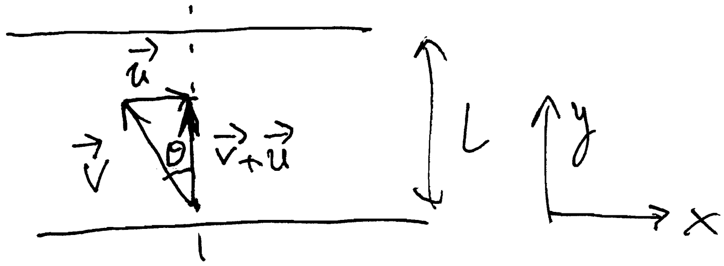
First midterm Fall 2003.

Solutions.

(1.)



$\vec{v}$ must be directed as shown: $\vec{v}+\vec{u}$ points along y.
Determine the angle θ:

$$-v\sin\theta + u = 0,$$

so that ~~v sin θ~~ $\sin\theta = u/v$, $\cos\theta = \sqrt{1-u^2/v^2}$.

Then $\vec{v}+\vec{u} = (0, v\cos\theta) = (0, \sqrt{v^2-u^2})$.

It takes

$$\boxed{t = \frac{L}{|\vec{v}+\vec{u}|} = \frac{L}{\sqrt{v^2-u^2}}}$$

to cross the river.

(2.) In the absence of friction the center of mass remains in place (it wasn't moving to begin with). When they pull themselves together, the students meet at the center of mass.

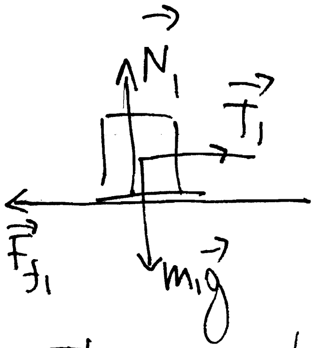
Directing the x axis from Jack to Jill we have

$$x_{Jack} = 0, \quad x_{Jill} = L, \quad x_{CM} = \frac{m_2 L}{m_1 + m_2}.$$

They meet at $\boxed{x = x_{CM} = \frac{m_2 L}{m_1 + m_2}}$.

(b) Jack is at rest:

$$\begin{cases} N_1 - m_1 g = 0, \\ T_1 - F_{f1} = 0. \end{cases}$$

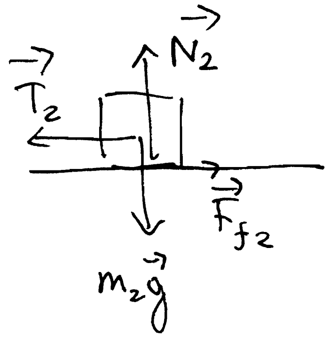


Jack

this force of friction does not exceed max. static value $\mu_s N_1$. Thus we have

$$T_1 = F_{f1} \le \mu_s N_1 = \mu_s m_1 g.$$

At the same time $T_2 = T_1$ must exceed the max friction generated by Jill:



$$T_2 > \mu_s N_2 = \mu_s m_2 g.$$

Thus $\boxed{\mu_s m_2 g < T \le \mu_s m_1 g.}$

3. First rock: $g' t_1^2 / 2 = h$.

Second rock takes $t_2/2$ to climb 1 m and $t_2/2$ to fall back to initial height. Hence

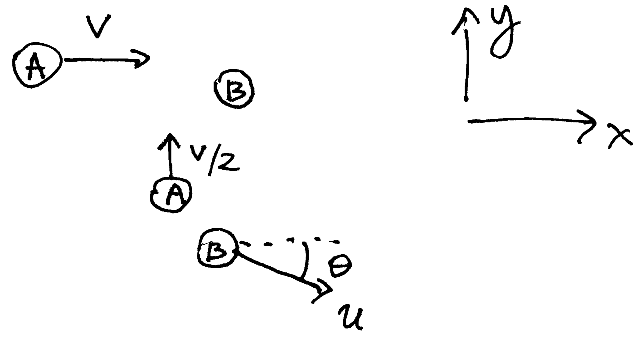
$$\frac{1}{2} g' (t_2/2)^2 = 1\,\text{m}.$$

Combining two equation yields

$$\boxed{\frac{h}{1\text{m}} = \left(2t_1/t_2\right)^2}$$

(4.) Before:

After:



Initial momenta are $\vec{P}_A = (m_A v, 0)$, $\vec{P}_B = (0, 0)$.

Final momenta are $\vec{P}_A' = \left(0, \frac{m_A v}{2}\right)$, $\vec{P}_B' = (m_B u \cos\theta, -m_B u \sin\theta)$.

Conservation of momentum yields

$$\begin{cases} m_A v = m_B u \cos\theta, \\ 0 = m_A v/2 - m_B u \sin\theta \end{cases}$$

$$\boxed{\tan\theta = \frac{m_A v/2}{m_A v} = \frac{1}{2}.}$$

(b) $u = \dfrac{m_A v}{m_B \cos\theta}$.

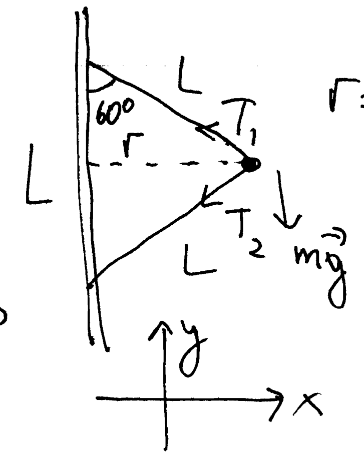
Determine $\dfrac{1}{\cos\theta} = \sqrt{1 + \tan^2\theta} = \dfrac{\sqrt{5}}{2}$. Then

$$\boxed{u = \frac{m_A v}{(m_A/2)} \frac{\sqrt{5}}{2} = v\sqrt{5}.}$$

5.

(a) $\boxed{a_{cp} = \frac{v^2}{r} = \frac{2v^2}{L\sqrt{3}}}$.

Directed horizontally towards the pole.



$r = L\sin 60°$
$= L\sqrt{3}/2$.

(b) $\vec{T}_1 + \vec{T}_2 + m\vec{g} = m\vec{a}_{cp}$

$x: \; -T_1\cos 30° - T_2 \cos 30° = -m a_{cp}$.

$y: \; T_1 \sin 30° - T_2 \sin 30° - mg = 0$.

From that, $T_1 + T_2 = \frac{2}{\sqrt{3}} m a_{cp}$, $T_1 - T_2 = 2mg$,

yielding $\boxed{T_1 = m\left(g + \frac{a_{cp}}{\sqrt{3}}\right), \; T_2 = m\left(g - a_{cp}/\sqrt{3}\right).}$

(c) T_2 vanishes when $g = \frac{a_{cp}}{\sqrt{3}} = \frac{2v^2}{3L}$. Therefore

$\boxed{v = \sqrt{3gL/2}}$