Physics 105 : Solutions to 2^{nd} midterm from 2003.

Nov 17, 05

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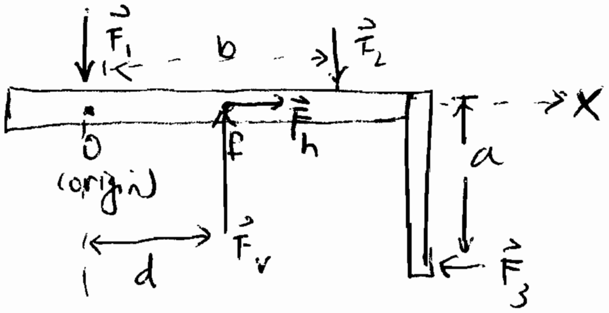
Pg 1

Problem 1.

Equilibrium $\Rightarrow$ $\vec{\tau}_{net} = 0$ (zero net torque)

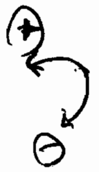
and $\vec{F}_{net} = 0$. (Net force zero)



$\vec{F}_{net} = 0 \Rightarrow F_{net,y} = 0$ and $F_{net,x} = 0$.

(1) $F_{net,y} = F_1 + F_2 - F_v = 0$

$\Rightarrow \boxed{F_v = F_1 + F_2} \leftarrow$ (b)



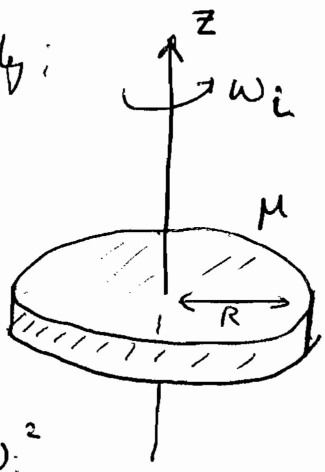
(2) $F_{net,x} = 0 = F_h - F_3 \Rightarrow \boxed{F_h = F_3} \leftarrow$ (a)

(3) $\tau_{net} = 0 = F_v d - F_2 b - F_3 a \Rightarrow d = \dfrac{F_2 b + F_3 a}{F_v}$

$\Rightarrow \boxed{d = \dfrac{F_2 b + a F_3}{F_1 + F_2}} \leftarrow$ (c)

Problem 2

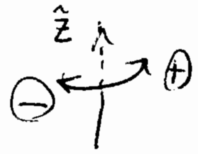
(a) Initially:



$\vec{L}_{initial} = I\vec{\omega}_i$

$= \boxed{\left(\dfrac{MR^2}{2}\right)\omega_i \hat{k}}$

direction determined by right hand rule.

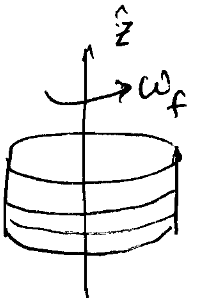


(b) $K_{initial} = \dfrac{1}{2} I\omega_i^2$

Initial rotational KE

$= \boxed{\dfrac{MR^2}{4}\omega_i^2}$

(c) Total $\vec{L}$ conserved.



$$\vec{L}_i = \vec{L}_f$$

$$\frac{MR^2}{2}\omega_i = (\underbrace{3\, I_{\substack{single\\ disk}}}_{= I_{tot.}})\omega_f = \frac{3MR^2}{2}\,\omega_f$$

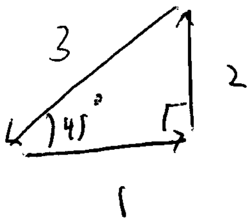
$$\Rightarrow \boxed{\frac{\omega_i}{3} = \omega_f}$$

(d) $K_f = \frac{1}{2} I_{tot}\,\omega_f^2 = \frac{3}{4}MR^2\,\omega_f^2 = \boxed{\frac{MR^2}{12}\omega_i^2}$

(e) $K_f = \frac{1}{3}K_i \Rightarrow K_i - K_f = \Delta K = \frac{2}{3}K_i$ dissipated as heat due to work done by friction between the disks.

