a}t^2}{2}, \quad \text{where } \vec{r}_0 = (0, h, 0),$$

$$\vec{v}_0 = (v_0 \cos\theta, v_0 \sin\theta, 0), \quad \vec{a} = (0, -g, 0).$$

Motion along x: $x = v_0 t \cos\theta$.

Along y:

$$y = h - v_0 t \sin\theta - gt^2/2.$$

Hits the ground when $y=0$:

$$-\frac{gt^2}{2} - v_0 t \sin\theta + h = 0.$$

Solve for t: $$t = \frac{v_0 \sin\theta \pm \sqrt{v_0^2 \sin^2\theta + 2gh}}{-g}.$$

One solution is negative, throw it away and take the positive one:

$$\boxed{t = \frac{\sqrt{v_0^2 \sin^2\theta + 2gh} - v_0 \sin\theta}{g} > 0.}$$

The horizontal range $\boxed{R = v_0 t \cos\theta.}$

(4.) (a) $v_1 = v_1' \cos\theta_1 + v_2' \cos\theta_2$,

$0 = - v_1' \sin\theta_1 + v_2' \sin\theta_2$

(b) From 2nd equation, $\boxed{\dfrac{v_1'}{v_2'} = \dfrac{\sin\theta_2}{\sin\theta_1}}$.

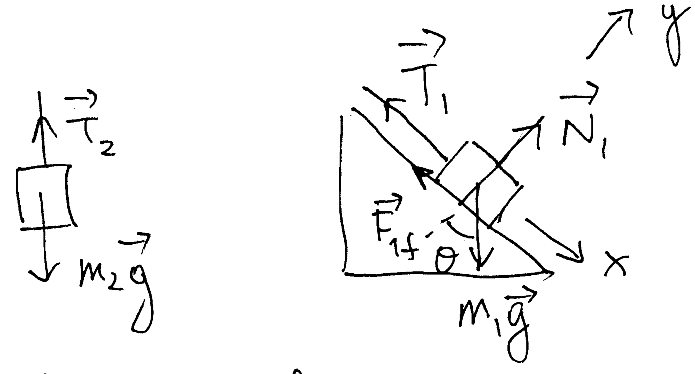
(c) If completely inelastic, $\vec{v}_1' = \vec{v}_2'$.
Coupled with momentum conservation,

$m\vec{v}_1 = m\vec{v}_1' + m\vec{v}_2'$

this yields $\vec{v}_1' = \vec{v}_2' = \vec{v}_1/2$.

Hence $\boxed{v_1' = v_2' = v_1/2, \quad \theta_1 = \theta_2 = 0.}$

(5.) (a)



Equilibrium of 2nd block means $|\vec{T}_2| = m_2 g$.
Tension is same : $|\vec{T}_1| = |\vec{T}_2|$.
Choose axes along slope (x) and normal to it (y):

$\vec{T}_1 + m_1\vec{g} + \vec{N}_1 + \vec{F}_{1f} = 0$.

y axis : $N_1 - m_1 g \cos\theta = 0$, $\quad N_1 = m_1 g \cos\theta$.

x : $-T_1 + m_1 g \sin\theta - F_{1f} = 0$,

hence

$F_{1f} = m_1 g \sin\theta - T_1 = m_1 g \sin\theta - m_2 g$.

Note that, depending on the masses and the angle, F_{1f} could be either positive or negative. The magnitude of F_{1f} must not exceed $\mu_s N_1$, hence

$$\boxed{\underbrace{|m_1 g\sin\theta - m_2 g|}_{|F_{1f}|} \le \underbrace{\mu_s m_2 g}_{\mu_s N_1}}$$

(b) No friction: both may be accelerating. If m_2 is accelerating upward with acceleration a, then m_1 is accelerating along x with the same acceleration a:

$$\begin{cases} T_2 - m_2 g = m_2 a, \\ -T_1 + m_1 g\sin\theta = m_1 a, \\ N_1 - m_1 g\cos\theta = 0 \\ T_1 = T_2 \end{cases}$$

Adding first two and taking into account fourth

$$m_1 g\sin\theta - m_2 g = (m_1 + m_2)a, \text{ or}$$

$$\boxed{a = \frac{m_1 g\sin\theta - m_2 g}{m_1 + m_2}}$$