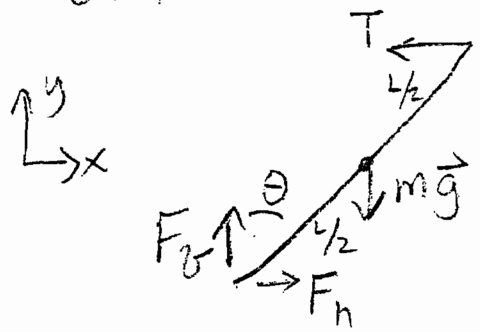
Classical Physics 171.105
Second Midterm Solutions 11/17/04

1. a. In equilibrium the total force and torque on the rod must vanish. Forces are drawn below.



The gravitational force is applied at the center of mass which is also the center of the stick

$$\sum \vec{F}_{ext} = 0 \Rightarrow F_h - T = 0 \quad (x\text{-comp})$$
$$F_v - mg = 0 \quad (y\text{-comp})$$

$$F_h = T, \quad F_v = mg$$

$$\sum \tau_{ext,z} = 0 \Rightarrow T \cdot \underbrace{L\cos\theta}_{\perp dist} - mg \underbrace{\frac{L}{2}\sin\theta}_{\perp dist} = 0$$

use hinge as axis but any will do

$$T = \frac{mg}{2}\tan\theta$$

b. If string cut $\quad I_{zz}\alpha_z = \sum \tau_{ext,z} = -mg\frac{L}{2}\sin\theta$

From last page case (f) $\quad I_{zz} = \frac{1}{3}mL^2$

$$\frac{1}{3}mL^2\alpha_z = -mg\frac{L}{2}\sin\theta$$

$$\alpha_z = -\frac{3}{2}\frac{g\sin\theta}{L} \quad < 0 \text{ means clockwise}$$

c. Here use conservation of energy

$$\Delta U = mg\,\Delta h = -mg \cdot \frac{L}{2}\cos\theta \quad \leftarrow -\frac{L}{2}\cos\theta = \text{height change of c}$$

$$\Delta K = -\Delta U = mg \cdot \frac{L}{2}\cos\theta = \frac{1}{2}I\omega_f^2 \quad \text{since } \omega_i = 0$$

$$mg \cdot \frac{L}{2}\cos\theta = \frac{1}{2}\cdot\frac{1}{3}mL^2\omega_f^2$$

$$\omega_f^2 = \frac{3g\cos\theta}{L}, \quad \omega_f = \sqrt{\frac{3g\cos\theta}{L}}$$

2. a. $\Delta U = -MgH$

$$\Delta K = \frac{1}{2}Mv_f^2 = -\Delta U \Rightarrow \frac{1}{2}Mv_f^2 = MgH$$

$$v_f = \sqrt{2gH}$$

b. The change in mechanical energy is the change in $U + K$.

$$\Delta E_{mech} = \Delta U + \Delta K = -MgH + \frac{1}{2}M\left(0.7\cdot\sqrt{2gH}\right)^2$$

$$= -MgH\left[1 - 0.7^2\right] = -0.51\,MgH$$

This energy went into internal energy in the boulder and the hillside, including motion of other rocks

c. If rolls without slipping must rotate with $|\omega| R = v$. Now $\Delta K = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2$

$$-\Delta U = \frac{1}{2}Mv_f^2 + \frac{1}{2}\cdot\frac{2}{5}MR^2\cdot\left(\frac{v_f}{R}\right)^2$$

$$MgH = Mv_f^2\left(\frac{1}{2} + \frac{1}{5}\right) = 0.7\,Mv_f^2$$

$$v_f = \sqrt{\frac{10}{7}gH}$$

3a. Use conservation of angular momentum.

Initially $\vec{L}_{tot} = 0$

Afterwards $0 = \vec{L}_{tot} = \underbrace{m_2 v R}_{\substack{\vec{r} \times \vec{p} \\ \text{for rock}}} + \underbrace{I \omega}_{\text{merry go round}}$

$$\omega = -\frac{m_2 v R}{I} \qquad I = \frac{1}{2} M_1 R^2$$

$$= -\frac{2 m_2}{M_1} \cdot \frac{v}{R} \qquad \text{Sign is clockwise}$$

$$|\omega| = \frac{2 m_2 v}{m_1 R} \qquad |\vec{L}| = m_2 v R.$$

b. $\vec{\omega}$ and $\vec{L}$ are both into page, by right hand rule

c. linear speed is $|\omega R| = \frac{2 m_2}{m_1} v$

4\. a. At each r only the parts of the earth that are at smaller r contribute to the force. They act like a sphere of the same mass centered on the origin. The mass is

$$M_{<r} = \underbrace{\frac{4\pi}{3} r^3}_{\text{volume}} \cdot \underbrace{\rho_{n}}_{\text{density}}.$$

For $r = R$ have $\frac{4\pi}{3} R^3 \rho = M_e \Rightarrow \rho = \frac{3 M_e}{4\pi R^3}$

$$M_{<r} = \frac{r^3}{R^3} \cdot M_e$$

$|\vec{F}| = G \frac{M_{<r}\, m}{r^2}$ directed toward center $|\vec{F}| = G \frac{m M_e r}{R^3}$

b. $U = -\int_0^r \vec{F} \cdot d\vec{s}$ As go away from origin $\vec{F} \cdot d\vec{s} = -F dr$ since pulled back to center

$$= + \int_0^r G \frac{M_e m r}{R^3} \cdot dr = \frac{G M_e m r^2}{2 R^3}$$

c. $$\underbrace{\frac{1}{2} m v^2}_{\Delta K} = -\left[\underbrace{\frac{G M_e m}{2 R^3} R^2}_{\text{initial } U} - \frac{G M_e m r^2}{2 R^3} \right]$$

$$v^2 = \frac{G M_e}{R^3}(R^2 - r^2) \qquad v = \sqrt{\frac{G M_e}{R^3}(R^2 - r^2)}$$