Problem 3

(a)



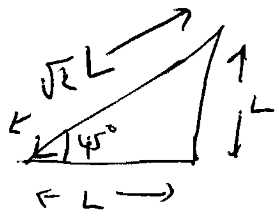
On the 1st leg;

$$W_1 = \vec{F}_1 \cdot \vec{d}_1 = (f\,\hat{i})\cdot(L\,\hat{i}) = \boxed{fL}$$

(b) On 2nd leg: $W_2 = \vec{F}_2\cdot\vec{d}_2 = (f\,\hat{i} - \frac{f}{2}\,\hat{j})\cdot(L\,\hat{j})$

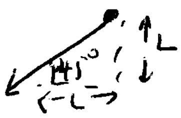
$$= \boxed{-\frac{fL}{2}} \quad (\text{using } \hat{i}\cdot\hat{j} = 0)$$

(c) On 3rd leg:



$$W_3 = \vec{F}_3\cdot\vec{d}_3 = (-f/2,\ f/2)\cdot(-L,-L)$$

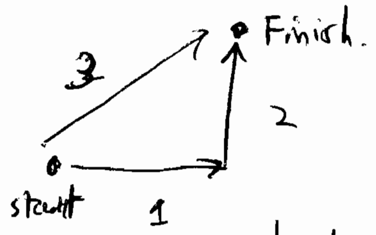
$\vec{d}_3 = (-L, -L)$



$$= \frac{fL}{2} - \frac{fL}{2} = \boxed{0}$$

(d) If this force were conservative, the total work done should depend only on starting position and final position (independent of path ~~the~~ joining the 2 points)

Hence,



$W_{(1+2)}$ and $\tilde{W}_3$ should be same.

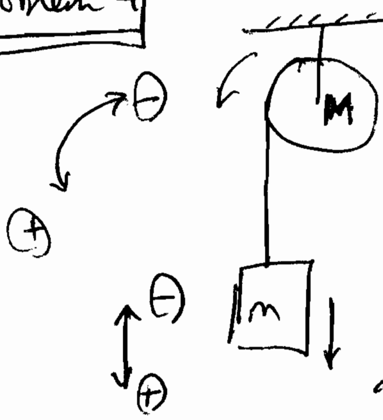
$$W_{(1+2)} = W_1 + W_2 = \frac{fL}{2}$$

but $\tilde{W}_3 = -W_3 = \underbrace{-(0)}_{\text{found in (c)}} = 0$.

Thus $W_{(1+2)} \neq \tilde{W}_3$

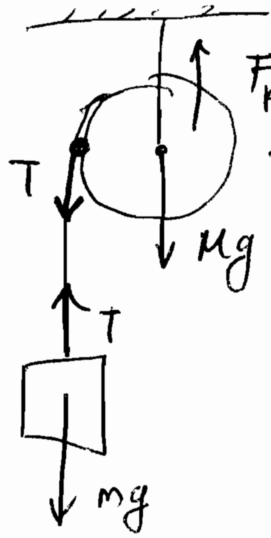
$\therefore$ $\boxed{\text{Force is Not conservative}}$

Problem 4



← Note that my 2 sign conventions here are consistent (important)

(a)



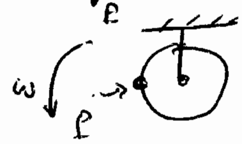
F_{pull} due to ceiling $= \underbrace{Mg + T}_{\text{since com of pulley at rest.}}$

(b)

$$\boxed{R\alpha = a.}$$

accel. of mass m.

← since $V_P = R\omega$ (V_P = speed of point P on the rim of wheel.)



Hence taking time derivative of both sides, we get

$$a = R\alpha.$$

1(c) Using the 2 sign conventions (linear and angular) stated in (a):

① $mg - T = ma$

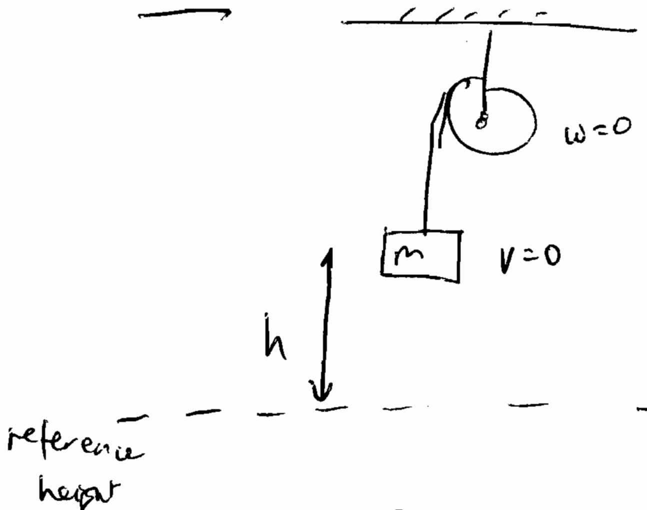
② $I\alpha = RT$ $\Rightarrow$ $I\alpha = m(g-a)R$

$\Rightarrow$ $I\alpha = m(g - R\alpha)R$

$\Rightarrow$ $\alpha(I + mR^2) = mRg$ $\Rightarrow$ $\boxed{\alpha = \dfrac{mRg}{I + mR^2}}$

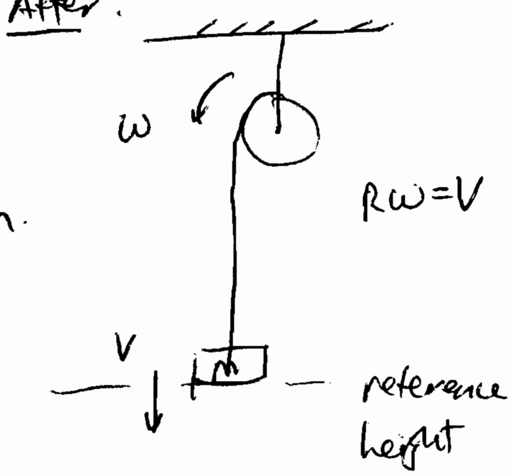
1(d)

Initially:



$E_i = mgh$

After:



$E_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{mv^2}{2} + \frac{1}{2}I\left(\frac{V}{R}\right)^2$

$\therefore$ $E_i = E_f$

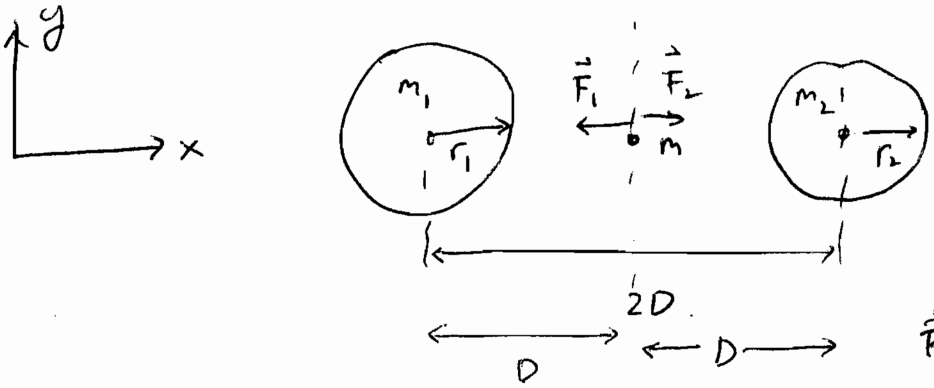
$mgh = \frac{V^2}{2}\left[m + \frac{I}{R^2}\right]$ $\Rightarrow$ $\frac{2mghR^2}{I + mR^2} = V^2$

$\Rightarrow$ $\boxed{V = R\sqrt{\dfrac{2mgh}{I + mR^2}}}$

so $V \propto \sqrt{h}$.

Problem 5

(a)



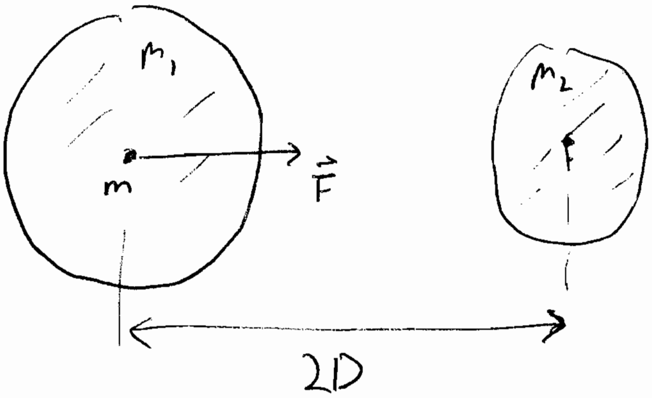
$$\vec{F}_1 = \frac{-Gm_1 m}{D^2}\,\hat{i}$$

$$\vec{F}_2 = \frac{Gm_2 m}{D^2}\,\hat{i}$$

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$

$$= \boxed{\frac{Gm}{D^2}\,\hat{i}\,[m_2 - m_1]}$$

(b)



$$\vec{F} = \frac{Gm_2 m}{(2D)^2}\,\hat{i}$$

$$= \boxed{\frac{Gm_2 m}{4D^2}\,\hat{i}}$$

* (The sphere m_1 doesn't affect m.)

(c) Similarly:

$$\vec{F} = \frac{-Gm_1 m}{(2D)^2}\,\hat{i} = \boxed{\frac{-Gm_1 m}{4D^2}\,\hat{i}}$$